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Assignment - 1

Numerical Methods and Algebra.

Q1. Find the value of :-

$$\sqrt{7+\sqrt{7+\sqrt{7+\cdots}}}$$

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$$x = \sqrt{7+x}$$

$$x^2 = 7 + x$$

$$x^2 - x - 7 = 0$$

$a = 1 \quad b = -1 \quad c = -7$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{1^2+28}}{2}$$

$$x = \frac{1 \pm \sqrt{29}}{2}$$

$\therefore \quad x = \frac{1+\sqrt{29}}{2}$ OR $x = \frac{1-\sqrt{29}}{2}$

Q2. Find the value of $\frac{x}{y}$ in the following equation.

$$7x^2 - 6y^2 = -11x^2 - 4y^2$$

$$7x^2 + 11x^2 - 6y^2 + 4y^2 = 0$$

$$18x^2 - 2y^2 = 0$$

$$18x^2 = 2y^2$$

$$\frac{x^2}{y^2} = \frac{2}{18}$$

$$\sqrt{\left(\frac{x}{y}\right)^2} = = \sqrt{\frac{2}{18}}$$

$$\therefore \quad \frac{x}{y} = \frac{1}{3}$$

7. What ordered pairs satisfy the system

$$x + 3y = 4$$
$$x^2 + y^2 - 4y = 12$$

let,

$$x = 4 - 3y$$

Now,

$$(4-3y)^2 + y^2 - 4y = 12$$
$$16 - 2\times4\times3y + 9y^2 + y^2 - 4y - 12 = 0$$
$$10y^2 - 28y + 4 = 0$$

$a = 10 \quad b = -28 \quad c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-28) \pm \sqrt{(28)^2 - 4(10)(4)}}{2(10)}$$

$$x = \frac{28 \pm 4\sqrt{39}}{20} = \frac{28}{20} + \frac{4\sqrt{39}}{20}$$

$$\therefore x = \frac{7}{5} \pm \frac{\sqrt{39}}{5} \cong 2.649$$

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Q4. Given:

$$d = 2r^2 - 16r + 34$$

(a) Here,

$$r = 3.5$$

$$\therefore d = 2(3.5)^2 - 16(3.5) + 34$$
$$\therefore d = 7 - 56 + 34$$
$$\therefore d = -15 \text{ m/f}^3$$

(b)

$$d' = 4r - 16$$

$$4r - 16 = 0$$
$$4r = 16$$
$$\therefore r = \frac{16}{4}$$
$$\therefore r = 4$$

Minimum output $= 2 \times 16 - 16 \times 4 + 34$
$= 2 \text{ m/f}^3$

(c) $d = 100 \text{ m/f}^3$

$$d = 2r^2 - 16r + 34$$

$$100 = 2r^2 - 16r + 34$$

$$50 = r^2 - 8r + 17$$

$$r^2 - 8r - 33 = 0$$

$$r^2 - 11r + 3r - 33 = 0$$
$$r(r-11) + 3(r-11) = 0$$

$$\therefore \quad r = -3, 11$$

∴ The value of r cannot be negative.

∴ Speed of engine = 11,000 rph.

Q5.

a) $C_1 = 1500 + 25n$

b) $C_2 = 2100 + 22n$

c) $2100 + 22n < 1500 + 25n$

$600 < 3n$

$\therefore 200 < n$

d) 200

$n \geq 200 \rightarrow C_2$

$n \leq 200 \rightarrow C_1$

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Let,

Q6. $f(x) = x^3 - 7x^2 + 14x - 6$

(a) $[0, 1]$

Iteration (i)	a	b	pi	f(a)	f(b)	f(pi)
1	0	1	0.5	-6	2	-0.625
2	0.5	1	0.75	-0.625	2	0.984375
3	0.5	0.75	0.625	-0.625	0.984375	0.259766
4	0.5	0.625	0.5625	-0.625	0.259766	-0.16187
5	0.5625	~~0.59375~~ 0.625	0.59375	-0.16187	0.259766	0.054047
6	0.5625	0.59375	0.578125	-0.16187	0.054042	-0.05262
7	0.578125	0.59375	0.585938	-0.05262	0.054042	0.001031
8	0.578125	0.585938	0.582031	-0.05262	0.001031	-0.02522
9	0.582031	0.585938	0.583984	-0.02522	0.001031	-0.01232

