Numerical method and Algebra

1) $\sqrt{7\sqrt{7\sqrt{7}\ldots}}$

$= 7^{1/2} \cdot 7^{1/4} \cdot 7^{1/8} \cdot 7^{1/16} \ldots\ldots$

$= 7^{1/2 + 1/4 + 1/8 + 1/16 \ldots}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \ldots$ is in form of infinite GP

Sum of infinite GP - $\frac{a}{1-r}$

$= \frac{1/2}{1 - 1/2}$

$= 1$

$\therefore 7^{1/2 + 1/4 + 1/8 \ldots}$

$= 7^1$

$= 7$

2) $7x^2 - 6y^2 = -11x^2 - 4y^2$

$18x^2 = 2y^2$

$\frac{x^2}{y^2} = \frac{2}{18}$

$\frac{x}{y} = \sqrt{\frac{2}{18}}$

3 $x+y=4 \quad \rightarrow (1)$

$x^2+y^2-4y=12 \quad \rightarrow (2)$

$x = 4-y$ — from (1) in 2

$(4-y)^2+y^2-4y=12$

$16+y^2-8y+y^2-4y=12$

$16+2y^2-12y=12$

$y^2-6y+8=6$

$y^2-6y+2=0$

$y = \frac{6 \pm \sqrt{36-8}}{2}$

$= \frac{6 \pm \sqrt{28}}{2}$

$= 3 \pm \sqrt{7}$

$x_1 = 4-y_1$

$= 4-3+\sqrt{7}$

$= 1+\sqrt{7}$

$x_2 = 4-y_2$

$= 4-3-\sqrt{7}$

$= 1-\sqrt{7}$

the ordered pairs are

$(1+\sqrt{7}, 3-\sqrt{7})$ and $(1-\sqrt{7}, 3+\sqrt{7})$

4) $d = 2r^2 - 16r + 34$ => density

i) $r = 3.5$
$d = 2.5$ mill particles / ft^3

ii) $d \leftarrow 2r^2 - 16r + 34 = 0$
differentiation
$4r - 16 = 0 \rightarrow (1)$
$r = 4$ (400 RPM)

differentiation of (1)
4

minima
$d = 2(4)^2 - 16(4) + 34$
$d = 2$

∴ the no. of revolution for minimum
smoke = 4

iii) $d = 100$

$100 = 2r^2 - 16r + 34$
$50 = r^2 - 8r + 17$
$0 = r^2 - 8r + (-33)$
$r = (r - 11)(r + 3)$
$\boxed{r = 11}$

the speed must be 11 revolution per minute

5) $C_1 = 1500 + 25(km)$
$C_2 = 2100 + 22(km)$

i) Cost to rent car from Co. 1
$C_1 = 1500 + 25n$

ii) Cost to rent car from Co. 2
$C_2 = 2100 + 22n$

iii) $C_2 < C_1$
$1500 + 25n > 2100 + 22n$

iv) $3n > 600$
$n > 200$

it is less expensive to take C_2 when the number of km is more than 200.

6) $x^3 - 7x^2 + 14x - 6 = 0$

a) $a = 0$, $b = 1$

a	b	$\frac{a+b}{2}$	f(a)	f(b)	$f(\frac{a+b}{2})$
0	1	0.5	-6	2	-0.625
0.5	1	0.75	-0.625	2	0.9844
0.5	0.75	0.625	-0.625	0.9844	0.2598

$x = 0.625$

b) $a=1 \quad b=3.2 \quad f(x) = x^3 - 7x^2 + 14x - 6$

a	b	$a+b/2$	f(a)	f(b)	$f(a+b/2)$
1	3.2	2.1	2	1.791	-0.112
2.1	3.2	2.65	1.791	-0.112	0.5521
2.65	3.2	2.925	0.5521	-0.112	0.08582

$x = 2.925$

c) $[3.2, 4]$

a	b	$a+b/2$	f(a)	f(b)	$f(a+b/2)$
3.2	4	3.6	-0.112	2	0.336
3.2	3.6	3.4	-0.112	0.336	-0.016
3.4	3.6	3.5	-0.016	0.336	0.125
3.4	3.5	3.45	-0.016	0.125	0.046125
3.4	3.45	3.425	-0.016	0.0461	0.01380

$x = 3.425$

7 $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$f'(x) = 920x^3 + 54x^2 + 18x - 221$

a)

i	x_i	$f(x)$	$f'(x)$	$x_{i+1} = x_i - \frac{f(x)}{f'(x)}$
1	-1	156.1398	-1105	-0.608141
2	-0.6081	43.996	-418.898	-0.2354
3	-0.2354	1.537	-234.246	-0.04759
4	-0.04759	0.0004	-221.704	-0.04066
5	-0.04066	~0	-221.70443	-0.04066
	-0.04066	~0	-221.70443	-0.04066

$x = -0.04066$

b)

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
1	0.9	-36.585	509.62	0.97179
2	0.97178	6.37668	691.80	0.96254
3	0.9625	0.11523	666.87	0.9624
	0.9624	0.3267	666.411	0.9624
	0.9624			

$x = 0.9624$

8 a) Average $= \frac{70+86+81+83+x}{5}$

$= \frac{320+x}{5}$

b) ~~Average~~ Average $= B$

$\therefore \quad 80 < \frac{320+x}{5} < 90$

c $80 < \frac{320+x}{5} < 90$

$400 < 320 + x < 450$

$880 < x < 130$

$\boxed{80 < x < 130}$

9 $x_1 = 0.04 \quad x_2 = 0.2 \quad x = 0$

$f(x_1) = 8.54 \quad f(x_2) = -11.81 \quad f(x) = ?$

$$\frac{x - x_1}{x_2 - x_1} = \frac{f(x) - f(x_1)}{f(x_2) - f(x_1)}$$

$$\frac{-0.04}{0.16} = \frac{f(x) - 8.54}{-20.35}$$

$$f(x) = 13.6275$$

10 $R(x) = 32x - 0.21x^2$

$C(x) = 195 + 12x$

for profit $R(x) > C(x)$

$\therefore 32x - 0.21x^2 > 195 + 12x$

$20x - 195 - 0.21x^2 > 0$

$0.21x^2 - 20x + 195 < 0$

$0.21x^2 - 20x + 195 = 0$

$$x = \frac{20 \pm \sqrt{212.8}}{0.42}$$

$x = 82.35, 12.88$

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$(x - 82.38)(x - 12.88) < 0$

$x \in (12.88, 82.38)$