Pricing & Reserving for life insurance Products - 2

ASSIGNMENT

1

i) To prove: $(Ia)_{\overline{n}|} = (\ddot{a}_{\overline{n}|} - nv^n)/n$

LHS: $(Ia)_{\overline{n}|} = v + 2v^2 + 3v^3 + \dots + nv^n$ — (A)

multiplying both sides by $(1+i)$

$\Rightarrow (1+i)(In)_{\overline{n}|} + 1 + 2v + 3v^2 + \dots + nv^{n-1}$ — (B)

(B) - (A)

$i\,(Ia)_{\overline{n}|} = 1 + v + v^2 + v^3 + \dots + v^{n-1} - nv^n$

$= \ddot{a}_{\overline{n}|} - nv^n$

$\therefore (Ia)_{\overline{n}|} = (\ddot{a}_{\overline{n}|} - nv^n)/i$

ii) X = PV of contributions

$= 2{,}00{,}000\, \ddot{a}_{\overline{10}|} + X(Ia)_{\overline{9}|}$

$i = 10\%$, $v = 0.9091$, $d = \frac{1}{1+i} = 9.091\%$

$\ddot{a}_{\overline{10}|} = \frac{(1-v^{10})}{d} = 6.7590$

$(Ia)_{\overline{9}|} = (\ddot{a}_{\overline{9}|} - 9v^9)/i$ or $(1-v^9)/d$

$= 6.3349$

$\Rightarrow (Ia)_{\overline{9}|} = (6.3349 - 9(0.4241))/0.1$

$= 25.1804$

PV $= 200000(6.759026) + X(25.18046)$

$= 13{,}51{,}805.20 + X(25.18046)$

PV of higher ed. expense

$= 30{,}00{,}000\,(1.06)^{10} \cdot v^{10}$ @ 10%

$= 20{,}71{,}350$

substituting & equating values:

$20{,}71{,}350 = 13{,}51{,}805.20 + X(25.18046)$

$\Rightarrow X = 28{,}575$ approx

② : $100000 \int_0^{20} v^t \, {}_tp_{40}^{h} (\mu_{40+t} + \sigma_{40+t}) dt$

$= 100000 \int_0^{20} e^{-\ln(1.05)t} \, {}_tp_{40}^{hh} \, (0.2)06 \, dt$

$\Rightarrow {}_tp_{40}^{hh} = {}_tp_{40}^{hh} = \exp(-\int_{40}^{40+t} (\mu_s + \sigma_s) ds$

$= e^{-0.006t}$

$\therefore$ value $= 100000 \times 0.06 \int_0^{20} e^{-\ln(1.05)t} e^{-0.006t} dt$

$= 600 \int_0^{20} e^{-\ln(1.05)t} e^{-0.06t} dt$

$= 600 \int_0^{20} e^{--5479 t} dt$

$= 600 \left(-e^{--5479.t} / 0.5479\right)_0^{20}$

$= 600 (-6.10097 + 18.25151)$

$= 7290.3$

③ Assuming claim is paid only at the end

Premium $= P\ddot{a}^{(12)}_{40:\overline{20}|}$

$= P\left[\left[\ddot{a}_{40:\overline{20}|} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{40}}\right)\right]\right]$

$= P\left[\left[13.927 - \frac{11}{24}\left(1 - \frac{882.85}{2052.96}\right)\right.\right.$

$= 13.6652 6724 P$

Claim cost :

$7500000 \, A^1_{40:\overline{20}|} + 50{,}00{,}000 A^1_{[40]:\overline{20}|}$

$= 4{,}27{,}714.965824$

Claim investigation expense : 0.03×436185.3914

$= 13{,}085.56174$

Mgmt expense $= 0.1P \times \ddot{a}^{(12)}_{40:\overline{20}|} \times 1.04^{0.5}$

$= 0.1P \times 13.6657 6724 \times 1.04^{0.5}$

$= 1.39364 P$

Commission expense : $0.3P \ddot{a}^{(12)}_{40:\overline{1}|} + 0.05 \ddot{a}^{(12)}_{40:\overline{20}|}$

$= 0.3P \times 0.942596 + 0.683288 \cdot$

$= 0.9660 6 7P$

UW expense $= 300$

Monthly premium:

$13.66576724P = 427714.96 + 12931 + 0966 + 1.3906r + 500$

$\Rightarrow P = 39009.7$

ii) Calculation of reserves

→ policies w/ accidental claim made:

