Probability & Statistics

S1] i) Mean $= \frac{\Sigma x}{n} = \frac{134}{10} = 13.4$

Median $= \left(\frac{n+1}{2}\right)^{th} = \left(\frac{10+1}{2}\right)^2 = 5.5^{th}$

$= 11 + (15-11)(0.5) = 13$

Mode $= 18$

S.D $= \sqrt{\frac{\Sigma(\cancel{\Sigma}x^{\cancel{2}} - \bar{x})^2}{n-1}}$

$= \sqrt{\frac{(88.36 + 40.96 + 19.36 + 19.36 + 5.76 + 2.56 + 21.16 + 21.16 + 21.16 + 134.56)}{9}}$

$= \cancel{31.836}\ 6.45$

IQR $= Q_3 - Q_1$

$= 3\left(\frac{n+1}{4}\right)^{th} - \frac{1}{4}(n+1)^{th}$

$= (8.25)^{th} - (\cancel{3}2.75)^{th}$

$= 18 - [7 + (9-7)0.75]$

$= 18 - 8.5 = 9.5$

ii) Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{120}{60} = 2$

[Normal dist]
mean = median = ~~mode~~

Median $= \frac{(n+1)^{th}}{2} = 30.5^{th} = 2$

Mode $= 2$

$2 \times x \times \frac{\sum x}{n}$ $\cancel{x^2 - 2x\bar{x} + \bar{x}^2}$

Page :
Date :

$$S.d = \sqrt{\frac{1}{n-1} \sum f x^2 - \bar{x}^2 n} \quad // \quad \sqrt{\frac{1}{n-1} (\sum f (x - \bar{x})^2}$$

$$= \sqrt{58/59} = 0.9915$$

$$IQR = Q_3 - Q_1$$
$$= \frac{3}{4}(n+1)^{th} - \frac{1}{4}(n+1)^{th}$$
$$= \frac{3}{4}(61)^{th} - \frac{1}{4}(61)$$
$$= 45.75^{th} - 15.25^{th}$$
$$= 3 - 1 = 2$$

Q2] [No. of claims per year] $X \sim Poi(\lambda = 0.5)$

i) $P(X=0) = \frac{e^{-0.5} \times 0.5^0}{0!} = e^{-0.5} = 0.60653$

ii) $[P(X \geq 1) \cdot P(X=0) \cdot P(X=0)] \times 3$ if ~~claim~~ one or more claim in 1 year

$$[(0.60653)^2 \times [1 - P(X \leq 0)]] \times 3$$

$$[(0.60653)^2 \times [1 - 0.60653]] \times 3$$

$$= 0.43425$$

iii) As one claim has occurred and we have to find time till the next claim we will use exponential distribution

~~P(2.F.Q.F | IMFF)~~
~~= 7|51~~
~~10|11 · 8|18~~

Page :
Date :

$$X \sim Exp(\cancel{\lambda = 0.5}) \quad \lambda = 0.5$$

~~E(X) = 1/λ~~
~~1/λ = 0.5~~
~~λ = 1/0.5~~

$$P(X \geq 2) = 1 - P(X \leq 2)$$
$$= 1 - [1 - e^{-\lambda x}]$$
$$= e^{-\lambda x}$$
$$= e^{-0.5 \times 2} = \cancel{0.018316} \; 0.367879$$

Q3] $X \sim Gamma(\alpha = 2, \lambda = 1/2)$ $\quad 0 < x < \infty$

i) $E(X) = \dfrac{\alpha}{\lambda} = \dfrac{2}{1/2} = 4$

$V(X) = \dfrac{\alpha}{\lambda^2} = \dfrac{2}{(1/2)^2} = 8$

$S.D(X) = \sigma = \sqrt{8} = 2.8284$

ii) The distribution seems to be skewed due to a high variance of 8 and as the mean = 4 is to close to range $0 < x < \infty$ we can assume it to be positively skewed.

