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PROBABILITY & STATISTICS

SAKSHI TRIPATHI   ROLL 81   FYBSC - SECT B

Q.1 $\frac{\hat{\lambda} - \lambda}{\sqrt{\hat{\lambda}/n}} \approx N(0,1)$

$\hat{\lambda} = \frac{\Sigma xi}{n}$

$\hat{\lambda} = 200$

$P(-1.96 < z < 1.96) = 0.95$

$P\left(-1.96 < \frac{\hat{\lambda} - \lambda}{\sqrt{\hat{\lambda}/n}} < 1.96\right) = 0.95$

$P\left(-1.96\sqrt{\frac{\hat{\lambda}}{n}} - \hat{\lambda} < -\lambda < 1.96\sqrt{\frac{\hat{\lambda}}{n}} - \hat{\lambda}\right) = 0.95$

$P\left(-1.96\sqrt{\frac{\hat{\lambda}}{n}} + \hat{\lambda} < \lambda < 1.96\sqrt{\frac{\hat{\lambda}}{n}} + \hat{\lambda} = 0.95\right.$

95% confidence interval of λ when $\hat{\lambda} = 200$, $n = 1$

$(200 - 1.96\sqrt{200}, 200 + 1.96\sqrt{200})$
$(172.28141, 227.7185)$

Q.2 $xi = i = 1, 2, \ldots 4$ are n
mean $= \mu$ variance $= \sigma^2$

$\bar{x} = \frac{\Sigma xi}{n}$ (Given)

$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (xi - \bar{x})^2.$

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1. $E[\bar{x}] = E\left[\frac{\Sigma x_i}{n}\right]$

$= \frac{1}{n} E[\Sigma x_i]$

$= \frac{1}{n} E[x_1 + x_2 + x_3 - - - . x_n]$.

$= \mu$.

2. $V[\bar{x}] = V\left[\frac{\Sigma x_i}{n}\right] = \frac{1}{n^2} V[\Sigma x_i]$

$= \frac{1}{n^2}[V[x_1 + x_2 \ldots x_n]$.

$= \frac{1}{n^2} \times k\sigma^2$

$= \sigma^2/n$

ii) To show $E[S^2] = \sigma^2$

$E[S^2] = \frac{1}{n-1} E\left[\sum_{1}^{n}(x_i - \bar{x})^2\right]$

$\frac{1}{n-1} E[\Sigma x_i^2 - n\bar{x}^2]$

$\frac{1}{n-1}\left[E[\Sigma x_i^2] - nE[\bar{x}^2]\right]$

$\frac{1}{n-1}[E[x_1^2 + x_2^2 + \ldots + x_n^2] - nE[\bar{x}^2]]$

Since

$E[x^2] = V(x) - E[x]]^2$

$E[x^2] = \sigma^2 - \mu^2$

Since

$E[\bar{x}^2] = V(\bar{x}) - E[\bar{x}]]^2$

$= \frac{\sigma^2}{n} - \mu^2$

$$\frac{1}{n-1}\left[n\sigma^2 - n\upsilon^2 - \sigma^2 + n\upsilon^2 \right]$$

$$= \frac{\sigma^2 \cancel{(n-1)}}{\cancel{n-1}} = \sigma^2.$$

iii) $X_i \sim N(\mu, \sigma^2)$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$V[S^2] = V[S^2]$$

$$= V\left[\frac{(n-1)S^2}{\sigma^2}\right] \times \frac{\sigma^4}{(n-1)^2}$$

$$= V\left[\chi^2_{n-1}\right] \times \frac{\sigma^4}{(n-1)^2}$$

$$= \frac{2\cancel{(n-1)}\sigma^4}{(n-1)^{\cancel{2}}}$$

$$= \frac{2\sigma^4}{n-1}$$

iv) $P[-0.5\sigma^2 < S^2 < 0.5S^2]$

$$P[-0.5(100) < S^2 < 0.5(100)]$$

$$P[-50 < S^2 < 50]$$

$$P\left[\frac{-50(n-1)}{\sigma^2} < \chi^2_9 < \frac{\cancel{50}\times 9}{\cancel{100}\,2}\right]$$

$$P[-4.5 < \chi^2_9 < 4.5]$$

$$P[\chi^2_9 < 4.5] - P[\chi^2_9 \leq -4.5]$$

$$= \underline{0.1245}$$

3) $X \sim \text{Binom}(4, p)$

$x = 180.$

$$MLE = (P(X=0))^{86} \times P(X=1)^{75} + (P(X=2))^{16} \times (P(X=3))^{2} \times (P(X=4))^{1}$$

