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Probability and Statistics - Assignment 1

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Q1) (i) Mean $= \dfrac{4+7+9+9+11+15+18+18+18+25}{10}$

$= \dfrac{134}{10} = \boxed{13.4}$

Median $= \dfrac{11+15}{2} = \dfrac{26}{2} = \boxed{13}$

Mode $= \boxed{18}$

S.D. = ?

$\Sigma xi^2 = 16+49+81+81+121+225+324+324+324+625$

$= 2170$

$n = 10$

$\therefore$ S.D. $= \sqrt{\dfrac{1}{10-1} \times 2170 - 10\,(13.4)^2}$

xi	$xi-\bar{x}$	$(xi-\bar{x})^2$
4	-9.4	88.36
7	-6.4	40.96
9	-4.4	19.36
9	-4.4	19.36
11	-2.4	5.76
15	1.6	2.56
18	4.6	21.16
18	4.6	21.16
18	4.6	21.16
25	11.6	134.56
		374.4

$\therefore$ S.D. $= \sqrt{\dfrac{\Sigma(xi-\bar{x})^2}{N-1}}$

$= \sqrt{\dfrac{374.4}{9}}$

$= \boxed{6.449806199}$

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ term $= \frac{11}{4}^{th}$ term $= 2.75^{th}$ term

$= 2^{nd}$ term $+ 0.75$ (3^{rd} term $- 2^{nd}$ term)

$= 7 + 0.75(9-7)$

$= \boxed{8.5}$

$Q_3 = \left[\frac{3(n+1)}{4}\right]^{th}$ term $= 3 \times \frac{11}{4}^{th}$ term $= 3 \times 2.75 = 8.25^{th}$ term

$= 8^{th}$ term $+ 0.25$ (9^{th} term $- 8^{th}$ term)

$= 18 + 0.25(18-18)$

$= 18 + 0.25(0)$

$= \boxed{18}$

$\therefore$ Inter-quartile range $= Q_3 - Q_1 = 18 - 8.5 = \boxed{9.5}$

(ii)

No. of households x_i	No. of people f_i	$x_i f_i$	c.f.	$x_i - \bar{x}$	$f_i(x-\bar{x})^2$
0	5	0	5	-2	20
1	11	11	16	-1	11
2	26	52	42	0	0
3	15	45	57	1	15
4	3	12	60	2	12
	60	120			58

$\therefore$ Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{120}{60} = \boxed{2}$

Median $= \left(\frac{60}{2}\right)^{th}$ term $= 30^{th}$ term $= \boxed{2}$

Mode $= \boxed{2}$

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}} = \sqrt{\frac{58}{60}} = \boxed{0.983192080}$$

$$Q_1 = \left(\frac{\cancel{n+1}}{4}\right)^{th} \text{term} = \frac{60}{4}^{th} \text{term} = 15^{th} \text{term}$$

$\therefore Q_1 = 1$

$$Q_3 = \left(\frac{3n}{4}\right)^{th} \text{term} = \frac{180}{4}^{th} \text{term} = 45^{th} \text{term}$$

$\therefore Q_3 = 3$

$\therefore$ Inter-quartile range $= Q_3 - Q_1 = 3 - 1 = \boxed{2}$

2) $X \sim Poi(0.5) \quad \therefore \lambda = 0.5$

(i) $P(X \leq 0) = \boxed{0.60653}$

(ii) $P(X \geq 1) = 1 - P(X < 1)$

$$= 1 - \frac{e^{-\lambda} \lambda^x}{x!} = 1 - \frac{e^{-0.5} \, 0.5^1}{1} = \boxed{0.69673467}$$

(iii) $P(X = x) = 3\left[P(X \geq 1) + P(X=0) + P(X=0)\right]$

$$= 3(0.69673467 + 0.60653 + 0.60653)$$

$$= 3(1.90979467)$$

$$= \boxed{5.729384010}$$

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Q3) $X \sim \text{Gamma}(2, 0.5)$. Range: $0 < x < \infty$

$\alpha = 2$

$\lambda = 1/2 = 0.5$

(i) Mean $= E(x) = \frac{\alpha}{\lambda} = \frac{2}{0.5} = \boxed{4}$

$S.D = \sqrt{V(x)} = \sqrt{\frac{\alpha}{\lambda^2}} = \sqrt{\frac{2}{0.25}} = \boxed{2.828427}$

