Assignment (Homework).

1) $\lim_{h\to 0} \dfrac{2(-3+h)^2 - 18}{h}$

$\lim_{h\to 0} \dfrac{2(9-6h+h^2)-18}{h}$

$\lim_{h\to 0} \dfrac{\cancel{18} - 12h + 2h^2 - \cancel{18}}{h}$

$\lim_{h\to 0} \dfrac{2h^2 - 12h}{h}$

$\lim_{h\to 0} \dfrac{\cancel{h}(2h-12)}{\cancel{h}}$ $\because h \to 0$

$h \neq 0$

$\lim_{h\to 0} 2h - 12$

$= 2(0) - 12$

$\boxed{-12}$

2 Determine ~~th~~ if the following function is continous or discontinous at a) $x=4$ b) $x=6$

$g(x) = \begin{cases} 2x & x<6 \\ x-1 & x \geq 6 \end{cases}$

$f(4) = 2x$
$= 2(4)$
$= 8.$

$\lim_{x\to 4^-} 2x$
$= 2(4)$
$= 8$

$\lim_{x\to 4^+} 2x$
$= 2(4)$
$= 8$

$\therefore f(x) = \lim_{x\to 4^-} = \lim_{x\to 4^+}$ $\boxed{\therefore \text{function is continous at } x=4}$

$f(6) = x-1$
$= 6-1$
$= 5$

$\lim_{x\to 6^-} 2x$
$= 2(6)$
$= 12$

$\lim_{x\to 6^+} x-1$
$= 6-1$
$= 5$

$\therefore f(6) \neq \lim_{x\to 6^-} \neq \lim_{x\to 6^+}$ $\boxed{\therefore \text{function is discontinous at } x=6.}$

3 Solve the following limit $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$.

$$\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$$

$$\lim_{x\to 9} \frac{(x-9)(3+\sqrt{x})}{(3-\sqrt{x})(3+\sqrt{x})}$$

$$\lim_{x\to 9} \frac{(x-9)(3+\sqrt{x})}{(9-x)}$$

$$\lim_{x\to 9} -\frac{\cancel{(9-x)}(3+\sqrt{x})}{\cancel{(9-x)}} \quad \because x\to 9 \quad 9-x\neq 0.$$

$$\lim_{x\to 9} -(3+\sqrt{x})$$

$$-(3+\sqrt{9})$$

$$-(3+3)$$

$$\boxed{-6}$$

4) Determine if the following function is continuous or discontinuous at a) $x=-1$, b) $x=0$ c) $x=3$

$$f(x) = \frac{4x+5}{9-3x}$$

a) $\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{4x+5}{9-3x} = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{+1}{12} = \frac{-1}{6}$

$f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-1}{12}$ $\because \lim_{x\to -1} f(x) = f(-1)$ $\therefore$ function is continous at $x=-1$

b) $f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$ $\because \lim_{x\to 0} f(x) = f(0)$ $\therefore$ function is continous at $x=0$.

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{4x+5}{9-3x} = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

c) $f(3) = \frac{4(3)+5}{9-3(3)} = \frac{12+5}{9-9} = \frac{17}{0}$ = Not defined

$\therefore f(3)$ is Not Defined function is discontnous at $x=3$

5 ~~Determine~~ Evaluate each of the following limits.

$$\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$\div$ by e^{4x}

$$= \frac{\lim_{x\to\infty} 6 - \frac{1}{e^{6x}}}{8 - \frac{1}{e^{2x}} + \frac{3}{e^{5x}}}$$

$$= \frac{\lim_{x\to\infty} 6 - \frac{1}{e^{6x}}}{\lim_{x\to\infty} 8 - \frac{1}{e^{2x}} + \frac{3}{e^{5x}}}$$

$$= \frac{\lim_{x\to\infty} 6 - e^{-6x}}{\lim_{x\to\infty} 8 - e^{-2x} + 3e^{-5x}}$$