$$= \left({}^nC_0\, p^0 (1-p)^n\right)^{86} \left({}^nC_1\, p(1-p)^{n-1}\right)^{75} \left({}^nC_2\, p^2 (1-p)^{n-2}\right)^{16} \left({}^nC_3\, p^3 (1-p)^{n-3}\right)^{2} \left({}^nC_4\, p^4 (1-p)^{4n-4}\right)$$

$$(1-p)^{86n+75n-75+16n-32+2n-6+n-4} \times \text{constant}$$

$$L = (1-p)^{180n-117}\, p^{117} \cdot \text{constant}$$

taking log on both sides

$$\log L = (180n-117)\log(1-p) + 177\log p + \log \text{const}$$

diff wrt p.

$$\frac{dL}{dp} = \frac{-(180n-117)}{1-p} + \frac{117}{p} = 0.$$

$$\frac{117}{p} = \frac{180n-117}{1-p}$$

$$\frac{117}{180n} = p$$

$$\frac{117}{180 \times 4} = \hat{p}$$

$$\hat{p} = 0.1625$$

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Goodness of fit test

H_0 : No of claims confirm to binomial mode

H_1 : No. of claims does not confirme to the binomial model.

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.542	68.724	19.998	2.574	0.108

$P(X=0) = {}^4C_0\, p^0 (1-0.1625)^4 = 0.4919$

$P(X=1) = {}^4C_1 (0.1625)(1-0.1625)^3 = 0.3818$

$P(X=2) = {}^4C_2 (0.1625)^2 (1-0.1625)^2 = 0.1111$

$P(X=3) = {}^4C_3 (0.1625)^3 (1-0.1625)^1 = 0.0143$

$P(X=4) = {}^4C_4 (0.1625)^4 (1-0.1625)^0 = 0.0006$

O_i	86	75	19
E_i	88.542	68.724	22.68

$TS = \frac{\Sigma (O_i - E_i)^2}{E_i} \sim \chi^2_{5-1-2-\cancel{2}}$

$= 1.24322 \sim \chi^2_1$

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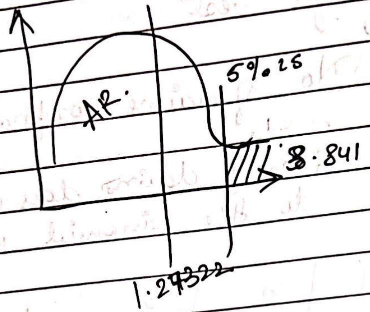

we have insufficient evidence to reject H_0 at LS at 5%. Therefore No of claims confirm to the binomial model.

Q4

Q5 $X \sim U(a,b)$

Sample mean = $\bar{x}$
Sample stand dev = S

$E[x] = \frac{a+b}{2}$ $\qquad V(x) = \frac{(a-b)^2}{12}$

Sample mean = population mean

$\bar{x} = \frac{a+b}{2}$

$2\bar{x} = a+b$

Sample var = Population var

$S^2 = \frac{(a-b)^2}{12}$

$(12S)^{1/2} = a-b$

$a+b = 2x$

$a-b = 2\sqrt{3S^2}$

$2a = 2x - 2\sqrt{3S^2}$

$$\hat{a} = \bar{x} - \sqrt{3}\,s.$$

$$b = 2\bar{x} + a$$
$$\hat{b} = 2\bar{x} + \hat{a}$$
$$\hat{b} = 2\hat{x} - \bar{x} + \sqrt{3}S$$
$$\hat{b} = \bar{x} + \sqrt{3}S$$

iii) Sample of 5 observation

1, 2, 8, 50, 150

$$\frac{\Sigma x_i}{n} = 42.2 \qquad \sigma = 63.57$$

$$\hat{a} = \bar{x} - \sqrt{3}S$$
$$= 42.2 - \sqrt{3}\,(63.57)$$
$$\hat{b} = 152.30684$$

hence we can conclude that value of $\hat{b}$ is approx very close to sample value but $\hat{a}$ is very far from the sample value so it is the potential weakner of MOM.