(ii) The graph is right skewed

(iii) $F_x(x) = ?$

$$F_x(x) = \int_0^{x} f_x(x)\,dx = \int_0^{x} \frac{\lambda^{\alpha} \cdot e^{-\lambda x} \, x^{\alpha-1}}{\Gamma(\alpha)}$$

$$= \int_0^{x} \frac{(0.5)^2 \cdot e^{-0.5 x} \cdot x^{2-1}}{(2-1)!}$$

$$= \int_0^{x} \frac{0.25 \cdot e^{-0.5} \cdot x}{1}$$

$$= 0.25 \int_0^{x} x \cdot e^{-0.5}\,dx$$

$$= 0.25 \left[x \cdot e^{-0.5} - \int e^{-0.5}\,dx \right]$$

$$= 0.25 \left(x e^{-0.5} - e^{-0.5} \right)$$

$$= 0.25 e^{-0.5} (x-1). = \boxed{0.151632665\,(x-1)}$$

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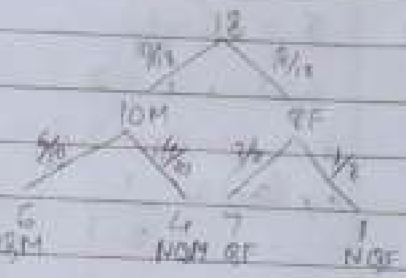
Q4) $n = 18 < \begin{matrix} 10M < \begin{matrix} 6Q \\ 4NQ \end{matrix} \\ 8F < \begin{matrix} 7Q \\ 1NQ \end{matrix} \end{matrix}$

~~PF~~

$P(M) = \frac{10}{18}$ $\quad P(Q \cap M) = \frac{6}{10}$

$P(F) = \frac{8}{18}$ $\quad P(Q \cap F) = \frac{7}{8}$



$\therefore P(X) =$

Q5) $P(L1) = 40\% = 0.4$ $\quad P(L/L1) = 20\% = 0.2$ $\quad$ ~~$P(L1 \cap L) = 0.2 \times 0.4 = 0.08$~~

$P(L2) = 50\% = 0.5$ $\quad P(L/L2) = 40\% = 0.4$ $\quad$ ~~$P(L2 \cap L) = 0.4 \times 0.5 = 0.2$~~

$P(L3) = 10\% = 0.1$ $\quad P(L/L3) = 10\% = 0.1$ $\quad$ ~~$P(L3 \cap L) = 0.1 \times 0.1 = 0.01$~~

By Baye's Theorem:

$$P(L2/L) = \frac{P(L_2) \cdot P(L/L2)}{P(L1) \cdot P(L/L1) + P(L_2) \cdot P(L/L_2) + P(L3) \cdot P(L/L3)}$$

$$= \frac{(0.5)(0.4)}{(0.4)(0.2) + (0.5)(0.4) + (0.1)(0.1)}$$

$$= \frac{0.2}{0.08 + 0.2 + 0.01}$$

$$= \frac{0.2}{0.29} = \boxed{0.689655172}$$

Q6) $X \sim U(1,2)$

$a=1$, $b=2$

$$f_X(x) = \frac{1}{2-1} = \frac{1}{1} = 1$$

$$Y = \frac{3}{6-X}$$

$$6-X = \frac{3}{Y}$$

$$X = 6 - \frac{3}{Y} = w^{-1}(y)$$

$$x = 6 - 3\left(\frac{1}{y}\right)$$

$$\frac{dx}{dy} = \frac{3}{y^2}$$

$$f_Y(y) = f_X(w^{-1}(y)) \left| \frac{d}{dy}(w^{-1}(y)) \right|$$

$$= f_X\left(6 - \frac{3}{y}\right) \cdot \frac{3}{y^2}$$

$$= 1 \cdot \frac{3}{y^2} = \boxed{\frac{3}{y^2}}$$

Q7) ~~$n=5$~~

x	$P(X=x)$	$x \cdot P(X=x)$	$F(x)$	$x^2 \cdot P(X=x)$
1	0.05	0.05	0.05	0.05
2	0.15	0.3	0.35	0.6
3	0.35	1.05	1.48	3.15
4	0.4	1.6	3.0	6.4
5	0.05	0.25	3.25	1.25
		3.25		11.45

