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Probability & Statistics

Ans-1 Let x be claim size on a home insurance

$X \sim N(800, 100^2)$

Let Y be claim size on a car insurance

$Y \sim N(1200, 300^2)$

$P(y_1 + y_2 + y_3 > x_1 + x_2 + x_3 + x_4 + 800)$

$P(y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 > 800)$

$y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 \sim N\left(3(1200) - 4(800)\right), 3 \times 300^2 + 4 \times 100^2$

$\therefore \sim N(400, 310000)$

$P\left(Z > \dfrac{800-400}{\sqrt{310000}}\right) = P(Z > 0.718)$

$= 1 - P(Z < 0.718)$

$= 0.23576$

Ans-2 N = Number of claims

$N \sim$ negative binomial $(3, 0.9)$

X = Claim Amount

$X \sim$ Gamma $(6, 2)$

Y = Total Claim Amount

$$M_y(t) = M_n \quad \log(M_x(t))$$

$$M_x(t) = \left(1 - \frac{t}{2}\right)^{-6}$$

$$M_y(t) = M_n\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$\left(M_n(t) = \left(\frac{pe^t}{1-qe^t}\right)^k\right)$$

$$= \left(\frac{(0.9)\left(1-\frac{t}{2}\right)^{-6}}{1-(0.1)\left(1-\frac{t}{2}\right)^{6}}\right)^3$$

$$E(N) = \cancel{\frac{0.3}{0.9}} = \frac{3}{0.9} = 3.33$$

$$V(N) = \frac{k(0.1)}{(0.9)^2} = 3.33 \times \frac{1}{9} = 0.37$$

$$E(x) = \frac{6}{2} = 3 \qquad (\alpha/\lambda)$$

$$V(x) = \frac{6}{4} = 1.5 \qquad (\alpha/\lambda^2)$$

$$V(Y) = E(N)\cdot V(x) + E(x)^2 \cdot V(N)$$

$$= 8.333$$

$$SD(Y) = \sqrt{\frac{25}{3}} = 2.88$$

Ans-34 $G_v(t) = \cancel{\left(\frac{p}{1-qt}\right)} \left(\frac{p}{1-qt}\right)^K$, $p+q=1$

$\therefore$ Distribution is -ve binomial

$P(v=4) = \frac{\sqrt{K+4}}{\sqrt{5}\,\sqrt{K}}\, p^4 q^4$

Ans-9 $Z = x + y$

$E(x) = 3$ $E(y) = 4$

$V(x) = 4$ $V(y) = 1$

i) $Cov(X, Z) = Cov(x, x+y)$

$= Cov(x,x) + Cov(x,y)$

$= V(x) + (-0.6)$

$= 4 - 0.6$

$= \underline{3.4}$

ii) $Var(Z) = Var(x+y)$

$= V(x) + V(y) + 2\,Cov(x,y)$

$= 4 + 1 + 2(-0.6)$

$= \underline{3.8}$

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Ans-10 $f(x,y) = k\, x^{-\alpha} e^{-y/\beta}$

$$1 = k \int_1^{\infty} \int_1^{\infty} x^{-\alpha} e^{-y/\beta}\, dx\, dy$$

$$1 = k \left(\frac{1}{\alpha+1}\right) \left(-\beta e^{-1/\beta}\right)$$

$$k = \frac{-1+\alpha}{\beta e^{-1/\beta}}$$

Ans-8 $\alpha = 3$, $\lambda = 2$

$X \sim \text{Gamma}(3,2)$

$E(Y/X=x) = 3x+1$ $\quad$ $Var(Y/X=x) = 2x^2+5$

$$E(Y) = E(E(Y/X=x))$$
$$= 3E(X) + 1$$
$$= 3\left(\frac{3}{2}\right) + 1 = \underline{11/2}$$

$$V(Y) = E(V(X/Y)) + V(E(X/Y))$$
$$= E(2x^2+5) + V(3x+1)$$
$$= 2E(x^2) + 5 + 9V(x)$$
$$= (2\times 3) + 5 + 9(3/4)$$
$$= \underline{71/4}$$

$V(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \frac{3}{4} + \frac{9}{4}$

$E(x^2) = 3$

Ans-6 $X \sim Poi(\lambda)$, $Y \sim Poi(\mu)$

$M_X(t) = e^{\lambda(e^t-1)}$ $\quad M_Y(t) = e^{\mu(e^t-1)}$

$$M_{X-Y}(t) = E\left(e^{t(X-Y)}\right)$$

$$= \frac{E(e^{tX})}{E(e^{tY})}$$

$$= \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$$

$$= e^{\lambda-\mu(e^t-1)}$$

Hence Proved

$$M_{X+Y}(t) = E\left(e^{t(X+Y)}\right)$$

$$= E(e^{tX}).E(e^{tY})$$

$$= e^{\lambda(e^t-1)}.e^{\mu(e^t-1)}$$

$$= e^{\lambda+\mu(e^t-1)}$$

Hence Proved

Ans - 7

		$\longleftarrow$	X	$\longrightarrow$	
		1	2	3	4
Y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

i) $v(x/y=2) = E(x^2/y=2) - [E(x/y=2)]^2$

• $E(x^2/y=2) = \sum x^2 \frac{P(x=x, y=2)}{P(y=2)}$

$\Rightarrow \underset{(x=1)}{(1^2)} \frac{P(x=1, y=2)}{P(y=2)} + \underset{(x=2)}{4} \frac{P(x=2, y=2)}{P(y=2)} + \underset{(x=3)}{9} \frac{P(x=3, y=2)}{P(y=2)}$

$+ \underset{(x=4)}{16} \frac{P(x=4, y=2)}{P(y=2)}$

$\Rightarrow \frac{1 \times 0}{0.6} + \frac{4 \times 0.3}{0.6} + \frac{9 \times 0.1}{0.6} + \frac{16 \times 0.2}{0.6}$

$\Rightarrow$ 8.8333

• $E(x/y=2) = \sum_x x \frac{P(x=x, y=2)}{P(y=2)}$

$\Rightarrow \frac{1 \times 0}{0.6} + \frac{2 \times 0.3}{0.6} + \frac{3 \times 0.1}{0.6} + \frac{4 \times 0.2}{0.6} = 2.833$

$$V(X/Y=2) = 8.8333 - (2.8333)^2$$
$$= 0.80571110$$

ii) $$f(u,v) = \frac{48}{67}(2uv - v^2)$$

$$E(U/V=v) = ?$$

$$\frac{f(u,v)}{f(v)} = f(u/v=v)$$

$$f(v) = \frac{48}{67}\int_0^1 (2uv - v^2)\,du$$

$$= \frac{48}{67}\left[u^2 v - \frac{v^3}{3}\right]_0^1$$

$$f(v) = \frac{16}{67}[3v-1]$$

$$\frac{f(u,v)}{f(v)} = \frac{\cancel{67}\times 48}{\cancel{67}} \frac{(2v^2 u - v^2)}{(3v-1)16}$$

$$= \frac{3}{(3v-1)}(2uv^2 - v^2)$$

$$E(U/V=v) = \frac{3}{(3v-1)}\int_0^1 u(2uv^2 - v^2)\,du$$

$$= \frac{3}{3v-1}\left[\frac{2v^3v^2}{3} - \frac{v^4}{4}\right]_0^1$$

$$= \frac{3}{3v-1}\left[\frac{8v^2-3}{12}\right]$$

$$= \frac{8v^2-3}{4(3v-1)}$$