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Financial Mathematics

Assignment 2

Loan Amt = 20,000

Monthly Pay. = 427.9

Time = 5yrs.

$$20000 = 427.9\,(12)\, a_{\overline{5|}}^{(12)} \ @\ i\%$$

$$20000 = 427.9\,(12)\left(\frac{1-(1+i)^{-5}}{\left((1+i)^{1/12}-1\right)12}\right)$$

At $i = 10\%$ RHS $= 20341$

$i = 11\%$ RHS $= 19916$

By interpollation $i = 10.8\%$

$\therefore$ APR $= 10.8\%$

$$PV_1 = 427.9\,(12)\, a_{\overline{4|}}^{(12)} \ @\ i = 10.8\%$$

$$= 427.9\,\cancel{(12)}\left(\frac{1-(1.108)^{-4}}{\left((1.108)^{1/12}-1\right)\cancel{12}}\right)$$

$$= 427.9\,(39.205,369) = 16775.97728$$

$$16775.97 = 274.49\,(12)\, a_{\overline{t|}}^{(12)} \ @\ 10.8\%$$

$$(1)-(1.108)^{-t} = 0.524566$$

$$-t = -7.2499$$

$$\boxed{t = 7.2499}$$

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iii) Total interest of initial loan $= 427.9 \times 48 - 16775.97728$

$= 3763.22$

Total int. of restructured loan $= 7.2499 \times 274.49 \times 12 - 16775.97728$

$= 7104.3233$

∴ They will pay 3341.43 more interest.

Q2. (i) a. Discounted Payback Period:

- It is the number of years after which the cumilative discounted cash inflows cover the initial investment.

b. Payback Period:

- Payback period is the number of years necessary to recover funds invested in the project without considering the interest, both on the borrowing and investing.

(ii) Unlike the NPV, neither the DPP nor the PP gives any indication of how profitable a project is, as they ignore cashflows after the accumulated value of zero is reached.

The PP can give misleading results, as it does not take into account the time value of money. Hence, NPV is a superior measure as compared to DPP or PP.

(iii) a. Proj. A

PV of outlay $= -170 - 20v - 10v^2$
PV of inlay $= 20v + 20v^2 + 200v^3$

$$NPV = -170 - 20v - 10v^2 - (20v + 20v^2 + 200v^3)$$
$$D = -170 + 10v^2 + 200v^3$$

$$IRR = 0 = -1\cancel{7}0 + 10v^2 + 200v^3$$
$$170 = 10(1+i)^{-2} + 200(1+i)^{-3}$$

At $i = 7\%$, LHS $= 171.99$
$i = 8\%$, LHS $= 167.33$

By interpollation, $i = 7.4\%$

Proj B

PV of outlay $= -200$
PV of inlay $= 14a_{\overline{6}|} + 200v^6$

$$NPV = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$$

$$IRR = 0 = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$$

At $i = 0.07$ LHS $= 200$

By interpollating $i = 7\%$

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b. $NPV_A = -170 + 10v^2 + 200v^3$

$d = 6\%$

$NPV_A = -170 + 10(1-0.06)^2 + 200(1-0.06)$

$NPV_A = 4.9528$

$NPV_B = -200 + 14a_{\overline{6|}} + 200v^6$

$= -200 + 14\left(\frac{1-(0.94)^6}{0.06}\right)\left((0.94)\right) + 200(0.94)^6$

$NPV_B = 5.9958509$

c. For Proj A, the int. rate should be below 7.4%, if above this then the proj. is unprofitable.
For Proj B, the int. rate should be below 7%. if above this, then project is unprofitable.

Other factors,
- Project A (no net income during FY)
- comparision time period
- lower IRR for Proj B for a larger period of both projs.

Q3. PV of annuity 1 $= 200\ddot{a}_{\overline{22}|}(v^{3/12})$

$= (1.08)^{-1/4}(200)(10.2007)(0.08)$

$= 2161.3635534$

PV of annuity 2 $= 320 \cdot \ddot{a}^{(4)}_{\overline{16\frac{1}{4}}|}\, v^{2/12}$

$= 320\left(\dfrac{1-(1.08)^{-16.25}}{0.08}\right)(1.049519)(1.08)^{-1/5}$

$= 2996.048870$

PV of annuity 3 $= \left(15\,\ddot{a}_{\overline{226}|} \text{ @ } \dfrac{d^{(12)}}{12} = 0.006392917\right)$

