Financial Mathematics Assignment - [Unit 1 & 2]

1. $d^{(4)} = 8\%$

$$d = 1 - \left(1 - \frac{d^{(p)}}{p}\right)^p$$

$$= 1 - (1 - 2\%)^4$$

$$= 7.763184\%$$

(i) $\delta = -\ln(1-d)$

$$= 8.0810829\%$$

(ii) $$i = \frac{1}{\left(1 - \frac{d^{(p)}}{p}\right)^p} - 1$$

$$= 8.4165784\%$$

(iii) $d^{(12)} = p\left[1 - (1-d)^{1/p}\right]$

$$= 8.053934\%$$

2. $\delta(t) = 0.004t + 0.0002t^2$

(i) Amt due $= C = 1000$

$$PV_0 = C \cdot V(t)_{10}$$

$$= 1000 \cdot e^{-\int_0^{10} 0.004t + 0.0002t^2 \, dt}$$

$$= 1000 \cdot e^{-\int_0^{10} 0.002t^2 + 0.0002t^3/3 \, dt}$$

$$= 1000 \cdot e^{-0.266667}$$

$$= 765.93$$

(ii) $(1+i)^{10} = e^{\delta}$

$$(1+i) = e^{\delta/10}$$

$$i = e^{0.2666667/10} - 1$$

$$i = 2.7025403\%$$

3. (i) The relationship $d = iv$, has an interesting verbal interpretation. Interest earned on an investment, paid at the beginning of the period is d. Interest earned on an investment, paid at the end of the period is i. Therefore, if we discount i from the end of the period with the discounting factor v, we obtain d.

(ii) (a) $\frac{d^{(4)}}{4} = 2.25\%$

$$d = 1 - \left(1 - \frac{d^{(p)}}{p}\right)^{p}$$

$$= 8.7007806\%$$

$$d^{(2)} = p\left[1 - (1-d)^{1/p}\right]$$

$$= 2\left[1 - (1-d)^{1/p}\right]$$

$$= 8.8987499\%$$

(b) $d^{(1/2)} = 5\%$

$$d = 1 - \left(1 - \frac{d^{(p)}}{p}\right)^{p}$$

$$= 5.1316701\%$$

$$d^{(2)} = p\left[1 - (1-d)^{1/p}\right]$$

$$= 5.1992506\%$$

(iii) $$v(0,1) = \frac{1}{(1+i)^{91/365}} = 1 - \frac{91}{365}d$$

$$\frac{1}{(1.05)^{91/365}} = 1 - \frac{91}{365}d$$

$$d = \left[1 - \frac{1}{(1.05)^{91/365}}\right]\frac{365}{91}$$

$$d = 4.8494621\%$$

4. (i) The relationship $d = iv$, has an interesting verbal interpretation. Interest earned on an investment, paid at the beginning of the period is d. Interest earned on an investment, paid at the end of the period is i. Therefore, if we discount i from the end of the period with the discounting factor v, we obtain d.

(ii) $i = 0.08$

(a) $d^{(12)} = p[1 - (1+i)^{-1/p}]$

$= 7.671478\%$

(b) $i^{(365)} = p[(1+i)^{1/p} - 1]$

$= 7.6969155\%$

(c) $\delta = \ln(1+i)$

$= 7.696104\%$

(d) $i^{(0.5)} = p[(1+i)^{1/p} - 1]$

$= 8.32\%$

5. (i) $d^{(4)} = 6\%$

~~(a)~~ $d = 1 - \left(1 - \frac{d^{(p)}}{p}\right)^p$

$= 1 - (1 - 1.5\%)^4$

$= 5.8663449\%$

(a) $i = \frac{d}{1-d}$

$= 6.2319314\%$

$i^{(2)} = p[(1+i)^{1/p} - 1]$

$= 6.137752\%$

PAGE No.
DATE / /

(b) $d^{(12)} = p[1-(1-d)^{1/p}]$
$= 6.0802524\%$

(ii) $i = \frac{20000}{200000} \times 100$
$= 10\%$

No. of days = 93
$AV_{93} = 200000 \times (1.1)^{93/365}$
$= 2,04,916.3564$

6. (i) Let the retail price be x
If retailer pays cash = 70% x
then he has to pay, $PV = 0.7x$
If retailer ~~has to~~ uses the 6 month credit = 75% x
then he has to pay, $AV = 0.75x$ in 6 months time
we need to find the discounting done on 6 month credit made for the retailers who pay in cash [i.e. cash is preffered rather than credit mode]
Time 6 months = ½ year
$PV_0 = AV(1-d)^n$
$0.7x = 0.75x(1-d)^{1/2}$
$d = 1-\left(\frac{0.7}{0.75}\right)^2$

