Assignment 2

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1 $S = 65$ $\sigma = 25\%$ p a $r = 2\%$. $T = 1/2$
$K = 55$

i> call :- $C_t = S_0 \Phi(d_1) - X \cdot e^{-rt} \Phi(d_2)$

$$d_1 = \frac{\log\left(\frac{65}{55}\right) + \left(0.02 + \frac{1}{2}(0.25)^2\right) \times \frac{1}{2}}{0.25 \times \sqrt{1/2}}$$

$= 1.089957$

$d_2 = d_1 - \sigma\sqrt{T} = 1.089957 - 0.25\sqrt{\frac{1}{2}}$

$= 0.913186$

$\Phi(d_1) = 0.85993$ $\Phi(d_2) = 0.81854$

$C_t = 65 \times 0.8993 - 55 \times e^{-0.02 \times \frac{1}{2}} \times 0.81859$

$= 11.32098$

Q2 delta (Δ) : change in value of options w.r.t change in price of underlying asset

vega (V) : change in option w.r.t change in the volatility

$$V = \frac{dc}{d\sigma}$$

iii> $S = 55$ $K = 50$ $\sigma = 25\%$. $r = 5\%$. $T = 1$

By BS

European call option :

$C_t = S_0 \Phi(d_1) - Xe^{-rt} \Phi(d_2)$

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$c_t = S_0 \Phi(d_1) - Xe^{-rt} \Phi(d_2)$

$$d_1 = \frac{\log\left(\frac{55}{50}\right) + \left(0.05 + \frac{1}{2}(0.25)^2\right)}{0.25}$$

$d_1 = 0.7062407$
$d_2 = 0.4562407$

$\Phi(d_2) = 0.75804 \qquad \Phi(d_2) = 0.67364$

$C_t = S\Phi(d_1) - Ke^{-rt}\Phi(d_2)$

$= 55 \times 0.75804 - 50 \cdot e^{-0.05 \times 1} \times 0.67364$

$C_t = 9.652890$

By put call parity

$S_0 + P = C_t + Xe^{-rt}$

$P = 9.65289 - 55 + 50\, e^{-0.05 \cdot 1/2}$

$= 2.214361$

ii) To show the vega of european call option and european Put option to be identical

using put call parity

$S + P = C + Xe^{-rt}$

differentiating w r t σ

$$\frac{dP}{d\sigma} = \frac{dC}{d\sigma}$$

ie vagas are identical

iv delta hedged :-
portfolio for which overall delta ie weighted sum of delta of all individual assets aggregates to zero is called as delta hedged or delta neutral.

vega hedged:
Portfolio for which overall vega ie weighted sum of vega of all individual assets equals to zero is equates to zero is called as vega hedged or vega Neutral.

Q3 i) Price derivative in terms of expectation under risk neutral measure

$$C_t = E\left[C_T \cdot e^{-r(T-t)} \mid F_t \right]$$

C_T = Payoff of derivative
C_t = PV of derivative

$S=50$ $k=49$ $r=5\%$ $\sigma=25\%$ $T=1/2$

ii) european call option :

$$C_t = S_0\, \Phi(d_1) - X e^{-rt}\, \Phi(d_2)$$

$d_1 = 0.3441$ $d_2 = 0.1073$
$\Phi(d_1) = 0.6346$, $\Phi(d_2) = 0.5564$

$$C_t = 50 \times 0.6346 - 49 - e^{-0.05 \times 1/2} \times 0.5564$$
$= 4.66$

iii) Price of an american call will be same as european call option - 4.66

As it is not beneficial to exercise american call before maturity. bcz investor will loose future profit potential

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iv] $S_0 + P = C + Xe^{-rt}$

$P = 4.66 - 50 + 49 \times e^{-0.05 \times 1/2}$

$P = 2.45018$

v] • dividend paying stock will cause stock price to fall

- Buyer of the underlying asset will recieve the dividend not the holder of option.
- European call option price will decrease b/c by buying the option instead of underlying they far goes in came. So european call option buyer pays less.
- European put option price will increase as put buyer will get benefilled from the fall in price.
- American call option will be less than european due to optiortunity of early excercise

Q5] i] delta under Black scholes model

$\Delta = \Phi(d_1)$

ii] $S_0 = 40 \quad r = 2\% \quad K = 45.91 \quad t = 5 \quad \Delta_C = 0.6179$

$\sigma = ?$

Assuming $\sigma = 20\%$

$$C_t = S_0 \Phi(d_1) - Ke^{-rt} \Phi(d_2)$$

$$d_1 = \frac{\ln\left(\frac{40}{45.91}\right) + \left(0.02 + \frac{1}{2} \times (0.20)^2\right) \times 5}{0.2 \times \sqrt{5}}$$

