Probability & Statistics

Assignment - 1

1) i) 4, 7, 9, 9, 11, 15, 18, 18, 25

$$\text{Mean} = \bar{x} = \frac{4+7+9+9+11+15+18+18+25}{9}$$

$$= \frac{116}{9} = 12.88$$

$$\text{Median} = \left(\frac{9+1}{2}\right)^{th} = \left(\frac{10}{2}\right)^{th} = 5^{th} = 11$$

Mode $\rightarrow$ 9 and 18

$$\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{N}} = \cancel{\sqrt{8.88+5.88+3.88+3.88+1.88+}}$$

$$\cancel{(4-\bar{x})}, \quad \Sigma(x_i-\bar{x})^2 = (-8.88)^2 + (-5.88)^2 + (-3.88)^2 + (-3.88)^2 + (-1.88)^2 + (2.12)^2 + (5.12)^2 + (5.12)^2 + (12.12)^2$$

$$= 350.8896$$

$$\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{N}} = \sqrt{\frac{350.8896}{9}} = 6.244$$

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} = (2.5)^{th} = \frac{7+9}{2} = 8$$

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{th} = (7.5)^{th} = \frac{18+18}{2} = 18$$

$$IQR = Q_3 - Q_1 = 18 - 8 = 10$$

ii)

x_i	f_i	$x_i f_i$	C.F	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
0	5	0	5	2.3	11.5
1	11	11	16	1.3	14.3
2	26	52	42	0.3	7.8
3	15	45	57	0.7	10.5
4	13	52	70	1.7	22.1
	70	160			66.2

$$\bar{x} = \frac{\Sigma x_i f_i}{N} = \frac{160}{70} = 2.2857 \approx 2.3$$

$$\text{Median} = 2\left(\frac{N+1}{4}\right)^{th} = \left(\frac{N+1}{2}\right)^{th} = \left(\frac{71}{2}\right)^{th}$$

$$= (35.5)^{th} = 2$$

Mode $\rightarrow 2$

$$Q_1 = \left(\frac{N+1}{4}\right)^{th} = \left(\frac{71}{4}\right)^{th} = (17.75)^{th} = 2$$

$$Q_3 = 3\left(\frac{N+1}{4}\right)^{th} = 3\left(\frac{71}{4}\right)^{th} = (53.25)^{th} = 3$$

$$IQR = Q_3 - Q_1 = 3 - 2 = 1$$

$$\sigma = \frac{\Sigma f_i \lvert x_i - \bar{x}\rvert}{\Sigma f_i} = \frac{66.2}{70} = 0.94571$$

2) i) $X \sim Poi(0.5)$

$$P(N=0) = \frac{e^{-0.5} \times (0.5)^0}{0!}$$

$$= e^{-0.5}$$

$$= 0.606530$$

ii) $P(N \geq 1) = 1 - P(N=0)$

$$= 1 - 0.60653$$

$$= 0.39347$$

$X \sim Bin(3, 0.39347)$

$$P(X=1) = {}^3C_1 \times 0.39347 \times (0.60653)^2$$

$$= 0.43425$$

iii) $P(T>2) = 1 - F(2)$

$$= 1 - [1 - e^{-0.5 \times 2}]$$

$$= 1 - [1 - e^{-1}]$$

$$= e^{-1} = 0.367879$$

4) $$P(Q|F) = \frac{P(Q \cap F)}{P(F)} = \frac{{}^7C_1 / {}^{18}C_1}{8/18}$$

$$= \frac{7}{18} \times \frac{18}{8}$$

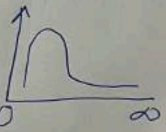
$$= \frac{7}{8}$$

3) i) $X \sim \text{Gamma}(2, 1/2)$

$$S.D = \sqrt{V(X)} = \sqrt{\frac{\alpha}{\lambda^2}} = \sqrt{\frac{2}{\left(\frac{1}{2}\right)^2}} = \sqrt{8} = 2\sqrt{2} = 2.828427$$

$$\text{Mean} = E(X) = \frac{\alpha}{\lambda} = \frac{2}{0.5} = 4$$

ii) Since, X cannot take -ve values, therefore, it is +vely skewed.



iii) $F_X(x) = P(X \leq x) = \int_0^x$

5) $$P(L/2) \Rightarrow \frac{\frac{50}{100} \times \frac{40}{100}}{\frac{40}{100} \times \frac{20}{100} + \frac{50}{100} \times \frac{40}{100} + \frac{10}{100} \times \frac{10}{100}}$$

$$\Rightarrow \frac{20}{8+20+1} = \frac{20}{29}$$

$$\left(\frac{P(2) \cdot P(2/L)}{P(1) \times P(1/L) + P(2) \times P(2/L) + P(3) \times P(3/L)} \right)$$

5) $X \sim U(1,2) \Rightarrow \cancel{x}\ 0 < X < 1$

$Y = \dfrac{3}{6-X}$ $\quad \cancel{y} \Rightarrow X = 0,\ Y = \dfrac{1}{2}$

$\quad X = 1,\ Y = \dfrac{3}{5}$

PDF of Y = ?