$= 15\left(\dfrac{1-(1.08)^{-226/12}}{0.006392917}\right)$

$= 1795.651419$

Total PV $= 6953.065823$

Revised annuity $= 6953.065823 = x \cdot \ddot{a}^{(2)}_{\overline{265}|} \text{ @ } d^{(2)}$

$6953.065823 = x \cdot \left(\dfrac{1-(1.08)^{-21.5}}{0.08}\right)(1.059615)$

$x = 649.0135657$

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Q4. To derive $(I\ddot{a})_{\overline{n|}} = \frac{\ddot{a}_{\overline{n|}} - nv^n}{d}$

$\rightarrow$ $PV = 1 + 2v + 3v^2 + \dots nv^{n-1}$ — (1)

$(1-d)(PV) = v + 2v^2 + 3v^3 + \dots (n-1)v^{n-1} + nv^n$ — (2)

(1) − (2)

$PV - (1-d)PV = 1 + v + v^2 + v^3 + \dots + v^{n-1} - nv^n$

$d(PV) = \ddot{a}_{\overline{n|}} - nv^n$

$I\ddot{a}_{\overline{n|}} = \frac{\ddot{a}_{\overline{n|}} - nv^n}{d}$

Q5. For Property Fund

i) $(1+i) = \frac{\cancel{13.1}}{12.4} \times \frac{\cancel{14.8}}{\cancel{13.1}} \times \frac{\cancel{15.8}}{\cancel{14.8}} \times \frac{16.4}{\cancel{15.8}}$

$(i) = 32.2580645\%$

TWRR, $i = 32.258\%$

For, Equity Fund,

$(1+i) = \frac{9.2}{12.1} \times \frac{10.3}{9.2} \times \frac{13.1}{10.3} \times \frac{15.5}{13.1}$

TWRR, $i = 28.0991736\%$

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ii) a. $12.4(1+i) + 13.1(1+i)^{3/4} + 14.8(1+i)^{1/2} + 15.8(1+i)^{1/4} = 4(16.4)$

$i = 29.32\%$

iii) a. $(12.1)(1+i) + 9.2(1+i)^{3/4} + 10.3(1+i)^{1/2} + 13.1(1+i)^{1/4} = 4 \times 15.5$

$\boxed{i = 67.31\%}$

Q6. i) $160000 = X \cdot a_{\overline{10|}}$ @ 8%

$X = \dfrac{160000}{6.7101} = 23{,}844.65201$

ii) $PV_4 = 23844.65201 \cdot a_{\overline{6|}}$ @ 8%

$= 110231.4422$

$110231.4422 = X' a_{\overline{6|}}$ @ 10%

$X' = 25309.72428$

iii) $PV_7 = 25309.7428$ @ $_{\overline{3|}}$ 10%

$= 62942.75331 = X'' a_{\overline{3|}}$ @ 9%

$X'' = \dfrac{62947.75331}{2.5313} = 24865.78174$

Q8.

i) PV of outlay $= 2.7 + 0.2 a_{\overline{3|}}$

PV of inlay $= \bar{a}_{\overline{10|}} \, (0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3$

$NPV = 0 = \bar{a}_{\overline{10|}} \, (0.25v + 0.2v^2 + 0.1v^3) - 2.7 - 0.2a_{\overline{3|}}$

$0 = \dfrac{1-(v^{10})}{-\ln(v)} \, (0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2\left(\dfrac{1-v^3}{v^{-1}-1}\right)$

$\boxed{i = 9.3\%}$

ii) $NPV = 0.1v^3 + 0.85\,\bar{a}_{\overline{10|}} v^3 - 2.7 - 0.2 a_{\overline{3|}}$ @ 16%

$= 1.0089$

Q9. i) $(1+i)^3 = \dfrac{33}{30.5} \times \dfrac{38.5}{33+6} \times \dfrac{41.05 - 4.5}{38.5} \times \dfrac{45}{41.05} \times \dfrac{45.6}{45} \times \dfrac{47}{45.6-2.5}$

$(1+i)^3 = 1.228313$

$i = 7.0951\% = TWRR$

$30.5\,(1+i)^3 + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{1.5} - 2.5(1+i)^{0.5} = 47$

