b] Initial Exposed to Risk for popⁿ aged 54 last b'day:-
Assuming

Q.3 Given data:-

- Policy Issue date
- Calender year of policy surrendering.
- Policy Maturity date
- Date of Exit, ~~exits~~ due to unknown reasons

Investigate:- RATE OF SURRENDER in relⁿ to DURATION of policy.

a] Left Censoring:-

This is a type of censoring wherein censoring / missing data ~~is present~~ exists even before the start of the investigation.

Classic example would be when a person is already exposed to a co-morbidity disease* when the ~~st~~ point of investigation too is morbidity analysis.

*even before the investigation begins.

This censoring isn't present.

b] **Right Censoring:-**

This is the most common type of censoring observed wherein the lives are censored or cut short during the observation. The reason could either be lapsation, unknown reason of ~~exit~~ exit or end of investigation

This censoring is present for all those policies that were ~~sur~~ lapsed [Known from date of exit with reason being other than surrender]

c] **Internal Censoring:-**

As the ~~na~~ name suggests, this censoring takes place when data is censored or cut-short off between two points of investigation within an ongoing investigation.

This type of censoring is present since the calender year of surrender tells us the year of surrendering but not the exact point in time when it took place.

d] **Informative**

This censoring exists when the surrender gives some useful information or adds meaning to the ~~inves~~ interest of investigation. This censoring basically impacts & influences the investigation.

This ~~exists~~ exists if some of the date of exits explicitly states that the life surrendered due to certainty about death probability/chances.

Q4 i] Mathematical formula for Complete expectation of life & what does it mean?

→ Complete Expectation of life as the name suggests, helps us find the exact estimation ~~approximation~~ of future life from age x onwards. It is represented as $\mathring{e}_x$.

Survival is found as:-

$$ {}_tp_x \cdot \mu_{x+t} = f(x+t) = \frac{dF_x(t)}{dt} = \frac{d}{dt}\,{}_tq_x = \frac{d}{dt}(1-{}_tp_x) $$

$$ = 0 - \underbrace{\frac{d}{dt}\,{}_tp_x}_{\text{we found this}} $$

∵ we want ans in terms of survival

$$ \therefore \mathring{e}_x = \int_0^{\omega-x} \underbrace{t}_{u} \cdot \underbrace{{}_tp_x\,\mu_{x+t} \cdot dt}_{v} $$

$$ \because {}_tp_x\,\mu_{x+t} = -\frac{d}{dt}({}_tp_x) $$

$$ \therefore = \int_0^{\omega-x} t \cdot \left(-\frac{d}{dt}\,{}_tp_x\right) \cdot dt $$

Using $\int u \cdot v$ rule;

$$ t\int_0^{\omega-x} -\frac{d}{dt}\,{}_tp_x \cdot dt - \int\left(\frac{dt}{dt}\int -\frac{d}{dt}\,{}_tp_x \cdot dt\right) dt $$

→

$$\therefore \left[t\,(-{}_tp_x)\cancel{dt}\right]_0^{\omega-x} - \int 1 - {}_tp_x \cdot \cancel{dt} \cdot dt$$

[Taking Constants out]

$$\therefore -\left[t \cdot {}_tp_x\right]_0^{\omega-x} + \int_0^{\omega-x} {}_tp_x \cdot dt$$

① ②

[Since part ① would become 0, thus]

$$\mathring{e}_x = \int_0^{\omega-x} {}_tp_x \cdot dt$$

⇒ This in words represents, complete future lifetime of age "x" is atleast for 't' years from ~~now~~ age x with death possible any time after the same. The range of t perhaps is from "0" to "ω−x or ∞".

ii] GIVEN:- $\mu_x = 0.0325$ {constant for all ages}

Curtate Expectation of life - $e_x = E[K_x]$

$$\therefore E[K_x] = \sum_{k=1}^{\omega-x} {}_kp_x$$

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Q.5

i] The Gompertz function perhaps describes the age dynamics of human mortality rather accurately in the age range 35 years & above until 80 years.