$$= \frac{6 - e^{-\infty}}{8 - e^{-\infty} + 3e^{-\infty}}$$

$$= \frac{6-0}{8-0}$$

$$= \frac{6}{8}$$

$$\boxed{= \frac{3}{4}}$$

6 Given the function compute the following limits

$$g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$$

a) $\lim_{y \to 6} g(y)$

b) $\lim_{y \to -2} g(y)$

a) $\lim_{y \to 6} g(y)$

$\lim_{y \to 6} 1 - 3y$

$= 1 - 3(6)$

$= 1 - 18$

$\boxed{= -17}$

b) $\lim_{y \to -2^-} g(y)$

$\lim_{y \to -2^-} y^2 + 5$

$(-2)^2 + 5$

$4 + 5$

$\boxed{9}$

$\lim_{y \to -2^+} \overset{g(y)}{1 - 3y}$

$\lim_{y \to 2^+} 1 - 3y$

$1 - 3(-2)$

$1 + 6$

$\boxed{7}$

7 Use the Product Rule to find the Derivate of

$f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

Differentiate wrt t

$\frac{d f(t)}{dt} = 4t^2 - t \frac{d}{dt}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12)\frac{d}{dt} 4t^2 - t$

$= [4t^2 - t(3t^2 - 16t)] + (t^3 - 8t^2 + 12)(8t - 1)$

$= 12t^4 - 64t^3 - 3t^3 - 16t^2 + [8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12]$

$= \cancel{20t^4 - 128t^3 - 12t^2 + 32t}$

$\boxed{f'(t) = 20t^4 - 132t^3 + 24t^2 + 96t - 12}$

8 Use the quotient Rule to find the derivate of

$R(w) = \frac{3w + w^4}{2w^2 + 1}$

$R(w) = \frac{3w + w^4}{2w^2 + 1}$

Differentiate wrt w.

$\frac{dR(w)}{dw} = \frac{2w^2 + 1 \frac{d}{dw}(3w + w^4) - (3w + w^4)\frac{d}{dw} 2w^2 + 1}{(2w^2+1)^2}$

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$$\frac{dR(w)}{dw} = \frac{(2w^4+1)(3+4w^3) - (3w+w^4)(8w^3)}{(2w^2+1)^2}$$

$$= \frac{6w^4+3+8w^7+4w^3-(24w^4+8w^7)}{(2w^2+1)^2}$$

$$= \frac{6w^4+3+8w^7+4w^3-24w^4-8w^7}{(2w^2+1)^2}$$

$$\frac{dR(w)}{dw} = \frac{[2w^2+1\,(3+4w^3)] - [(3w+w^4)(4w)]}{(2w^2+1)^2}$$

$$\frac{dR(w)}{dw} = \frac{[6w^2+3+8w^5+4w^3] - [12w^2+4w^5]}{(2w^2+1)^2}$$

$$\frac{dR(w)}{dw} = \frac{6w^2+3+8w^5+4w^3-12w^2-4w^5}{(2w^2+1)^2}$$

$$\frac{dR(w)}{dw} = \frac{4w^5-6w^2+4w^3+3}{(2w^2+1)^2}$$

$$\boxed{\frac{dR(w)}{dw} = \frac{4w^6+4w^3-6w^2+3}{(2w^2+1)^2}}$$

9 Differentiate

$f(x) = 2e^x - 8^x.$

$f(x) = 2e^x - 8^x$

$\boxed{f'(x) = 2e^x - 8^x \log(8)}$

10 Differentiate $y = z^5 - e^z \ln(z)$

$y = z^5 - e^z \ln z$

Differentiate wrt z

$\frac{dy}{dz} = 5z^4 - \left[e^z \frac{d}{dz}\ln z + \ln(z) \frac{de^z}{dz}\right]$

$\frac{dy}{dz} = 5z^4 - \left[\frac{e^z}{z} + e^z \ln(z)\right]$

$\boxed{\frac{dy}{dz} = 5z^4 - e^z \left[\frac{1 + z\ln(z)}{z}\right]}$

11 Using chain rule differentiate the following

a) $f(x) = (6x^2 + 7x)^4$

$f(x) = (6x^2 + 7x)^4$

Differentiate wrt x

$f'(x) = 4(6x^2 + 7x)^3 \frac{d}{dx}(6x^2 + 7x).$

$f'(x) = 4(6x^2 + 7x)^3 (12x + 7)$

$\boxed{f'(x) = 4(12x + 7)(6x^2 + 7x)^3}$

b) $f(t) = 5 + e^{4t + t^7}$

$f(t) = 5 + e^{4t + t^7}$

Differentiate wrt t.