iii) $F_X(x) = \int_0^{x} f_X(x)\,dx$

$$= \int_0^{x} \frac{0.5^{(2)}}{\lceil 2} x^{(2-1)} e^{-x/2}$$

$$0.5^{(2)} \int_0^{x} x \cdot e^{-x/2}\,dx$$

Camlin

$$u\int v\,du - \int \frac{du}{dx}\cdot\int v\,du$$

$$= 0.5^{(2)} \cancel{\int_0^\infty}\left[x\int_0^\infty e^{-x/2}\,dx - \int_0^\infty \frac{d(x)}{dx}\cdot\int_0^\infty e^{-x/2}\,dx\right]_0^x$$

$$= 0.5^2\left[\cancel{x\cdot[e}\; x\cdot\frac{e^{-x/2}}{-1/2} - \int(1)\,\frac{e^{-x/2}}{-1/2}\,dx\right]_0^x$$

$$= 0.5^2\left[x\cdot e^{-x/2}\cdot(-2) - \frac{e^{-x/2}}{-1/2\times-1/2}\right]_0^x$$

$$= 0.5^2\left[-2xe^{-x/2} - 4e^{-x/2}\right]$$

$$= 0.5^2\left[(-2xe^{-x/2} - 4e^{-x/2}) - (-4)\right]$$

$$= 0.5^2\left[-2xe^{-x/2} - 4e^{-x/2} + 4\right]$$

$$= 0.5^2\left[\frac{-2x}{\sqrt{e^x}} - \frac{4}{\sqrt{e^x}} + 4\right]$$

$$= 1 - \frac{0.5x}{\sqrt{e^x}} - \frac{1}{\sqrt{e^x}}$$

Q4 ?

~~10 = male~~ ~~8 = female~~ $M = 10$ $F = 8$

~~6 = f.q~~ ~~7 = f.q~~ $P(M) = 10/18$ $P(F) = 8/18$

$P(F.Q|M) = 6/10$ $P(F.Q|F) = 7/8$

~~$P(Q.F|F) = \frac{P(F\cap QF)}{P(F)}$~~ $P(2Q.F.F|{}^2F)$

1F 1M ~~$= \frac{7/18\times 6/10}{8/18} = 7/8\times 6/187 = 42$~~ $= \frac{7}{\cancel{8}3}\times\frac{\cancel{6}^1}{17}$

7/51 $P(2F) = \frac{8}{18}\times\frac{7}{17}$ $= \frac{7/51}{28/153}$

$= 56/306 = 28/153$ $= 3/4$

$\boxed{=0.75}$

Q5] $P(L1) = 0.4$ $\qquad P(L|L_1) = 0.2$

$P(L2) = 0.5$ $\qquad P(L|L_2) = 0.4$

$P(L3) = 0.1$ $\qquad P(L|L_3) = 0.1$

$$P(L2|L) = \frac{P(L|L_2) \cdot P(L_2)}{\sum_{i=1}^{3} P(L|L_i) \cdot P(L_i)}$$

$$= \frac{0.4 \times 0.5}{(0.4 \times 0.2) + (0.5 \times 0.4) + (0.1 \times 0.1)}$$

$$= \frac{0.2}{0.08 + 0.2 + 0.01}$$

$$= \frac{0.2}{0.29} = 20/29$$

$$= 0.689655$$

Q6] $X \sim U(1,2)$ $\qquad \cancel{X = f_X(x) = \frac{1}{b-a}}$ $a<x<b$

$Y = 3/6-x$

$\cancel{f_Y(y) = ?}$

$\cancel{F_X(x) = \frac{x-a}{b-a}}$ $\cancel{a<x<b}$

$=$

$F_Y(y) = P(Y \le y) = P\left(\frac{3}{6-x} \le y\right)$ $\qquad \cancel{3 = x = 6y}$

$\cancel{x = 3\ 6y}$

$= P(x \le 6 - 3/y)$ $\qquad \frac{3}{y} = 6 - x$

$6 - \frac{3}{y}$

$\therefore F_Y(y) = F_X\,\cancel{(x)}\,(6 - 3/Y)$

To find PDF we differentiate

$\cancel{F_X'(6-3/y)} = \cancel{\frac{d}{d}\,6}$ $\quad F_Y'(y) = f_Y(y)$ $\quad \frac{6y-3}{y}$

$= F_X'(g^{-1}(y))$

$x = 6 - 3y$

$$\frac{dx}{dy} = 0 - \frac{3(-1)}{y^2} = 3/y^2$$

$$\therefore f_X(x) = \frac{1}{2-1} = 1$$

$$\therefore f_Y(y) = (1) \cdot \frac{3}{y^2}$$

$$= 3/y^2$$

Q7] i) $F(3)$

$$= P(X=1) + P(X=2) + P(X=3)$$

$$= 0.05 + 0.15 + 0.35$$

$$= 0.55$$

ii) $$E(X) = \sum_{x=1}^{5} x\,P(X=x)$$

$$= 3.25$$

iii) $$V(X) = E[X^2] - (E[X])^2$$

$$= 11.45 - (3.25)^2$$

$$= 0.8875$$

iv) ~~Skew~~ (coeffecient of skewness)

$$= \frac{3(\text{Mean} - \text{Median})}{\sigma}$$

$$= \frac{3(3.25 - 3)}{\sqrt{0.8875}} = 0.796117$$

Q8]

