Assignment-1

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Subject: Calculus
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① $\lim\limits_{h\to 0} \dfrac{2(-3+h)^2-18}{h}$

Solⁿ $\lim\limits_{h\to 0} \dfrac{2(h^2+9-6h)-18}{h}$

$= \lim\limits_{h\to 0} \dfrac{2h^{\cancel{2}}+\cancel{18}-12\cancel{h}-\cancel{18}}{\cancel{h}}$

$= \lim\limits_{h\to 0} (2h-12)$

$\boxed{=(-12)}$

② $g(x) = \begin{cases} 2x, & x<6 \\ x-1, & x\geq 6 \end{cases}$

Solⁿ (i) 4

$\lim\limits_{x\to 4^-} g(x) = 2(4) = 8$

$\lim\limits_{x\to 4^+} g(x) = 2(4) = 8$

$g(4) = 8$

$$\lim_{x\to 4^-} g(x) = \lim_{x\to 4^+} g(x) = g(4)$$

Hence, $g(x)$ is Continuous at $x=4$.

(ii) 6

$$\lim_{x\to 6^-} g(x) = 2(6) = 12$$

$$\lim_{x\to 6^+} g(x) = 6-1 = 5$$

$$\lim_{x\to 6^-} g(x) \neq \lim_{x\to 6^+} g(x)$$

Hence, $g(x)$ is discontinuos at $x=6$.

(3) $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$

Sol^n $\lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{3-\sqrt{x}}$

$\lim_{x\to 9} -(\sqrt{x}+3)$

$\boxed{= -6}$

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④ $f(x) = \frac{4x+5}{9-3x}$.

Sol^n ~~fd~~

i) for $x = -1$

$f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{1}{12}$

$\lim_{x\to -1} f(x) = \lim_{x\to -1}\left(\frac{4x+5}{9-3x}\right) = \frac{1}{12}$

here, $\lim_{x\to(-1)} f(x) = f(-1)$

Hence function is Continuous at $x = -1$

ii) for $x = 0$

$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left[\frac{4x+5}{9-3x}\right] = \frac{5}{9}$

here, $\lim_{x\to 0} f(x) = f(0)$

Hence, function is Continuous at $x = 0$.

(iii) $f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0} =$ not defined

Hence, the function is discontinuous at $x=3$

(5) $\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

Sol^n - $\lim_{x \to \infty} \frac{\frac{6e^{4x} - e^{-2x}}{e^{4x}}}{\frac{8e^{4x} - e^{2x} + 3e^{-x}}{e^{4x}}}$

$= \lim_{x \to \infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$

$= \frac{\lim_{x \to \infty} 6 - e^{-6x}}{\lim_{x \to \infty} 8 - e^{-2x} + 3e^{-5x}}$

$= \frac{6 - e^{-6(\infty)}}{8 - e^{-2(\infty)} + 3 \times e^{-5(\infty)}}$

$= \frac{6 - e^{-\infty}}{8 - e^{-\infty} + 3e^{-\infty}}$

$= \frac{6-0}{8-0-0}$

3. $= \frac{6}{8}$

$\boxed{= \frac{3}{4}}$

(6) $g(y) = \begin{cases} y^2+5, & y < -2 \\ 1-3y, & y \geq -2 \end{cases}$

Soln

a) $\lim_{y \to 6} g(y)$

$= \lim_{y \to 6} [1-3y]$

$\boxed{= -17}$

b) $\lim_{y \to (-2)^-} g(y)$

$= \lim_{y \to (-2)^-} (y^2+5)$

$\boxed{= 9}$

$\lim_{y \to (-2)^+} g(y)$

$= \lim_{y \to (-2)^+} (1-3y)$

$\boxed{= 7}$

⑦ $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

Sol^n $f'(t) = (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$f'(t) = [12t^4 - 64t^3 - 3t^3 + 16t^2] + [8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12]$

$\boxed{f'(t) = 20t^4 - 132t^3 + 24t^2 + 96t - 12}$

⑧ $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

Sol^n $R'(w) = \dfrac{(2w^2+1)(3+4w^3) - (3w+w^4)(4w)}{(2w^2+1)^2}$

$R'(w) = \dfrac{[6w^2 + 8w^5 + 3 + 4w^3] - [12w^2 + 4w^5]}{(2w^2+1)^2}$

$\boxed{R'(w) = \dfrac{[4w^5 + 4w^3 - 6w^2 + 3]}{[2w^2+1]^2}}$

⑨ $f(x) = 2e^x - 8^x$

Sol^n $\boxed{f'(x) = 2e^x - 8^x \log(8)}$

(10) $y = z^5 - e^z \ln(z)$

Solⁿ $\frac{dy}{dz} = 5z^4 - \left[\frac{e^z}{z} + \ln(z) \cdot e^z\right]$

$$\boxed{\frac{dy}{dz} = 5z^4 - \frac{e^z}{z} - \ln(z) \cdot e^z}$$

(11)

