Daweeka Ranawat
Roll No. : 59

Calculus Assignment
Chapter 5, 6, 7

Answer 1 -

Let $w^{-1} = t$

Then $\frac{-1}{w^2} dw = dt$

limit changes from 2 to $(2)^{-1} = \frac{1}{2}$ (~~lower~~ upper limit)

$1/50$ to $(1/50)^{-1} = 50$ (lower limit)

$$\therefore \int_{1/50}^{2} \frac{e^{2/w}}{w^2} dw = -\int_{50}^{1/2} e^{2t} dt = \int_{1/2}^{50} e^{2t} dt = \left[\frac{e^{2t}}{2}\right]_{1/2}^{50} = \frac{e^{100} - e^{1}}{2}$$

Ans : $\frac{e^{100} - e^{1}}{2} = 1.34405 \times 10^{43}$

Answer 2 -

$$\int \frac{2t^3 + 1}{(t^4 + 2t)^3} dt$$

Let $t^4 + 2t = u$

$4t^3 + 2 \, dt = du$

$$\frac{1}{2} \int \frac{1}{u^3} du = \frac{1}{2}\left[\frac{u^{-2}}{-2}\right] = -\frac{1}{4u^2} + c = \frac{-1}{4(t^4+2t)^2} + c$$

Answer 3 -

$f'(x) = x^4 + 3x - 9$

$$f(x) = \int f'(x) dx = \int x^4 + 3x - 9 \, dx = \frac{x^5}{5} - \frac{3x^2}{2} - 9x + c$$

Answer 4 -

$$f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$$

$$\int_{-2}^{3} f(x) dx = \int_{-2}^{1} f(x) dx + \int_{1}^{3} f(x) dx = \int_{-2}^{1} 3x^2 dx + \int_{1}^{3} 6 \, dx = \left[x^3\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= [1 - (-8)] + 12 = 21$$

Answer 5 - $\int 3(8y-1)\,e^{4y^2-y}\,dy$

Let $4y^2-y=t$

$(8y-1)\,dy = dt$

$\int 3e^t\,dt = 3e^t + c = 3e^{4y^2-y}+c$

Answer 6 - $f''(x) = 15\sqrt{x}+5x^3+6$

$f'(x) = \int f''(x)\,dx = \int 15\sqrt{x}+5x^3+6\,dx = 10x^{3/2}+\frac{5}{4}x^4+6x + k$

$f(x) = \int f'(x)\,dx = \int 10x^{3/2}+\frac{5}{4}x^4+6x+k\,dx = 4x^{5/2}+\frac{x^5}{4}+3x^2+kx+c$

Given $f(1) = -\frac{5}{4}$ and $f(4)=404$,

$f(1) = 4+\frac{1}{4}+3+c+k = -\frac{5}{4}$

$k+c = -\frac{5}{4} - \frac{29}{4} = -\frac{34}{4} = -\frac{17}{2}$ —— (1)

$f(4) = 4(32)+256+48+c+4k = 404$

$4k+c = -28$ —— (2)