$d = 12.888889\%$

(ii) (a) $d^{(12)} = \cancel{12}\ p[1-(1-d)^{1/p}]$
13.719544%

(b) $i = \frac{d}{1-d} = 14.79591837\%$

$i^{(2)} = p[(1+i)^{1/p} - 1]$
$= 14.285714\%$

(c) $\delta = -\ln(1-d)$
$= 13.798574\%$

7 Amt invested = C = 20000
$AV_{17 months} = 26000$
t = 17 months = $1 + 5/12$ years
$= 1.416666667$ years
$AV = C(1+i)^n$
$26000 = 20000(1+i)^{1.416666667}$
$i = 20.3457067\%$

(i) $i^{(4)} = p[(1+i)^{1/p} - 1]$
$= 18.9552545\%$

(ii) $d^{(2)} = p[1-(1+i)^{-1/p}]$
$= 17.6882352\%$

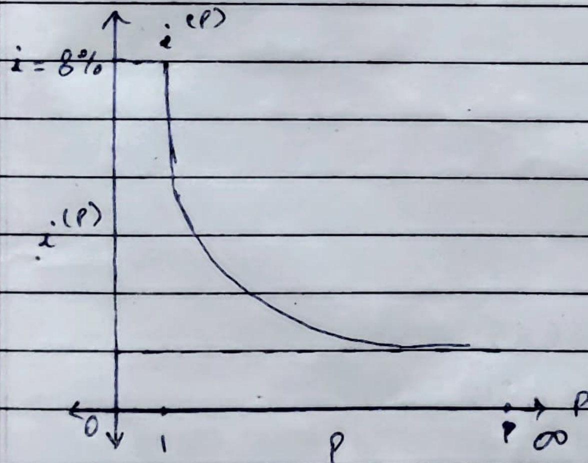
(iii) $AV = C(1+ni)$
$26000 = 20000(1 + 1.416666667i)$
$i_s = 21.1764705\%$

(iv) Numerically, in terms of compounding, i is the highest numerical value & d is the lowest numerical value. This is just numerically, as both will yeild the same accumulative & present value. The simple interest, i_s is greater than i because since there is no interest on interest accumulation in simple interest, it has to compensate for the compounding effect & hence it has to be greater than i

8. $i = 8\%$

$i^{(p)} = p[(1+i)^{1/p} - 1]$

p	$i^{(p)}$
1	8%
2	7.84609%
3	7.79567%
4	7.77062%
12	7.72084%
100	7.69907%
365	7.69691%
∞	7.696104%



→ More frequently the compounding takes place (i.e. as p increases), the smaller is the equivalent nominal annual rate

$i^{(p)} \propto \frac{1}{p}$

→ when continuous compounding takes place i.e. when p approaches infinity the $i^{(p)}$ has a limiting value known as the force of interest or δ.

9. (i) Force of interest - The measure of interest at individual moments of time or at any instant of time is known as force of interest.

(ii) $i^{(2)} = 0.0775$

$$i = \left(1 + \frac{i^{(P)}}{P}\right)^{P} - 1$$

$= 7.9001562\%$

$\delta = \ln(1+i)$

$= 7.6036133\%$

$d^{(12)} = p[1 - (1+i)^{-1/p}]$

$= 7.5795746\%$

$i^{(1/2)} = p[(1+i)^{1/p} - 1]$

$= 8.2122185\%$

10. $\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$

when, $0 \le t \le 5$

$V(t) = e^{-\int_0^t 0.08\,dt}$

$= e^{-0.08t}$

when $5 \le t \le 10$

$V(t) = e^{-\int_0^5 0.08\,dt - \int_5^t 0.06\,dt}$

$= e^{-[0.08t]_0^5 + [0.06t]_5^t}$

$= e^{-0.4 + 0.3 - 0.06t}$

$= e^{-0.06t - 0.1}$

when, $t \geq 10$

$$v(t) = e^{-\int_0^5 0.08\,dt - \int_5^{10} 0.06\,dt - \int_{10}^{t} 0.04\,dt}$$