Q.6.

$$H_0 = p = 0.1$$
$$H_1 = p > 0.1$$

1 P(Type I error = P(Rejecting H_0 / H_0 is true)

X be the no of hits in first 12 missile and Y be no. of hits in step 12 missile

$$P(\text{Type 1 error}) = P(X \geq 3 / p = 0.1)$$
$$+ P(X=0 / P=0.1)\, P(Y \geq 5 / p=0.1)$$
$$+ P(X=1 / P=0.1)\, P(Y \geq 4 / p=0.1)$$
$$+ P(X=2 / p=0.1)$$
$$P(Y \geq 3 / p=0.1)$$

$$= (1-0.8891) + 0.2824(1-0.9957)$$
$$+ (0.6590 - 0.2824)$$
$$(1-0.9744) + (0.8891-0.659)(1-0.889)$$
$$= 0.14727 \approx 15\%$$

iii) Prob of type II error

Accepting H_0 when H_0 is false

$$1 - 0.91253 = 0.08747 \approx 9\%$$

when $(q = 0.3)$

v) $p > 0.278$ at 10% LOS

$H_0 : p = 0.278$ vs $H_1\ p > 0.278$

One sample binomial model

$$\frac{x - np_0}{\sqrt{nq_0 p_0}} \sim N(0,1)$$ with cont correlation

$$\frac{39.5 - 120(0.278)}{\sqrt{120\,(0.278)(0.722)}} = 1.2811$$

we dont have sufficient evidence to reject H_0 at 10% LS.

vi) Lower found of confidence interval implies that there is only a 10% chance of $p \leq 0.2782$ whereas from the hypothesis test 'p' could be less than or equal to 0.2780 with prob more than 10%

vii) $H_0 : \mu_1 = \mu_2$ VS $\mu_1 \neq \mu_2$

$p = \theta$ $\quad \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$

Given

$\bar{x}_1 = 65 \quad \bar{x}_2 = 70 \quad S_1 = 54 \quad S_2 = 70$

$n_1 = 12 \quad n_2 = 15$

$$S_p^2 = \frac{1}{25}\left(11(2916) + 14(4900)\right)$$

$$= 4027.04$$

$$\frac{65 - 70}{63.459\sqrt{\frac{1}{12} + \frac{1}{15}}} = -0.2034$$

There is ± 2.060 ($= t_{25}$; 2.5%) so we have insufficient evidence to reject H_0 at 5% level.

Unbiased estimator

An ubiased estimator is an estimate where experted value is equal to the parameter

Estimator is sd to be unbiased if its bias is equal to zero for all value of parameter θ

Bised

A baised estimator is one which delivers an estimate which is consistently different from the parameter to be estimated

The mean sq error of an estimator $\hat{\theta}$ is defined by $MSE(\hat{\theta}) = Var + Bias^2$

The estimator having less MSE is more consistent

They both would considered be consistant when MSE minimiges such that when $n \to \infty$ $MSE \to 0$

MOM

Maximum likelihood estimator

Q.9

i) Variable t_k is defined as $t_k = \frac{N(0,1)}{\sqrt{\chi^2_k / k}}$

where k denotes the degree of freedom & the two ~~res~~ random var $N(0,1)$ and χ^2_k are independent

ii) Mean and var t_k for $k>2$ are 0 var $= \frac{k}{k-2}$

iii) a) we know that for a sample from a normal population $\frac{\bar{x}-\mu}{S/\sqrt{n}} \sim t_{n-1}$

For given interval $t_9 = 3.25$
confidence interval for μ is this

$$\left(50 - 3.25\sqrt{\frac{48.667}{10}},\ 50 + 3.25\sqrt{\frac{48.667}{10}}\right)$$

i.e $(42.83, 57.17$

b) From (ii)
$$t_{n-1} \sim N\left(0, \sqrt{\frac{n-1}{n-3}}\right) \sim N\left(0, \sqrt{\frac{9}{7}}\right)$$

i.e
$$\frac{\bar{x}-\mu}{S/\sqrt{n}} \sim N\left(0, \sqrt{9/7}\right)$$
$$\frac{\bar{x}-\mu}{S/\sqrt{n}/n} \sim N\left(6, \sqrt{\frac{9}{7}}\right)$$

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i.e

$$\cancel{\frac{\bar{x}-\mu}{s/\sqrt{n}} \sim N\left(0, \sqrt{\frac{9}{7}}\right)}$$

$$\cancel{\frac{\bar{x}-\mu}{s/\sqrt{}}}$$

$$\left(50 - 2.58\sqrt{\frac{48.667}{10}\times\frac{9}{7}} \; ; \; 50 + 2.58\sqrt{\frac{48.667}{10}\times\frac{9}{7}}\right)$$

$(43.54, 56.45)$

Q10 $n = 1000$
$\lambda = 3$

$X \sim Poi(n\lambda)$
$X \sim Poi(3000)$

i) $P(\text{Type 1 error}) = P(\text{Rejection} / H_0 \text{ is true})$
$= P(X > 3100 / X \sim Poi(3000))$

Usin