$= 7{,}500{,}000\, A'_{45:\overline{15}|} - 39{,}009.7\, \ddot{a}^{ac(12)}_{45:\overline{15}|}$

$= -1{,}65{,}363.4377$

→ policies w/ only illness claim

$= 50{,}00{,}000\, A'_{45:15} - 39{,}009\, \ddot{a}_{45:15}$

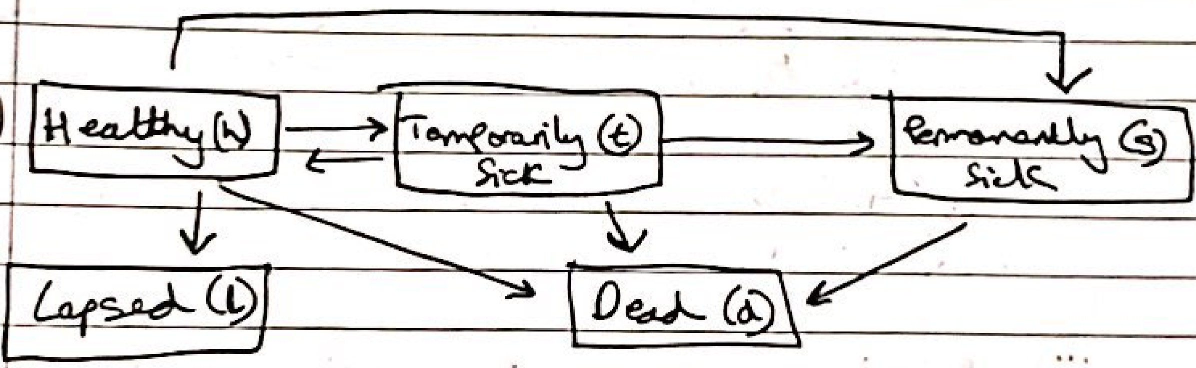
$= -2{,}55{,}159{,}1702$

→ policies in which no claim has been made

$= 1.25{,}00{,}000 \times (0.835918) - 434750.6352$

$= 5{,}621.1926$

(4) i)



ii) EPV (premiums) = EPV (benefits)

EPV of temporary sickness benefit

$10{,}000 \int_{r=0}^{15} v^r\, {}_rp_{50}^{ht}\, dr$

EPV of permanent sickness benefit

$20{,}000 \int_{r=0}^{15} v^r\, {}_rp_{50}^{hs}\, dr$

By setting EPV of benefits = EPV of premium we obtain

$P = \left\{10{,}000 \int_{r=0}^{15} v^r\, {}_rp_{50}^{ht}\, dr + 20{,}000 \int_{r=0}^{15} v^r\, {}_rp_{50}^{hs}\, dr + 100{,}000 \int_{r=0}^{15} v^r \left({}_rp_{50}^{hh}\, \mu_{50+r}^{hd} + {}_rp_{50}^{ht}\, \mu_{50+r}^{td} + {}_rp_{50}^{hs}\, \mu_{50+r}^{sd}\right) dr\right\} \Big/ \left\{\int_{r=0}^{15} v^r\, {}_rp_{50}^{hh}\, dr\right\}$

(5) (i) Gross premium for the compound reversionary policy

EPV premiums $= P\ddot{a}_{[40]:\overline{25}|} = 15.887P$

EPV expenses $= 1200 + 500\ddot{a}_{[40]} + 2\%(P)(\ddot{a}_{[40]:\overline{25}|} - 1)$

$= 1817.487 + 0.29774P$

EPV benefits $= 2{,}00{,}000(v^{15} \ldots + v^{15}(1.04) \ldots + v^{20}(1.04)^2 \ldots$

$\ldots]$

$= \dfrac{200000}{1.04^5} = 1{,}96{,}116.14.$

using equivalence principle,

$15.887 = 196{,}116.14 + 1817.487 + 0.29774P$

$P = 12{,}664$

(ii) Revised bonus rates for a simple reversionary policy:

since the company plans to charge the same premium rate, simple reversionary bonus rate can be calculated by equating the EPV of benefits of the 2 policy

$\Rightarrow (1.04)^{0.5}[2{,}00{,}000(x\%)(IA_{[40]}) + 20\,0000(1-x\%)A_{[40]}] = 196\,116.14$

$\Rightarrow x\% = 9\%$ (simple reversionary rate)