~~p=0.2~~ $X \sim Bin(15, 0.2)$

$P(X<3) = \cancel{{}^{15}C_2 \times 0.2}$

$P(X=0) + P(X=1) + P(X=2)$

$= {}^{15}C_0 \cdot (0.2)^0 (0.8)^{15}$

$+ {}^{15}C_1 \cdot (0.2)^1 (0.8)^{14}$

$+ {}^{15}C_2 \cdot (0.2)^2 (0.8)^{13}$

$= 0.035184 + 0.131941 + 0.230897$

$= 0.398022$

Q9]

$\cancel{X \sim Bin(7, 0.01)}$ $\cancel{P(X=3) = {}^7C_3 \cdot 0.01^3}$

$\cancel{P(X=7) = {}^6C_2 \; 0.01^3 (0.99)^4}$ 7 trials 3 success

$X \sim NBin(3, 0.01)$

$P(X=7) = {}^6C_2 \cdot (0.01^3) \cdot (0.99)^4$

$= 0.000014409 \;//\; 1.4409 \times 10^{-5}$

Q10]

$\lambda = 2$ per working week (5 days)

$\lambda = 2/5$ for one day

$X \sim Poi(\lambda = 2/5)$

$P(X>2) = 1 - P(X \leq 2)$

$= 1 - 0.99207 = 0.00793$

Page :
Date :

Q11] ~~P(X=M,F)~~

$P(\text{Next 3 are boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

as having a boy ~~and~~ or girl is independent of previously having a boy or girl and each other.

$= \frac{1}{8}$

Q12] $X \sim Poi$ (10 per day)

Waiting time between 2 claims hence,

$X \sim Exp(\lambda = 10)$

$P(X < 0.1) = 1 - e^{-\lambda x}$

$= 1 - e^{-10 \times 0.1}$

$= 1 - e^{-1}$

$= 0.63212$

Q13] $X \sim U(75, 120)$ as if we find indivival f_x is is constant due to equal probability density in unifor dist.

$P(80 < X < 100) = F(100) - F(80)$

$= \frac{100-75}{120-75} - \frac{80-75}{120-75}$

$= \frac{25-15}{45} = \frac{10}{45} = 0.2222\bar{2}$.

Q14] $X \sim Gamma(5, 0.4)$

$P(X < 22.89)$

$P(2\lambda X < 22.89 \times 2 \times 0.4)$

$P(\chi^2_{10} < 18.312) =$

0.94974

18	0.9450
18.5	0.9529

$\frac{0.9529(18.3-18) + 0.9450(18.5-18.3)}{(18.5-18)}$

Q15] i) $\int_0^{\infty} \frac{k}{(x+5)^4} dx = 1$

$k \int_0^{\infty} (x+5)^{-4} dx = 1$

$k \left[\frac{(x+5)^{-3}}{-3} \right]_0^{\infty} = 1$

$x+5 = t$

$1 = \frac{dt}{dx}$

$dx = dt$

$k \left[0 + \frac{(5)^{-3}}{3} \right] = 1$

$k [1/375] = 1$

$k = 375$

ii) $\int_0^{10} \frac{375}{(x+5)^4} dx$

$\int_0^{10} 375 \cdot (x+5)^{-4} dx$

$375 \left[\frac{(x+5)^{-3}}{-3} \right]_0^{10}$

$375 \left[\frac{(15)^{-3}}{-3} + \frac{(5)^{-3}}{3} \right]$

$\cancel{375[} = 0.96296$

iii) $E(x) =$

$\int_0^{\infty} x \cdot \frac{375}{(x+5)^{4}} dx$

$375 \int_0^{\infty} \frac{x}{(x+5)^{4}} dx$

~~375~~ $x+5 = t$ $\quad x = t-5$

$1 = \frac{dt}{dx}$

$dx = dt$

$375 \int_5^{\infty} \frac{t-5}{t^4} dt$

$375 \int_5^{\infty} t^{-3} - 5 \cdot t^{-4} dt$

$375 \left(\left[\frac{t^{-2}}{-2} \right]_5^{\infty} - \left[\frac{5t^{-3}}{-3} \right]_5^{\infty} \right)$

$375 \left(\left(0 + \frac{1}{2 \times 25}\right) - \left(0 + \frac{5 \times 1}{3 \times 125}\right) \right)$

$375 \left(1/50 - 1/75 \right)$

$= 2.5$