a) $f(x) = (6x^2 + 7x)^4$

Solⁿ $f'(x) = 4(6x^2+7x)^3 \frac{d}{dx}(6x^2+7x)$

$$\boxed{f'(x) = 4(6x^2+7x)^3(12x+7)}$$

b) $f(t) = 5 + e^{4t+t^7}$

Solⁿ $f'(t) = e^{4t+t^7} \frac{d}{dt}(4t+t^7)$

$$\boxed{f'(t) = (4+7t^6) \cdot e^{4t+t^7}}$$

(12)

a) $z = \ln(7 - x^3)$

Sol^n $$\frac{dz}{dx} = \frac{(-3x^2)}{(7-x^3)}$$

$$\frac{d^2z}{dx^2} = \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$\boxed{\frac{d^2z}{dx^2} = \frac{-(3x^4 - 42x)}{(7-x^3)^2}}$$

b) $$Q(v) = \frac{2}{(6+2v-v^2)^4}$$

Sol^n $$Q'(v) = \frac{(6+2v-v^2)^4(0) - (2)(4)(6+2v-v^2)^3(2-2v)}{(6+2v-v^2)^8}$$

$$Q'(v) = \frac{16(v-1)}{(6+2v-v^2)^5}$$

$$Q''(v) = \frac{(6+2v-v^2)^5(16) - 16(v-1)5(6+2v-v^2)^4(2-2v)}{(6+2v-v^2)^{10}}$$

$$Q''(v) = \frac{+16(6+2v-v^2) + 16\times10(v-1)^2}{(6+2v-v^2)^6}$$

e) $$\boxed{Q''(v) = \frac{(+16)(9v^2 - 18v + 16)}{(6+2v-v^2)^6}}$$

c) $f(t) = \ln(1+t^2)$

Sol^n $f'(t) = \dfrac{2t}{1+t^2}$

$$f''(t) = \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

$$\boxed{f''(t) = \frac{2-2t^2}{(1+t^2)^2}}$$

(13) $f(x) = x^3 - 3x^2 - 9x + 12$

Sol^n $f'(x) = 3x^2 - 6x - 9$

let $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

$f''(x) = 6x - 6$

$f''(-1) = -12$

$f''(3) = 12$

Hence the function is maxima at $x = -1$ and function is minima at $x = 3$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$

$f(3) = (3)^3 - (3)(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= -15$

Thus, the maximum value of function is 17 & minima value of the function is (-15).

(14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

Solⁿ $f'(x) = 12x^2 - 36x + 24$

let $f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1$ or $x = 2$

$f''(x) = 24x - 36$

$f''(1) = -12$

$f''(2) = 12$

Hence, the function is maximum at $x = 1$ & function is minimum at $x = 2$.

$f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7$

$= 4 - 18 + 24 - 7$

$= 3$

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 4(8) - 18(4) + 24(2) - 7$

$= 1$

Thus, the maximum value of function is 3 and minimum value of function is 1.