(iii) Premium as per net premium basis

$P\ddot{a}_{40:25} = 2{,}00{,}000\bar{A}_{40}$

$P = 2{,}960.54$

net premium reserve at time 10

${}_{10}V = 3{,}00{,}000\bar{A}_{50} - P\ddot{a}_{50:15}$

$= 300\,000(1.04)^{0.5}(0.32907) - 2960.54(\ldots)$

$= 67361.10$

(6) force of decrement due to death $= \mu_x$

force " " " " cancer diagnosis $= \sigma_x$

Value of benefits on death & due to cancer $=$

$V = 100000\int_0^{20} v^t\,{}_tp_{45}^{\alpha\alpha}(\mu_{45+t} + \sigma_{45+t})\,dt$

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$= \exp(-\int_{45}^{45+t}(0.008\,dx))$
$= e^{-0.008t}$

$\therefore V = 800 \int_0^{20} e^{-\ln(1.05)t} \cdot e^{-0.008t}\,dt$

$= 800 \int_0^{20} e^{-0.05679t}\,dt$
$= 800 \left[-e^{-0.05679t}/0.05679\right]_0^{20}$
$= 800\,(-5.65581 + 17.60873) = 9562.8$

Value of Return of Premium
$= P\left(e^{-\int_{45}^{65}(0.006+0.002)dx}\right)(v^{20})$
$= P\,(e^{-0.16})\,(0.37689)$
$= P\,(e^{-0.16})\,(0.37689)$
$\Rightarrow$ Premium $= P = 9562.8 + 0.321164(P) + 0.02(P) + 0.02P + 0.1P$
$= 17,747.15$

(7) PV of net CF $=$ PV of premium $-$ PV of benefits $-$ PV of expenses $-$
PV of commission

maximum initial commission $= I$
PV of premium $= 30,000(\ddot{a}_{[45]:\overline{20}|})$
$= 30,000\,(11.888)$
$= 356,640$

PV of benefits $= 1,000,000\,(A_{[45]:\overline{20}|})$
$= 1,000,000\,(0.32711)$
$= 327,110$

PV of expenses $= 5,000 + 2.5\%(30,000)(\ddot{a}_{[45]:\overline{20}|} - 1) +$
$500(p_{[45]})(1.06^{-1})(\ddot{a}_{[46]:\overline{19}|})$ @ 4%
$= 19,438$

PV of renewal commission: $= 2\%(30,000)(\ddot{a}_{[45]:\overline{20}|} - 1)$
$= 6,533$

PV of net CF $= 356,640 - 327,110 - 19438 - 6533 - I$
$= 3559 - I$

margin $= \dfrac{3559 - I}{30,000} = 10\% \Rightarrow I = 559$

8) i) The assumption of independence of decrements is used to derive single decrements table from a multiple decrement table.

$\Rightarrow$ $(a\mu)^j_x = \mu^j_x$ for all j and x

when looking at the transition intensities (forces of decrement) one is looking at infinitely small time intervals in which there is only time for one decrement.

ii) In the case of term assurance, it is likely that people who lapse their policies have a lower than average mortality.

iii) a) $1000\int_0^5 e^{-\delta t}\left({}_tp^{PI}_{45}\,\mu_{45+t} + {}_tp^{PP}_{45}\,\mu_{45+t}\right)dt$

b) $250\int_0^5 e^{-\delta t}\,{}_tp^{IP}_{45}\,\nu_{45+t}\,dt$

c) $150\int_0^5 e^{-2t}\,{}_tp^{II}_{45}\,dt$

d) $250\int_0^5 e^{-\delta t}\,{}_tp^{PP}_{45}\,\vartheta_{45+t}\,dt$

iv) Assuming decrements are independent $\therefore$ the given independent forces of decrement can be assumed to apply when both decrements occur together in the same population

$(aq)^c_{45} = \frac{0.05}{0.18}(1-e^{-0.18}) = 0.045758.$

$(aq)^s_{45} = 0.13/0.18\,(1-e^{-0.18}) = 0.118972$

$(ap)_{45} = 1-0.045758 - 0.118972 = 0.835270$

$(aq)^c_{46} = 0.06/0.16\,(1-e^{-0.16}) = 0.055446$

$(aq)^s_{46} = 0.1/0.16\,(1-e^{-0.16}) = 0.092410$

$(ap)_{46} = 1-0.055446 - 0.092410 = 0.852144$

$\Rightarrow$ probability of being in state inforce at the beginning of age 47 $= 0.835270\,(0.852144)$

$= 0.711770.$

(9) i) With profit products are sold by life insurance companies either as endowment assurance or whole life plans. At the end of the year, further amounts in the form of annual bonus may be credited as a percentage of the basic S.A. It makes sense to distribute bonuses through with profit products.

ii) → excess interest
→ savings from mortality
→ lapses & surrenders
→ bonus loading

iii) Terminal bonus is paid once in the life of the policy, when the policy has been in force for long times. It provides balance. It helps customers get treated fairly & earn some more.