$\cancel{F_X(x)} =$ $\dfrac{1}{2} < Y < \dfrac{3}{5}$; $Y \sim U\left(\dfrac{1}{2}, \dfrac{3}{5}\right)$

PDF of $f(y) = \dfrac{1}{\frac{3}{5} - \frac{1}{2}} = \dfrac{1}{\frac{6-5}{10}} = 10$

7) i) $F_X(3) = P(X \le 3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

ii) $E(X) = \sum_{i=1}^{5} x . P(X=x)$

$= 1 \times 0.05 + 2 \times 0.15 + 3 \times 0.35$
$+ 4 \times 0.4 + 5 \times 0.05$

$= 0.05 + 0.3 + 1.05 + 1.6 + 0.25$

$= 3.25$

iii) $V(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \sum_{i=1}^{5} x^2 . P(X=x)$

$= 1 \times 0.05 + 4 \times 0.15 + 9 \times 0.35$
$+ 16 \times 0.4 + 25 \times 0.05$

$= 0.05 + 0.60 + 3.15 + 6.40 + 1.25$

$= 11.45$

$V(X) = 11.45 - (3.25)^2$

$= 0.8875$

iv) ~~Bin(15, 0.2)~~ Coeff. of Skewness =

8) Bin(15, 0.2)

$P(X < 3) = P(X=0) + P(X=1) + P(X=2)$

$\Rightarrow {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)(0.8)^{14}$

$+ {}^{15}C_2 (0.2)^2 \times (0.8)^{13}$

$= (0.3)^{13} \left[(0.8)^2 + 15 \times 0.2 \times 0.8 + 15 \times 7 \times (0.2)^2 \right]$

$= (0.3)^{13} \times 7.24$

$= 1.1542899 \times 10^{-6}$

9) $n=7,\ x=3,\ p=0.01$

Using Negative Binomial

$$P(X=3) = {}^{6}C_{2}(0.01)^3(0.99)^4$$

$$= \cancel{0.0014408\,94} = 1.440894 \times 10^{-5}$$

15) $\cancel{P(\ T<0.1)} =$

i) $P.D.F = f_X(x) = \dfrac{k}{(x+5)^4},\ x>0$

$$\int_0^\infty \frac{k}{(x+5)^4}\,dx = 1 \Rightarrow k\int_0^\infty \frac{1}{(x+5)^4}\,dx = 1 \Rightarrow -5k\left[\frac{1}{(x+5)^5}\right]_0^\infty = 1$$

$$\Rightarrow -5k\left[\frac{1}{\infty} - 0 - \frac{1}{5^5}\right] = 1 \Rightarrow \frac{k}{5^4} = 1 \Rightarrow k = 5^4$$

ii) $F(10) = F(X \le 10) = \int_0^{10} f_X(x)\,dx = \int_0^{10} \frac{k}{(x+5)^4}\,dx$

$$\rightarrow \left[\frac{1}{(x+5)^5}\right]_0^{10} \cdot (-5k) = -5^5 \times \left[\frac{1}{15^5} - \frac{1}{5^5}\right] = 1 - \frac{1}{3^5}$$

11) $\dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{8} = 0.125$

12) $\lambda = 10$

$$P(T<0.1) = \int_{0.1}^{\infty} 10e^{-10x}\,dx = \frac{10}{10}\left[-e^{-10x}\right]_{0.1}^{\infty}$$

$$= -e^{-\infty} + e^{-1} = e^{-1} = \cancel{2.7182}$$

$$= 0.3678794$$

13) $X \sim U(75, 120)$

$PDF \to f(x) = \frac{1}{120-75} = \frac{1}{45}$ for $75 < x < 120$

~~P(80, 100~~

$P(80 < X < 100) = \frac{100-80}{45} = \frac{20}{45} = \frac{4}{9}$

$= 0.44$

15) iii) $E(X) = \int_0^\infty \frac{x \cdot k \cdot dx}{(x+5)^4} =$

$x+5 = t$
$dx = dt$

$= \int_5^\infty \frac{t-5}{t^4} dt = \int_5^\infty t^{-3} - 5t^{-4} \cdot dt$

$= \left[\frac{t^{-2}}{-2} - \frac{5t^{-3}}{-3}\right]_5^\infty$

$= [0-0] - \left[\frac{-1}{2 \times 5^2} + \frac{5}{3} \times \frac{1}{5^3}\right]$

$= \frac{1}{50} - \frac{1}{75} = \frac{1}{25}\left[\frac{1}{2} - \frac{1}{3}\right]$

$= \frac{1}{25} \times \frac{1}{6} = \frac{1}{150}$

13)

10) The No. of claims X per day has $Poi(\frac{2}{5})$

$$P(X>2) = 1 - P(X<2)$$

$$\Rightarrow 1 - [P(X=0) + P(X=1)]$$

$$= 1 - [e^{-2/5} + e^{-2/5} \times 2/5]$$

$$= 1 - e^{-2/5}[1 + 2/5]$$

$$= \cancel{1 - \frac{9}{7} e^{-2/7}} = 1 - \frac{7}{5} e^{-2/5}$$

$$= \cancel{0.03381490} = 0.06155193$$

$X \sim U(75, 120)$

14) $X \sim \text{Gamma}(5, 0.4)$; $\alpha = 5, \lambda = 0.4$

$$P(X < 22.89) = F_X(22.89) = \int_{L.L}^{x} \frac{\lambda^{\alpha} \cdot e^{-\lambda x} \cdot x^{\alpha-1}}{\Gamma(\alpha)} dx$$

$$F_X(22.89) = \int_{0}^{22.89} \frac{(0.4)^5 \cdot e^{-0.4x} \cdot x^4}{\Gamma(4)}$$

$\cancel{5.78}\ \overset{0.8}{\cancel{22.89}}\ X \sim \chi^2_{\nu=10}$ $\qquad \lambda = 0.4$

$\qquad 2\lambda = 0.8$

$$P(X < 22.89) = P(0.8X < 18.312)$$

$$= P(\chi^2_{10} < 18.312)$$

$18 \Rightarrow 0.945$

$18.5 \Rightarrow 0.9529$

$0.5 \Rightarrow 0.079$

$0.312 \Rightarrow \frac{0.079}{0.5} \times 0.312$

$\Rightarrow 0.049296$

$\Rightarrow 0.945 + 0.049296$

$\Rightarrow 0.994296$