(b) $[1, 3, 2]$

Iteration (i)	a	b	pi	f(a)	f(b)	f(pi)
1	1	3.2	2.1	2	-0.112	1.791
2	2.1	3.2	2.65	1.791	-0.112	0.552125
3	2.65	3.25	2.925	0.552125	-0.112	0.085828
4	2.925	3.2	3.0625	0.085828	-0.112	-0.05444
5	2.925	3.0625	2.99375	0.085828	-0.05444	0.006328
6	2.99375	3.0625	3.0028125	0.006328	-0.05444	-0.02652
7	2.99375	3.028125	3.010938	0.006328	-0.02652	-0.0107
8	2.99375	3.010938	3.002344	0.006328	-0.0107	-0.00233
9	2.99375	3.002344	2.998047	0.006328	0.00233	0.001961

c. [3, 2, 4]

Iteration (i)	a	b	pi	f(a)	f(b)	f(pi)
1	3.2	4	3.6	-0.112	2	0.336
2	3.2	3.6	3.4	-0.112	0.336	-0.016
3	3.4	3.6	3.5	-0.016	0.336	0.125
4	3.4	3.5	3.45	-0.016	0.125	0.046125
5	3.4	3.45	3.425	-0.016	0.046125	0.013016
6	3.4	3.425	3.4105	-0.016	0.013016	-0.002
7	3.4125	3.425	3.4188	-0.002	0.013016	0.005382
8	3.4125	3.41875	3.4156	-0.002	0.005382	0.00166
9	3.4105	3.415625	3.4141	-0.002	0.00166	-0.00018

Q7. $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

Intervals $[-1, 0]$ $[0, 1]$

Midpoint of the interval $[-1, 0]$ is $p_0 = \frac{-1}{2}$

Midpoint of the interval $[0, 1]$ is $p_0 = \frac{1}{2}$

With the newton method we have found the estimations $p_4 = -0.040639288$ for the interval $[-1, 0]$ and $p_6 = -0.040659288$ for the interval $[0, 1]$ after 4 and 6 iterations respectively.

Iterations	$-1 \leq x \leq 0$	$0 \leq x \leq 1$
0	-0.0500000	0.500000
1	-0.15045249	-0.20508982
2	-0.041816814	-0.32379111
3	-0.040659343	-0.064603131
4	-0.040659288	-0.040686151
5		-0.040659288
6		-0.040659288

In the following table,

For $p_0 = -0.5$

we have

$p_5 = -0.04$

Ans for $p_0 = 0.5$ we get,

$p_{21} = 0.96$.

Q8.

Let no. scored in fifth examination is x

$$\text{average} = \frac{70+86+81+83+x}{5}$$

$$\text{Average} = \frac{320+x}{5}$$

According to the given condition

① $$\frac{320+x}{5} \geq 80$$

$$320+x \geq 400$$

$$x \geq 400-320$$

$$x \geq 80$$

② $$\frac{320+x}{5} < 90$$

$$320+x < 450$$

$$x < 450-320$$

$$\therefore x < 130$$

Then

$$80 \leq x < 130$$

Ans. $80 \leq x \leq 130$

Q9.

x	y
4%	8.54
20%	-11.81

Required $y = 0$

$$x = x_1 + (y - y_1)\frac{(x_2 - x_1)}{(y_2 - y_1)}$$

$$= 0.04 + (18.54) \times \frac{0.16}{+20.35}$$

$$\therefore x = 10.71\%$$

Q10.

$$R(x) = 32x - 0.21x^2$$
$$C(x) = 195 + 12x$$

Profit = Revenue $[R(x)]$ − Cost $[C(x)]$

$$= 32x - 0.21x^2 - [195 + 12x]$$
$$= 32x - 0.21x^2 = 195 - 12x$$
$$= 20x - 0.21x^2 - 195$$

Profit $(\pi) = 20x - 0.21x^2 - 195$

Differentiating w.r.t x we get,

$$\frac{d(\pi)}{dx} = 20\frac{d(x)}{dx} - 0.21\frac{d(x^2)}{dx} - \frac{d(195)}{dx}$$

$$\frac{d\pi}{dx} = 20 - 0.21(2x) - 0$$

$$\frac{d\pi}{dx} = 20 - 0.42x$$

Considering,

For Maxima or Minima,

$$\frac{d\pi}{dx} = 0$$

$$20 - 0.42x = 0$$
$$20 - 0.42x =$$
$$20 = 0.42x$$
$$\therefore \frac{20}{0.42} = x$$
$$\therefore x = 47.62 \text{ units.}$$