(10)

(i) monthly premium = P

EPV of premiums = $12P\, \ddot{a}^{(12)}_{50:\overline{10}|}$ @ 6%

Now $\ddot{a}^{(12)}_{50:\overline{10}|} = \ddot{a}_{50:\overline{10}|} - (11/24)(1 - {}_{10}p_{50}(v^{10}))$

Now $\ddot{a}_{50:\overline{10}|} = 7.694$

$\Rightarrow \ddot{a}^{(12)}_{50:\overline{10}|} = 7.694 - (11/24)(1 - 1.06^{-10}(9287.2164/9712.0728))$

$= 7.694 - (11/24)(1 - 0.533968)$

$= 7.4804$

$\Rightarrow$ EPV of premiums $= 12P(7.4804)$

$= 89.7648\, P$

@ 6%

$\Rightarrow$ EPV of benefits $= 10{,}00{,}000(q_{50}\cdot v + 1.019231\, v^2({}_{1|}q_{50}) + \dots +$

$1.019231^{14} \cdot v^{15} \cdot {}_{14|}q_{50} + 1.019231^{6}\, v^{15}\, {}_{15}p_{50}$

EPV of benefits $= 10{,}00{,}000\,(1/1.019231)\,(A_{50:\overline{15}|}$ @ 4%$)$ +

$10{,}00{,}000\, v^{15}$ (at 6%) ${}_{15}p_{50}\,(1.019231^{15} - 1.019231^{14})$

Now $A_{50:\overline{15}|} = 0.56719$ we get

EPV of benefits $= 5{,}66{,}004.1$

EPV of Expense & commission:

$(40+35)/100\,(12P) + (2+1.5)/100\,({}_{1}p_{50})(1.06^{-1})(12P)(\ddot{a}^{(12)}_{51:\overline{9}|}) + 2{,}500 + 400\,{}_{1}p_{50}(1.06^{-1})(\ddot{a}_{51:\overline{19}|})$ @6%.

$= 9.3333 + 6115.06$

EPV of Premiums = EPV of benefits + EPV of Expense & commission

$\Rightarrow 89.7648P = 566004.1 + 9.3333P + 6115.06$

$\Rightarrow 80.4315P = 572119.16$

$\Rightarrow P = 7113.12$

(ii) Sum assured at the end of 10^{th} year + Accrued bonus

$x = 1000\,000\,(1.02^{10})$

$X = 12{,}18{,}994$

PV of future benefits at the end of the 10^{th} year =

$= X/1.05 * q_{60} + X/1.05^2(1.04)(1-q_{60}) + X/1.05^3 * (1.04)^2 \cdot {}_{2}q_{60}$

$= 0.952948X$

$= 1160774$

PV of expenses $= 400\ \ddot{a}_{60:\overline{5}|}$ @ 5%.

$= 400((1+1.05^{-1})(l_{61}/l_{60}) + 1.05^{-2}(l_{62}/l_{60}) + 1.05^{-3}(l_{63}/l_{60}) + 1.05^{-4}(l_{64}/l_{60}))$

$= 1787.39$

Reserves required = PV of benefits + PV of expenses

$= 11{,}62{,}561$

(ii) Net Premiums are required to calculate net premium reserves. Net Premium = P

$P(\ddot{a}_{30:\overline{20}|}) = 400000\ A_{30:\overline{20}|}$

$P = 4\,00\,000(0.315817)/12.08705$

$= 10451.42$

Total bonus added till date $= 7(4\%)(400000)$

$= 112\,000$

+16 000 added for each year gone by

_/_/_

Guaranteed SA now is $= 400000 + 112000$

$= 512000$

Therefore the prospective net premium reserves at time 7 is: $512\,000 (A_{37:\overline{13}|}) - 10451.42 (\ddot{a}_{37:\overline{13}|})$

$= 512000 (0.471638) - 10451.42 (9.333835)$

$= 143926.89$

—— x —— x ——