$f'(t) = e^{4t + t^7} \frac{d}{dt} 4t + t^7$

$\boxed{f'(t) = e^{4t + t^7} (4 + 7t^6)}$

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12 Determine the second derivative of.

a) $z = \ln(7-x^3)$

$z = \ln(7-x^3)$

Differentiate wrt x

$$\frac{dz}{dx} = \frac{1}{7-x^3} \frac{d}{dx} 7 - x^3$$

$$\frac{dz}{dx} = \frac{-3x^2}{7-x^3}$$

Differentiate wrt x.

$$\frac{d^2z}{dx^2} = \frac{7-x^3 \frac{d}{dx} -3x^2 - (-3x^2)\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{-42 + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{-42 - 3x^4}{(7-x^3)^2}$$

$$\boxed{\frac{d^2z}{dx^2} = \frac{-3(x^4+14)}{(7-x^3)^2}}$$

b) $$Q(v) = \frac{2}{(6+2v-v^2)^4}$$

$$Q(v) = 2(6+2v-v^2)^{-4}$$

Differentiate wrt v.

$$Q'(v) = 2(-4)(6+2v-v^2)^{-4-1} \frac{d}{dx}(6+2v-v^2)$$

$$Q'(v) = \frac{-8(2-2v)}{(6+2v-v^2)^5}$$

$$g'(v) = \frac{~~8~~16(v-1)}{(6+2v-v^2)^{5}}$$

$$g'(v) = 16(v-1)(6+2v-v^2)^{-5}$$

$$g''(v) = 16\left[v-1\frac{d}{dx}(6+2v-v^2)^{-5} + (6+2v-v^2)^{-5}\frac{d(v-1)}{dx}\right]$$

$$= 16\left[\frac{v-1}{(6+2v-v^2)^6}(-5)\frac{d\,6+2v-v^2}{dx} + \frac{1}{(6+2v-v^2)^5}\right]$$

$$= \frac{16}{(6+2v+v^2)^6}\left[(v-1)(-5)(2-2v) + 6+2v-v^2\right]$$

$$\boxed{= \frac{16(9v^2-18v+16)}{(6+2v+v^2)^6}}$$

c) $f(t) = \ln(1+t^2)$

$f(t) = \ln(1+t^2)$

Differentiate wrt t.

$$f'(t) = \frac{1}{1+t^2}\frac{d}{dt}(1+t^2)$$

$$f'(t) = \frac{2t}{(1+t^2)}$$

Differentiate wrt t

$$f''(t) = \frac{1+t^2\frac{d\,2t}{dt} - 2t\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$f''(t) = \frac{(1+t^2)(2) - 2t(2t)}{(1+t^2)^2}$$

$$f''(t) = \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$\boxed{f''(t) = \frac{2(1-t^2)}{(1+t^2)^2}}$$

13 Find the maximum and minimum value of function.

$x^3 - 3x^2 - 9x + 12$.

$f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$f'(x) = 0$

$0 = 3x^2 - 6x - 9$

$x^2 - 2x - 3 = 0$

$-3 \;\; +1$

$(x-3)(x+1) = 0$

$x - 3 = 0 \qquad x + 1 = 0$

$x = 3 \qquad x = -1$.

$f''(x) = 6x - 6$

$f''(3) = 6(3) - 6$

$= 18 - 6$

$= 12 > 0$ ∴ function is minimum at $x = 3$.

$f''(-1) = 6(-1) - 6$

$= -6 - 6$

$= -12 < 0$ ∴ function is maximum at $x = -1$

Maximum value of function.

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= \cancel{-13+12} \; -4 + 21$

$\boxed{= +17}$

Minimum value of function

$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$\boxed{= -15}$

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14 Find the maximum and minimum value of the function $4x^3 - 18x^2 + 24x - 7$

$f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$f'(x) = 0$

$0 = 12x^2 - 36x + 24$

$0 = x^2 - 3x + 2$

$-2 \quad -1$

$(x-2)(x-1) = 0$

$x - 2 = 0 \qquad x - 1 = 0$

$x = 2 \qquad x = 1$

$f''(x) = 24x - 36$

$f''(2) = 24(2) - 36$

$= 48 - 36$

$= 12 > 0$ & $\therefore$ function is minimum at $x = 2$.