$d_1 = 0.1390755 \qquad d_2 = -0.308138$

$\Phi(d_1) = 0.55172$

$\Phi(-d_2) = 1 - \Phi(0.308138)$

$= 0.38289$

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$= 0.38289$

$C_t = 6.196366$
$\Delta c = 0.55172$ for $\sigma = 0.2\%$

Assuming $\sigma = 35\%$

$d_1 = 0.3430$
$d_2 = -0.439615$

$\Phi(d_1) = 0.63307$
$\Phi(d_2) = 1 - \Phi(0.439011)$
$= 0.3335$

$\Delta c = 0.63307$ for $\sigma = 0.35$

$\Delta c_{20\%} = 0.55172$

$\Delta c_{35\%} = 0.63307$

$\Delta c_n = 0.6179$

By interpolation $\sigma = 32.2\%$

iii) As both stocks are independent of each other
By risk neutral measure

$$Q\left[\frac{S_1}{S_0} < k_s\right] \cdot Q\left[\frac{R_1}{R_0} < K^P_R\right] \cdot c\,e^{-r_0 t}$$

iv if stocks are perfectly correlated

$$Q\left[S_1/S_0 < k_s\right] c\,e^{-rt} \quad \text{— ①}$$
$$R\left[R_1/R_0 < K_R\right] \cdot c\,e^{-rt} \quad \text{— ②}$$

Q6 $S = 6.4$ $X = 24183$

X = Units of underlying

y = units of derivative

$T = 1$

$y = 100{,}000$

$K = 6.3$

$r = 3\%$ pa

i) a) delta of put option

$\Delta p = -1 + \Phi(d_1)$

$$d_1 = \frac{\ln(S/K) + (r + \tfrac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$\Delta p = \Phi(d_1) - 1$

b) $100{,}000\, \Delta p = -24830$

$\Delta p = -0.25$

ii) $\Delta p = -0.25\ (-0.24830)$

$\Delta p = \Phi(d_1) - 1$

$-0.24830 + 1 = \Phi(d_1)$

$\Phi(d_1) = 0.7517$

$d_1 = 0.68$

$$d_1 = \frac{\ln(S/K) + (r + \tfrac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$0.68\sigma = \ln\left(\frac{6.4}{6.3}\right) + \left(0.03 + \tfrac{1}{2}\sigma^2\right)$$

$$0.68\sigma = 0.045748 + \tfrac{1}{2}\sigma^2$$

$$0.5\sigma^2 - 0.68\sigma + 0.045748 = 0$$

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$$\frac{0.68 \pm \sqrt{0.6901}}{2\times 0.5} = 7.1\%.$$

iii) a) Price of eu put

$$P = Ke^{-rt}\,\Phi(-d_2) - S_0\,\Phi(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{6.4}{6.3}\right) + \left(0.03 + \frac{1}{2}(0.071)^2\right)}{0.071}$$

$d_1 \cdot 0.67984$ $\quad d_2 = 0.608863$

$$\Phi(-d_1) = 1 - \Phi(d_1)$$
$$= 1 - 0.74857$$
$$= 0.25143$$

$$\Phi(-d_2) = 0.27425$$

$$P_t = 6.3\times e^{-0.03}\times 0.27425 - 6.4\times 0.25143$$
$$= 0.0672595$$
$$= 6.7559p$$

Option price $= 6.7559p$

Q7) i) delta: change in option price w.r.t change in price of underlying

gamma: change in delta w.r.t change in underlying asset.

vega: change in Price option w.r.t change in volatility.

ii) $\Delta = \Phi(d_1)$

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$$d_1 = \frac{\ln\left(\frac{60}{80}\right) + \left[0.03 + \frac{1}{2}(0.25)^2\right] \times 3}{0.25\sqrt{3}}$$

$d_1 = 0.845406$
$d_2 = 0.41239$

$\Delta = \Phi(d_1) = 0.79955$

iii) delta hedging
delta $= 0.801$

$17.91 - 0.801 \times 60 = -30.15$
-30.1 short in cash

Q16 Portfolio which is long one call plus cash of Ke^{-rt} and short one put.

The portfolio has payoff at time of expiry of S_T

$$C + Ke^{-r(T-t)} - P_t = S_t$$