(15) $f(x) = x^2(x+3)$

Sol^n $f(x) = x^2(x+3)$

$f(x) = x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$

let $f'(x) = 0$

$3x^2 + 6x = 0$

$3x(x+2) = 0$

$x = 0$ or $x = -2$

increasing, decreasing, increasing

$-\infty$ $\quad -2 \quad 0 \quad$ ∞

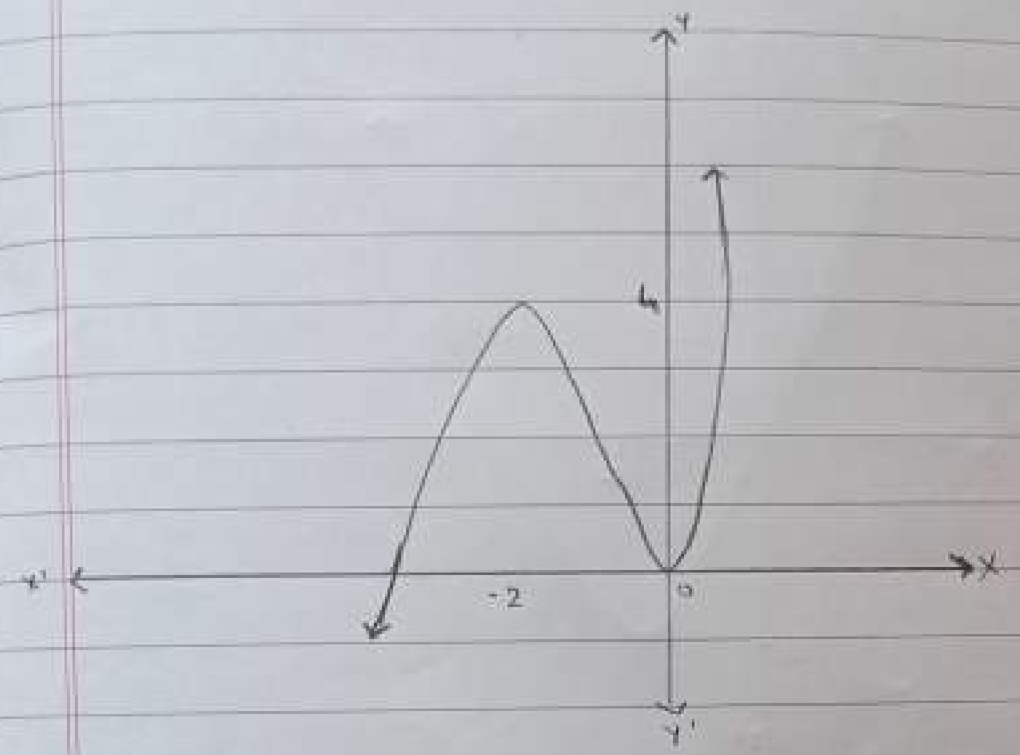
$f''(x) = 6x + 6$

$f''(0) = 6$ [Minima]

$f''(-2) = -6$ [Maxima]

$f(0) = 0$

$f(-2) = 4$



(16) $f(x) = 3x^2 - 6x$

Sol^n $f'(x) = 6x - 6$

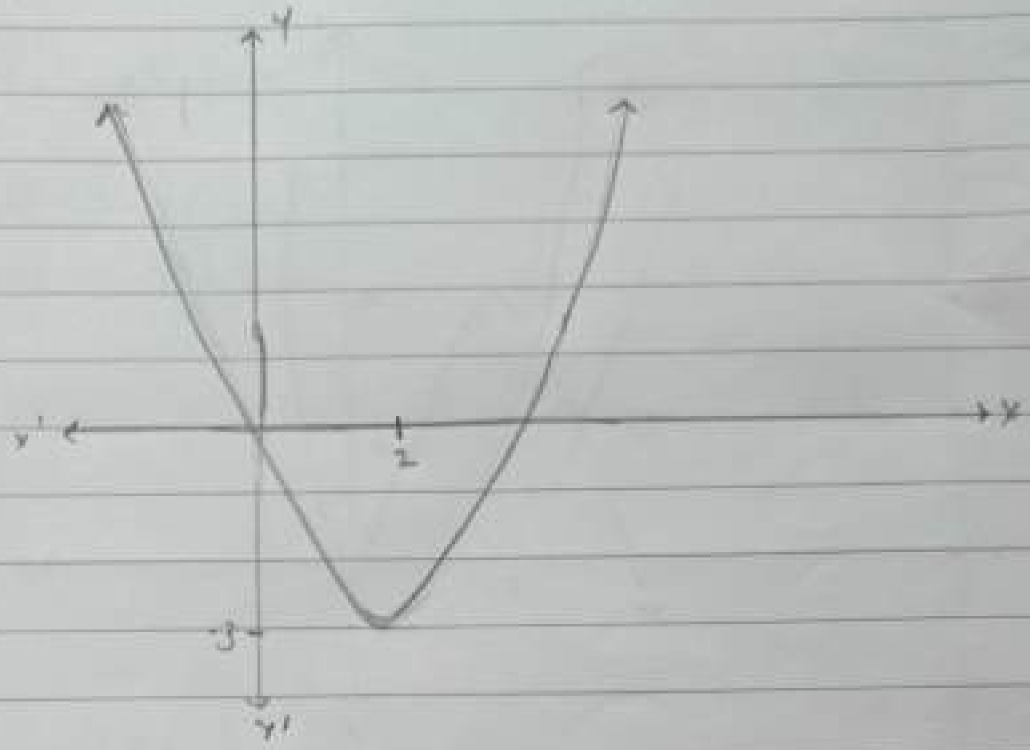
let $f'(x) = 0$

$x = 1$

$f''(x) = 6$

Minima at $x = 1$

$f(1) = -3$


Y
X'
X
2
-3
Y'