$f''(1) = 24(1) - 36$

$= 24 - 36$

$= -12 < 0$ $\therefore$ function is maximum at $x = 1$

Minimum
~~Maximum~~ value of function

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 32 - 72 + 48 - 7$

$\boxed{= 1}$

Maximum
~~Minimum~~ value of function

$f(1) = 4(1)^3 - 18(1) + 24(1) - 7$

$= 4 - 18 + 24 - 7$

$\boxed{= 3}$

15 Sketch the curve for the following function

$f(x) = x^2(x+3)$

$f(x) = x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$

Let $f'(x) = 0$

$0 = 3x^2 + 6x$

$3x(x+2) = 0$

$x = 0$ or $x = -2$.

Decreasing Increasing

$-2 \quad 0$

$(-1)^3 + 3(-1)^2$

$-1 + 3$

$2 > 0$

$f''(x) = 6x + 6$.

$f''(0) = 6(0) + 6$

$= 6 > 0$ [Minimum value at $x=0$]

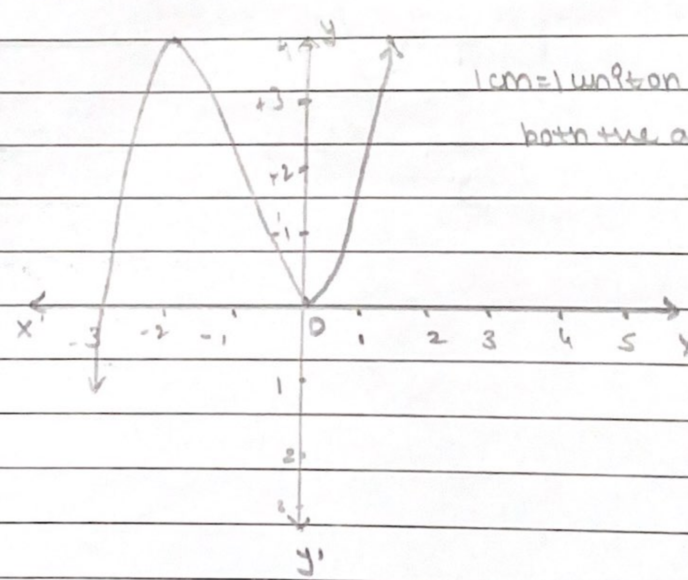
$f''(-2) = 6(-2) + 6$

$= -12 + 6$

$= -6 < 0$ [Maximum value at $x=2$]

Minimum value $= 0(0+3) = 0$ $(0,0)$

Maximum value $= (-2)^2(-2+3) = 4(1) = 4$. $(-2,4)$.



1cm = 1 unit on both the axis

$(1,4)$ $(-3,0)$.

16 Sketch a detailed graph for the following function.

$f(x) = 3x^2 - 6x$

$f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

let $f'(x) = 0$

$6x - 6 = 0$

$x = 1$

Increasing Decreasing

$-\infty$ 1 ∞

$3(-1)^2 - 6(-1)$ Minima

$3 + 6$

$9 > 0$

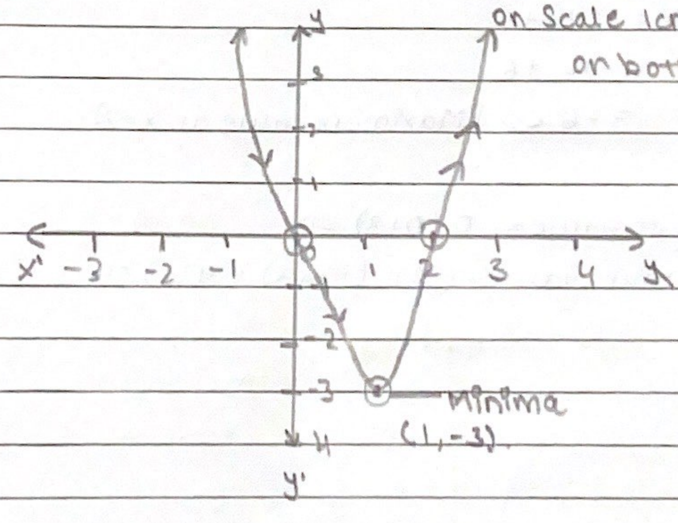
$f''(x) = 6$ M

$f''(1) = 6 > 0$ Minima at $x = 1$

$f(1) = 3(1)^2 - 6(1)$

$= 3 - 6$

$= -3.$



on Scale 1cm = 1unit on both axis.

(-1, 9)

(2, 0)

(3, 9)