$$= e^{-[0.08t]_0^5 + [0.06]_5^{10} + [0.04]_{10}^{t}}$$

$$= e^{-0.4+0.3-0.6+0.4-0.04t}$$

$$= e^{-0.04t-0.3}$$

11 a) $C = 50000$ = Amt due

$d = 18\%$

$n = \frac{9}{12} = \frac{3}{4}$ years

$PV_0 = C(1-d)^t$

$= 50000(1-18\%)^{3/4}$

$= 43085.42$

b) (i) $\delta = 6\%$

$d = 1 - e^{-\delta}$

$= 5.8235466\%$

$d^{(12)} = p[1-(1-d)^{1/p}]$

$= 5.9850249\%$

(ii) $d^{(4)} = 6\%$

$$i = \frac{1}{\left(1 - \frac{d^{(p)}}{p}\right)^p} - 1$$

$= 6.2319315\%$

$i^{(4)} = p[(1+i)^{1/p} - 1]$

$= 6.0913705\%$

$i^{(12)} = p[(1+i)^{1/p} - 1]$

$= 6.0607088\%$

c) Ascending order → $d < \delta < i^{(4)}$

d will always be the smallest numerical value as it is charged in the beginning of the term. so due to time value of money $d < \delta$ & $d < i^{(4)}$. δ refers to continuous compounding i.e more frequency than quaterly compounding. Due to more compounding than $i^{(4)}$, δ will yield a higher accumulated value than $i^{(4)}$ if $\delta = i^{(4)}$. Thus δ has to be smaller than $i^{(4)}$ to yield the same accumulated value. Thus $\delta < i^{(4)}$ & hence $d < \delta < i^{(4)}$

d) The relationship $d = iv$, has an interesting verbal ~~interpretati~~ interpretation. Interest earned on an investment, paid at the beginning of the period is d. Interest earned on an investment, paid at the end of the period is i. Therefore, if we discount i from the end of the period with the discounting factor v, we obtain d.

12 7049 $i_s^{(4)} = 1.5\%$

0 $i^{(12)} = 6\%$ 2 4.5 $d^{(12)} = 6\%$ 6

$C = 7049$ = Amt invested

$AV_6 = C \cdot A(0,6)$

$= C \cdot A(0,2) \cdot A(2,4.5) \cdot A(4.5,6)$

⇒ For $A(0,2)$

$i^{(12)} = 6\%$ $\frac{i^{(12)}}{12} = 0.5\%$

$\cancel{(12)\times 2}$ =

$A(0,2) = (1.005)^{24}$

⇒ For $A(2,4.5)$

$i_s^{(4)} = 1.5\%$

$A(2,4.5) = (1 + 10(0.015)) = 1.15$

PAGE No.
DATE / /

$\Rightarrow$ for $A(4.5, 6)$

$d^{(12)} = 6\%$ $\quad \frac{d^{(12)}}{12} = 0.5\%$

$A(4.5, 6) = \frac{1}{(1-0.005)^{18}}$

$\therefore AV_6 = 7049 \times (1.005)^{24} \times 1.15 \times \frac{1}{(1-0.005)^{18}}$

$= 9999.894$

13. Amt invested $= C = x$

$AV_n = 2x$

Time $= n$

(i) $i = 10\%$

$AV_n = C(1+i)^n$

$2x = x(1+10\%)^n$

$2 = 1.1^n$

$\ln 2 = n \ln 1.1$

$n = 7.27$ years

(ii) $d^{(12)} = 10\% \rightarrow \frac{d^{(12)}}{12} = 0.833333333\%$

$AV_n = \frac{C}{\left(1-\frac{d^{(12)}}{12}\right)^{n12}}$

$2x = \frac{x}{(1-0.833333333\%)^{n.12}}$

$(1-0.833333333\%)^{12n} = 1/2$

$12n . \ln(1-0.833333333\%) = \ln 1/2$

$n = 6.902$ years.