At $i = 7\%$ RHS $= 46.74$

$i = 7.5\%$ LHS $= 47.37$

By interpolation $\Rightarrow i = 7.208\% = MWRR$

iii) Linked rate of return,

2011 $\Rightarrow$ $30.5(1+i) + 6(1+i)^{0.25} = 38.5$

$i = 6.266\%$

2012 $\Rightarrow$ $38.5(1+i) + 4.5(1+i)^{0.5} = 45$

$i = 4.92\%$

2013 $\Rightarrow$ $45(1+i) + 2.5(1+i)^{0.5} = 47$

$i = 10.276\%$

$(1+i)^3 = 1.06266 \times 1.0492 \times 1.10276$

$i = 7.13\%$

Q10. i) $PV = 180000\, a_{\overline{15}|} + 20000\, (Ia)_{\overline{15}|}$ @ 12%

$180000(6.8109) + 20000(40.7313)$

$= 2040588$

$\therefore$ Total cost of house $= \frac{2040588}{0.8} = 2550735$

ii) $PV_9 = 340000\, a_{\overline{7}|} + 20000\, (Ia_{\overline{7}|})$ @ 12%

$= 340000(4.5638) + 20000\left(\frac{5.1114 - 7 \times 0.452}{0.12}\right)$

$= 1875850.33$

Year	Loan	Payment	Interest	Capital Repayment
9	1875850.33	360000	225102.04	134897.96
10	1740952.37	380000	208914.28	171085.72
11	1569866.65			

iii) $1569866.65 = X \cdot a_{\overline{5|}}$ @ 10%

$$X = \frac{1569866.65}{3.790786} = 414126.87$$

Total installments less loan outstandings at the end of 10th year = 631 033.35

Extra Payment made = 5 (414126.87) + 0.01 (1569866
− 1569866.65

= 51646 6.35

Q11 i) $TWRR = (1+i)^2 = \frac{500}{460} \times \frac{550-50}{500+40} \times \frac{600}{650} \times \frac{x}{600}$

$$1.44 = \frac{500}{460} \times \frac{500}{540} \times \frac{x}{550}$$

$X = 786.9312$

ii) $MWRR \Rightarrow 460(1+i)^2 + 40(1+i)^{21/12} - 50(1+i) = 786.9312$

$i = 30.909\%$

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Q12. i) Discounted Payback Period is the number of years after which the cumulative discounted cash inflows cover the initial payments.

ii) a. $-800(1+i)^{t} - 50(1+i)^{t-1} + 100\bar{s}_{\overline{t-2}|} = 0$ @ 7%

$$-800(1.07)^{t} - 50(1.07)^{t-1} + 100\left(\frac{(1.07)^{t-2}}{\ln(1.07)}\right) = 0$$

$$\therefore \boxed{t = 17.767}$$

b. $100\bar{s}_{\overline{4.233}|}$ @ 6%.

$$100\left(\frac{(1.06)^{4.233} - 1}{\ln(1.06)}\right) = 480$$

iii) $-800(1+i)^{t} - 50(1+i)^{t-1} + 102.97\,|\,s_{\overline{t-2}|}$ @ 7% $= 0$

$t = 18$

which will be the acc. amt after 22 years $= 462.79$

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Q13.

i) $988000 = X(12)\, a^{(12)}_{\overline{25|}}$ @ 7%

$988000 = 12 \cdot X \left(\dfrac{1-(1.07)^{-25}}{((1.07)^{1/12}-1)\,12} \right)$

$X = 6848$, monthly rep. = ₹6848

ii) $X\, a^{(12)}_{\overline{34/3|}} = 6848 \left(a^{(12)}_{\overline{34/3|}} @ 7\% \right) = 648600$

iii) PV after 10th Sept.'s payment $= X \cdot a^{(12)}_{\overline{83/6|}}$ @ 7%

PV after 10th October's payment $= X\, a^{(12)}_{\overline{13.75|}}$ @ 7%

Capital Repaid $= X \cdot a^{(12)}_{\overline{13.75|}} - X \cdot a^{(12)}_{\overline{83/6|}}$

$= X \left(a^{(12)}_{\overline{13.75|}} - a^{(12)}_{\overline{83/6|}} \right)$ @ 7%

$= 26.86$

iv) Capital repayment in these repayments in 12 installments ⇒

$X \left(a^{(12)}_{\overline{22/3|}} - a^{(12)}_{\overline{19/3|}} \right)$ @ 7%

$= 516.2$ $(12X - 51620)$

Total installments int. = 30556 //.