

Numerical method & Algebra

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$$1) \sqrt{7\sqrt{7\sqrt{7}\dots}}$$

$$= 7^{1/2} \cdot 7^{1/4} \cdot 7^{1/8} \cdot 7^{1/16} \dots$$

$$= 7^{1/2 + 1/4 + 1/8 + 1/16} \dots$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \dots \text{ is in form of infinite GP}$$

$$\text{Sum of infinite G.P} = \frac{a}{1-r}$$

$$= \frac{1/2}{1-1/2} = 1$$

$$\therefore 7^{1/2 + 1/4 + 1/8} \dots$$

$$= 7^1$$

$$= 7$$

$$2) 7x^2 - 6y^2 = -11x^2 - 4y^2$$

$$18x^2 = 2y^2$$

$$\frac{x^2}{y^2} = \frac{2}{18}$$

$$\frac{x}{y} = \sqrt{\frac{2}{18}}$$

$$3) \quad x + y = 4 \quad \text{--- (1)}$$

$$x^2 + y^2 - 4y = 12 \quad \text{--- (2)}$$

$$x = 4 - y \quad \text{--- from (1) in (2)}$$

$$(4 - y)^2 + y^2 - 4y = 12$$

$$16 + y^2 - 8y + y^2 - 4y = 12$$

$$y^2 - 6y + 8 = 6$$

$$y^2 - 6y + 2 = 0$$

$$y = \frac{6 \pm \sqrt{36 - 8}}{2}$$

$$= \frac{6 \pm \sqrt{28}}{2}$$

$$= 3 \pm \sqrt{7}$$

$$\begin{aligned} x_1 &= 4 - y_1 \\ &= 4 - 3 + \sqrt{7} \\ &= 1 + \sqrt{7} \end{aligned}$$

$$\begin{aligned} x_2 &= 4 - y_2 \\ &= 4 - 3 - \sqrt{7} \\ &= 1 - \sqrt{7} \end{aligned}$$

the ordered pairs are

$$(1 + \sqrt{7}, 3 - \sqrt{7}) \text{ and } (1 - \sqrt{7}, 3 + \sqrt{7})$$

$$4) \quad d = 2x^2 - 16x + 34 \rightarrow \text{density}$$

$$i) \quad x = 3.5$$

$$d = 2.5 \text{ mill particles/st}^3$$

$$ii) \quad d - 2x^2 - 16x + 34 = 0$$

differentiation

$$4x - 16 = 0 \quad \text{--- (1)}$$

$$x = 4 \quad (400 \text{ RPM})$$

differentiation of ①

$$\Rightarrow 4$$

Minima

$$d = 2(4)^2 - 16(4) + 34$$

$$d = 2$$

$\therefore$ the no. of revolution for minimum smoke = 4

iii) $d = 100$

$$100 = 2n^2 - 16n + 34$$

$$50 = n^2 - 8n + 17$$

$$0 = n^2 - 8n - 33$$

$$n = (n-11)(n+3)$$

$$\boxed{n=11}$$

the speed must be 11 ~~rev~~ revolution per minute

Q5) $C_1 = 1500 + 25(\text{km})$

$$C_2 = 2100 + 22(\text{km})$$

i) Cost to rent car from Co. 1

$$C_1 = 1500 + 25n$$

ii) Cost to rent car from Co. 2

$$C_2 = 2100 + 22n$$

iii) $C_2 < C_1$

$$1500 + 25n > 2100 + 22n$$

iv) $3n > 600$

$$n > 200$$

$\therefore$ It is less expensive to take C_2 when the no. of km is more than 200.

$$6) x^3 - 7x^2 + 14x - 6 = 0$$

$$a) a=0, b=1$$

a	b	$a + b/2$	$f(a)$	$f(b)$	$f\left(\frac{a+b}{2}\right)$
0	1	0.5	-6	2	-0.625
0.5	1	0.75	-0.625	2	0.9844
0.5	0.75	0.625	-0.625	0.9844	0.2598

$$x = 0.625$$

$$b) a=1 \quad b=3.2 \quad f(x) = x^3 - 7x^2 + 14x - 6$$

a	b	$a + b/2$	$f(a)$	$f(b)$	$f\left(\frac{a+b}{2}\right)$
1	3.2	2.1	2	-0.112	1.791
2.1	3.2	2.65	1.791	-0.112	0.5521
2.65	3.2	2.925	0.5521	-0.112	0.08582

$$x = 2.925$$

$$c) [3.2, 4]$$

a	b	$a + \frac{b}{2}$	$f(a)$	$f(b)$	$f\left(a + \frac{b}{2}\right)$
3.2	4	3.6	-0.112	2	0.336
3.4	3.6	3.4	-0.112	0.336	-0.016
3.4	3.6	3.5	-0.016	0.336	0.125
3.4	3.5	3.45	-0.016	0.125	0.046125
3.4	3.45	3.425	3.425 -0.016	0.0461	0.0130

$$x = 3.425$$

7. $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$
 $f'(x) = 920x^3 + 54x^2 + 18x - 221$

a)

i	x_i	$f(x_i)$	$f'(x_i)$	$x_i - \frac{f(x_i)}{f'(x_i)}$
1	-1	156.1398	-110.5	-0.608121
2	-0.6081	43.994	-418.898	-0.2354
3	-0.2354	1.534	-234.246	-0.04759
4	-0.04759	0.0004	-221.704	-0.04066
5	-0.04066	~ 0	-221.70443	-0.04066
	-0.04066	~ 0	-221.70443	-0.04066

$x = -0.04060$

b)

i	x_i	$f(x_i)$	$f'(x_i)$	$x_i - \frac{f(x_i)}{f'(x_i)}$
1	0.9	-36.585	509.62	0.97179
2	0.97178	6.37668	691.80	0.96254
3	0.9625	0.11523	666.84	0.9624
	0.9624	0.3267	666.411	0.9624
	0.9624			

$x = 0.9624$

8. a. Average = $\frac{70 + 86 + 81 + 83 + x}{5}$
 $= \frac{220 + x}{5}$

b. Average = β

$$\therefore 80 < \frac{320+n}{5} < 90$$

c. $80 < \frac{320+n}{5} < 90$

$$400 < 320 + n < 450$$

$$80 < n < 130$$

9. $x_1 = 0.04$

$$x_2 = 0.2$$

$$x = 0$$

$$f(x_1) = 8.54$$

$$f(x_2) = -11.81$$

$$f(x) = ?$$

$$\frac{x - x_1}{x_2 - x_1} = \frac{f(x) - f(x_1)}{f(x_2) - f(x_1)}$$

$$\frac{-0.04}{0.16} = \frac{f(x) - 8.54}{-20.35}$$

$$f(x) = 13.6275$$

10. $R(x) = 32x - 0.21x^2$

$$C(x) = 195 + 12x$$

for profit $R(x) > C(x)$

$$\therefore 32x - 0.21x^2 > 195 + 12x$$

$$20x - 195 - 0.21x^2 > 0$$

$$0.21x^2 - 20x + 195 < 0$$

$$0.21x^2 - 20x + 195 = 0$$

$$x = \frac{20 \pm \sqrt{212.8}}{0.42}$$

$$x = 82.38, 12.88$$

$$(x - 82.38)(x - 12.88) < 0$$

$$x \in (12.88, 82.38)$$