(i) $F(3) = \boxed{1.4}$

(ii) $E(X) = \Sigma x \cdot P(X=x) = \boxed{3.25}$

(iii) $V(X) = E(X^2) - E(X)^2$

$= 11.45 - (3.25)^2$

$= 11.45 - 10.5625$

$= \boxed{0.8875}$

(iv)

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5) ~~Let insurance claim amounts be X~~

~~$\therefore P(X > 1000) = 0.2$~~

8) $n = 15$

$p = 0.2 \Rightarrow p + q = 1 \Rightarrow q = 1 - 0.2 = 0.8$

$X \sim Bin(15, 0.2)$

$r = 3$

~~$P(X < 3) = {}^{n}C_r \cdot p^r \cdot q^{n-r} = {}^{15}C_3 \times 0.2^3 \times 0.8^{12}$~~

~~$= \boxed{0.250138895}$~~

$P(X < 3) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{15}C_0 \times 0.2^0 \times 0.8^{15} + {}^{15}C_1 \times 0.2^1 \times 0.8^{14} + {}^{15}C_2 \times 0.2^2 \times 0.8^{13}$

$= 0.035184372 + 0.131941395 + 0.230897442$

$= \boxed{0.398023209}$

Q9) $n = 7$

$p = 0.01 \Rightarrow p + q = 1 \Rightarrow q = 1 - 0.01 = 0.99$

$X \sim -ve\,Bin\,(7, 0.01)$

$r = 3$

$$P(X=3) = {}^{7-1}C_{3-1} \cdot (0.01)^3 (0.99)^{7-3}$$

$$= {}^{6}C_2 \cdot (0.01)^3 \cdot (0.99)^4$$

$$= \boxed{0.0000014409}$$

Q10) $\lambda = \frac{2}{5} = 0.4$

$X \sim Poi\,(2/5)$

$$P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \frac{e^{-2/5} \cdot \frac{2}{5}^0}{0!}$$

$$= 1 - (0.67032 + 0.93845 + 0.99207)$$

$$= 1 - 2.60084$$

$$= -1.60084$$

$$P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - 0.99207$$

$$= \boxed{0.00793}$$

Q11) $P(M) = P(F) = \frac{1}{2}$

Woman has 2 girls and 5 boys

$\therefore P(\text{next 3 are boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{8}}$

Q12) $X \sim Exp(10)$

$\lambda = 10$

$x = 0.1$ days

$f_X(0.1) = \lambda e^{-\lambda x} = 10 \cdot e^{-10(0.1)} = 10\, e^{-1} = \boxed{3.678794412}$

Q13) $X \sim U(75, 120)$ $\quad a = 75;\ b = 120$

$f_X(x) = \dfrac{1}{b-a} = \dfrac{1}{120-75} = \dfrac{1}{45}$

$\therefore P(80 < x < 100) = F_X(100) - F_X(80)$

$= \dfrac{100}{45} - \dfrac{80}{45}$

$= \dfrac{20}{45} = \dfrac{4}{9} = \boxed{0.44444}$

Q14) $X \sim gamma(5, 0.4)$ $\quad \lambda = 0.4,\ \alpha = 5$

$P(X < 22.89) = ?$

$2\lambda X \sim \chi^2_{10} \Rightarrow 0.8X \sim \chi^2_{10}$

$\therefore P(X < 22.89) = P(0.8X < 18.312)$

$= P(\chi^2_{10} < 18.312)$

$= \boxed{0.9499296}$

0.5 $\quad$ 0.007

$P(\chi^2_{10} < 18) = 0.9450$

$P(\chi^2_{10} < 18.312) = 0.9499296$

$P(\chi^2_{10} < 18.5) = 0.9529$

$0.5 \rightarrow 0.0079$

$1 \rightarrow \dfrac{0.0079}{0.5}$

$\therefore 0.312 \rightarrow \dfrac{0.0079}{0.5} \times 0.312$

$= 0.0049296$

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Q15) $f_X(x) = \frac{k}{(x+5)^4}$; $x > 0$

(i) $\int_0^\infty f(x)\,dx = 1$

$k \int_0^\infty (x+5)^{-4}\,dx = 1$

$\therefore \frac{k}{-3}\left[(x+5)^{-3}\right]_0^\infty = 1$

$k\left[\frac{1}{(\infty+5)^3} - \frac{1}{5^3}\right] = -3$

$\therefore k\ \underline{1}$