ii] Show that,

$$_tp_x = (g)^{c^x(c^t-1)}$$
where $\ln g = \frac{-B}{\log c}$

We are perhaps given that,

$$_tp_x = e^{-\int_0^t \mu_{x+s}\cdot ds} = e^{-\int_0^t Bc^{x+s}\cdot ds} \quad -① $$

Lets first understand & simplify the Exp. term.

$$\therefore Bc^{x+s} = B.c^x.c^s = B.c^x s\ln c \text{ (taking Exp of } 2^{nd} \text{ term)}$$
$$= Bc^x e^{s\ln c}$$

$$\therefore \int_0^t Bc^{x+s}\cdot ds = \int_0^t Bc^x e^{s\ln c} = Bc^x.\left[\frac{e^{s\ln c}}{\ln c}\right]_0^t$$

$$= \frac{Bc^x}{\ln c}\left[e^{s\ln c}\right]_0^t = \frac{Bc^x}{\ln c}\left(c^s\right)_0^t \quad \text{......taking the ln part down}$$

$$= \frac{Bc^x(c^t-1)}{\ln c}$$

We can also make use of the given parametric:-

$$\ln g = \frac{-B}{\log c}$$

$\therefore$ Substituting the ~~to~~ "lng" part in ~~the~~ ① we get;

$$\cancel{{}_tP_x = \int_0^t Bc^x e^{s\ln c}} - \int_0^t Bc^{x+s} \, . ds$$

$$= -\int_0^t \cancel{\ln g \ln c \, c^{x+s}} \frac{Bc^x}{\ln c}(c^t - 1) \quad \ldots \text{ where } B = \ln g \ln c$$

$$\therefore = \cancel{\int_0} \ln g \frac{\cancel{\ln c}}{\cancel{\ln c}} c^x (c^t - 1)$$

$$= \ln g \, c^x (c^t - 1)$$

$$\Rightarrow {}_tP_x = e^{(\ln g c^x (c^t - 1))} \quad \ldots \text{ taking log value down.}$$

$$\boxed{{}_tP_x = g^{c^x(c^t - 1)}}$$

Thus proved.

Q.6 Point of Study: Investigate effect of newly invented drug on Cancer suffering patients.

Given:- $y_i = 1$ if <u>non-smoker</u>

$= 0$ <u>otherwise</u>

$z_i = 1$ if <u>male</u>

$= 0$ if female

x_i = age at entry into the observation

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a) The class of lives to which the baseline hazard fn applies is:-

$x_i = 30$ years at entry

$y_i = 0$ ie. smoker &

$z_i = 0$ i.e female.

b) Given:-

$$h_i(t) = h_0(t) \times e^{(0.01(x-30)+0.2y-0.05z)}$$

Compare - $z_i=1, y_i=0, x=30$ with
$z_i=0, y_i=0, x=40$

$$\therefore h_{m,s}(t) = h_0(t) \times e^{(0.01(30-30)+0.2(0)-0.05)}$$

[male, smoker]

$$= h_0(t) \times e^{-0.05}$$

$$h_{f,s} = h_0(t) \times e^{(0.01(40-30)+0.2(0)-0.05(0))}$$

$$= h_0(t) \times e^{0.01(10)}$$

$$\therefore \frac{h_{m,s}}{h_{f,s}} = \frac{\cancel{h_0(t)} \times e^{-0.05}}{\cancel{h_0(t)} \times e^{0.1}}$$

$$= e^{-0.05-0.1}$$

$$= \boxed{0.86071}$$

⟹ Thus, a male smoker aged 30 has 13.93% less ~~risk~~ survival chances as compared to female smoker aged 40

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c) As found earlier,

$$h_{m,s,30} = h_0(t) \times e^{-0.05}$$

$$h_{m,s,40} = h_0(t) \times e^{(0.01(40-30)+0.2(0)-0.05)}$$

$$= h_0(t) \times e^{0.1-0.05}$$

$$\therefore \frac{h_{m,s,30}}{h_{m,s,40}} = e^{-0.05-0.05}$$

$$= \boxed{0.9048}$$

⇒ Perhaps a male smoker aged 30 has 90.48% chance of survival as compared to male smoker aged 40.

Q.7

i] Policyholders [life] classified by :- Age Next B'day

Deaths classified as:- age next bday + curtate duration at date of death