P&S Assignment

Q.1

→ i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25.

$$\text{Mean} = \bar{x} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$= 13.4$$

ii)

no. of households (x_i)	no. of people (f_i)	cf	$f_i x_i$
0	5	5	0
1	11	16	11
median class → 2	26	42	52
3	15	57	45
4	3	60	12
	$\Sigma f_i = 60 = N$		$\Sigma f_i x_i = 120$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{120}{60} = 2.$$

$$\text{Median} = l + \left(\frac{N/2 - cf}{f}\right) \times h$$

$$= 1.5 + \left(\frac{30-16}{26}\right) \times 1$$

$$= 1.5 + \frac{14}{26}$$

$$= 2.0384$$

$$\text{Mode} = 3\,\text{median} - 2\,\text{mean} = 3(2.0384) - 2(2)$$

$$= 2.1163.$$

$Q_1 \Rightarrow 15^{th}$ obs $\quad Q_1 = L_1 + \left(\frac{\frac{1}{4}N - F}{f_{Q_1}}\right) C$

$= 0.5 + \left(\frac{15 - 5}{11}\right) \times 1$

$= 1.409$

$Q_3 \Rightarrow 45^{th}$ obs $\quad Q_3 = L_3 + \left(\frac{\frac{3}{4}N - F_3}{f_{Q_3}}\right) C.$

$= 2.5 + \left(\frac{45 - 42}{15}\right) \times 1$

$= 2.7$

$\therefore IQR = Q_3 - Q_1 = 2.7 - 1.409 = 1.291$

32

→ i) 0 claims in 1 year $\quad X \sim Poi(0.5)$

$P(X=0) = \frac{e^{-\lambda}\lambda^x}{0!} = \frac{e^{-0.5}(0.5)^0}{1} = e^{-0.6} = 0.60653$

ii) $P(X \geq 1) = 1 - P(X=0) = 1 - 0.60653 = 0.39347$

Binomial dist →

$P(X < 1) = {}^nC_x \, p^x (1-p)^{n-x}$

$= {}^3C_1 \, p^1 (1-p)^{3-1}$

$= 6 \times (0.39347) \times (0.60653)^2$

$= 0.43424$

iii) $P(x>2) = 1 - P(x \leq 2) = 1 - (1 - e^{-\lambda x}) = e^{-\lambda x}$

$= e^{-0.5 \times 2}$

$= e^{-1}$

$= 0.36787$

Q.4

$\longrightarrow$ $P\left(\dfrac{\text{two qualified females}}{\text{at least one female}}\right)$

Q.3

$\longrightarrow$ $\alpha = 2$, $\lambda = 1/2$ $\qquad 0 < x < \infty$

$f(x)^2 = \text{variance}$

$f(x) = \sqrt{\text{var}}$

$= \sqrt{\dfrac{\alpha}{\lambda^2}}$

$= \sqrt{\dfrac{2}{(1/2)^2}}$

$= \sqrt{8}$

$= 2\sqrt{2}$

Q.5

$\longrightarrow$ $E_1 \rightarrow$ arrives through 1. $\qquad$ $A \rightarrow$ event that the package arrived

$E_2 \rightarrow$ arrives through 2.

$E_3 \rightarrow$ arrives through 3.

by bayes theorem $\rightarrow$

$$P(E_2|A) = \frac{P(A|E_2) \cdot P(E_2)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)}$$

$$= \frac{\frac{50}{100} \times \frac{40}{100}}{\frac{40}{100} \times \frac{20}{100} + \frac{50}{100} \times \frac{40}{100} + \frac{10}{100} + \frac{10}{100}}$$

$$= \frac{\frac{20}{100}}{\frac{29}{100}}$$

$$= \frac{20}{29}.$$

Q.7.

→

X	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

i) $F(3) = 0.35$

ii) $E(X) = \Sigma P(X=x) \cdot x dx$

$= 1 \times 0.05 + 2 \times 0.15 + 3 \times 0.35 + 4 \times 0.4 + 5 \times 0.05$

$= 0.05 + 0.3 + 1.05 + 1.6 + 0.25$

$= 3.25$.

iii) $V(X) = E(x)^2 - [E(x)]^2$

$= [1 \times 0.05 + 4 \times 0.15 + 9 \times 0.35 + 16 \times 0.4 + 25 \times 0.05] - (3.25)^2$

$= [0.05 + 0.6 + 3.15 + 6.4 + 1.25] - (3.25)^2$.

$= 0.8875$

Q.10

→ $\lambda = \frac{2}{5}$

$x = 2$

$\therefore P(x \geq 2) = \lambda e^{-\lambda x} = \frac{2}{5} e^{-2/5 \times 2}$.

$= \frac{2}{5} e^{-4/5} = 0.18$

Q.11

$\longrightarrow$ $P(M) = P(F) = \frac{1}{2}$.

Woman has 2 girls & 5 boys.

$P(\text{next 3 children are boys}) = \frac{1}{2} \times \left(\frac{1}{2}\right)^{3-1}$

$= \frac{1}{8}$.

Q.12.

$\longrightarrow$ $\lambda = 10$. $x = 0.1$

$P(x) = \lambda e^{-\lambda x} = 10e^{-1}$

$= 3.679$.

Q.13.

$\longrightarrow$ $\underset{(a)}{75} < x < \underset{(b)}{120}$.

$PDF = \frac{1}{b-a} = \frac{1}{120-75} = \frac{1}{45}$.

$P(80 < x < 100) = \frac{100-80}{45} = \frac{\cancel{20}^{4}}{\cancel{45}_{9}} = \frac{4}{9}$.