~~Answer 7~~

Using (1) and (2),

$\cancel{4k+c = 164}$

$\cancel{-k+c = -8.5}$

$\cancel{k = 57.5}$

$\therefore \cancel{c = -66}$

$4k+c = -28$

$-k+c = -8.5$

$3k = -19.5$

$k = 6.5$

$\therefore c = -8.5-6.5 = 15$

$\cancel{f(x) = 4x^{5/2}+\frac{x^5}{4}+3x^2+57.5x-66}$

$f(x) = 4x^{5/2}+\frac{x^5}{4}+3x^2+6.5x+15$

Answer 8-

$$\frac{dr}{d\theta} = \frac{r^2}{\theta}$$

$$r^{-2} dr = \theta^{-1} d\theta$$

Integrating both sides,

$$\int r^{-2} dr = \int \theta^{-1} d\theta$$

$$\frac{r^{-1}}{-1} + c = \ln|\theta|$$

$\because r(1) = 2$

$$\Rightarrow -\frac{1}{r} + c = \ln|\theta|$$

$$\Rightarrow -1 + c = \ln|\theta|$$

$$\Rightarrow c = \ln|\theta| + 1$$

$\therefore$

Answer 9 -

$$\frac{dv}{dt} = 9.8 - 0.196v$$

$$\int \frac{1}{9.8 - 0.196v} dv = \int dt$$

$$-\frac{1}{0.196} \int \frac{-0.196}{9.8 - 0.196v} dv = \int dt$$

$$-\frac{1}{0.196} \ln|9.8 - 0.1096v| = t + c$$

$$\ln|9.8 - 0.196v| = -0.196t - 0.196c$$

$$\ln|9.8 - 0.196v| = -0.196t + k \qquad \text{where } k = -0.196c$$

Answer 12 -

$f(x) = x^3 - 10x^2 + 6$ $\qquad f(3) = -57$

$f'(x) = 3x^2 - 20x$ $\qquad f'(3) = -33$

$f''(x) = 6x - 20$ $\qquad f''(3) = -2$

$f'''(x) = 6$ $\qquad f'''(3) = 6$

$$f(x) = f(3) + (x-3) f'(x) + \frac{(x-3)^2}{\lfloor 2} f''(3) + \frac{(x-3)^3}{\lfloor 3} f'''(3) + \cdots$$

$$f(x) = -57 + (x-3)(-33) + \frac{(x-3)^2}{\lfloor 2}(-2) + \frac{6(x-3)^3}{\lfloor 3} + \cdots$$

Answer 13 –

$$f'(x) = \mu(1+x)^{\mu-1}$$
$$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$$
$$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$$

$$f(x) = f(0) + \frac{x^1}{\lfloor 1} f'(0) + \frac{x^2}{\lfloor 2} f''(0) + \frac{x^3}{\lfloor 3} f'''(0) + \cdots$$

$$= 1 + \mu + (\mu-1)(\mu) + (\mu)(\mu-1)(\mu-2) + \cdots$$

Answer 14 -

$$f(x) = \sqrt{1+x}$$
$$f'(x) = \frac{1(1+x)^{-1/2}}{2} \qquad f'(0) = 1/2$$

$$f''(x) = -\frac{1}{4}(1+x)^{-3/2} \qquad f''(0) = -1/4$$

$$f'''(x) = -\frac{3}{8}(1+x)^{-5/2} \qquad f'''(0) = -3/8$$

$$f(x) = \sqrt{1+x} + \frac{x}{\lfloor 1}\left(\frac{1}{2}\right) + \frac{x^2}{\lfloor 2}\left(\frac{-1}{4}\right) + \frac{x^3}{\lfloor 3}\left(\frac{-3}{8}\right) + \cdots$$

Answer 15-

$$f(x) = e^{-6x} \qquad x = -4$$
$$f'(x) = e^{-6x}(-6) \qquad f'(-4) = e^{24}(6)$$
$$f''(x) = -6e^{-6x}(-6) \qquad f''(-4) = e^{24}(6)^2$$
$$f'''(x) = (-6)^3 e^{-6x} \qquad f'''(-4) = e^{24}(6)^3$$

$$f(x) = e^{24} + \frac{(x-4)}{\lfloor 1} e^{24}(6) + \frac{(x-4)^2}{\lfloor 2}(e^{24})(6)^2 + \frac{(x-4)^3}{\lfloor 3} e^{24}(6)^3 + \cdots$$

Answer 16 -

$f(x) = \ln(3+4x)$

$f'(x) = \frac{4}{3+4x}$ $\qquad f'(0) = 4/3$

$f''(x) = -4(4x+3)^{-2}(4)$ $\qquad f''(0) = (-1)(4)^2(3)^{-2}$

$f'''(x) = (8)(4x+3)^{-3}(4)^2$ $\qquad f'''(0) = (-1)(-2)(3)^{-2}(4)^3$

$$f(x) = \ln(3) + \frac{x}{\lfloor 1}\frac{4}{3} + \frac{x^2}{\lfloor 2}(-1)(4)^2(3)^{-2} + \frac{x^3}{\lfloor 3}(4)^3(2)(3)^{-2} + \cdots$$