Q1) $_u q_x = 1 - {_u p_x} = P[T_x \le u]$

$= P[K_x + S_x \le u] \Rightarrow P[K_x = 0 \,\&\, S_x \le u]$

$= P[K_x = 0] \times P(S_x \le u)$

$= q \times \int_0^u 1 \, dx \Rightarrow q \times u$ $\therefore$ hence proved

Q2) a) ~~k~~ at age 54, the life is at age 54 from 1-6-2000 to 25-10-2000, so $(30 + 31 + 3\cancel{1} + 30 + 2\cancel{5})/7 = 21$ weeks

b) initial risk will be until he/she turns 55, so 1 year = 52 weeks

Q3) 1) Left censoring :- this occurs when the event of interest happens before the start of investigation. in this case, if we are not aware of date of joining of company, it is left censored.

2) right censoring :- the event of interest happens after the investigation. If policy terminates, the data will be right censored.

3) interval censoring :- when we know only the interval when the event occured & not exact date. Herein, we only know year of surrender, it is interval censored.

4) informative censoring :- when data is lost due to reasons related to the study. it's not present here.

Q4) i) $\mathring{e}_x = E[T_x] =$ ~~$\sum$~~ $\int_0^{\omega - x} {_t p_x} \, dt$

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$\mathring{e}_x = \sum_{k=1}^{\omega-x} {}_kP_0 = \sum_{k=1}^{\omega-x} e^{-0.0325\times k}$

$= \dfrac{e^{-0.0325}}{1-e^{-0.0325}} = 30.27$

(iii) find ${}_9P_{30} \Rightarrow e^{-\int_0^9 0.0325\,dt} = e^{-0.2925}$

$= \underline{74.64\%}$

(iv) d_0 at age x, so T_0 for living x years, probablity $= 0.5$

so ${}_xP_0 = 0.5 \Rightarrow e^{-0.0325x} = 0.5$

$x = -\dfrac{\ln(0.5)}{0.0325} = 21.33$

Q5) (i) 30 to 80 years.

(ii) $\mu_x = B.C^x$. so ${}_tP_x = e^{-\int_0^t B.C^{x+s}\,ds}$.

$C^{x+s} = C^x \cdot e^{s\log C}$ so ${}_tP_x = e^{-\int_0^t B.C^x \cdot e^{s\log C}\,ds}$

$\Rightarrow \dfrac{B\cdot C^x}{\log C}\cdot[C^s]_0^t \Rightarrow \dfrac{B.C^x}{\log C}(c^t-1)$

$\therefore\ {}_tP_x = g^{c^x(c^t-1)}$

Q6) (a) smoker, female, age $= 30$.

(b) $\dfrac{\text{male, smoker, 30}}{\text{female, smoker, 40}} = \dfrac{\exp(0.01(0)+0.20-0.05)}{\exp(0.01(10)+0.20-0)}$

$= 0.8607$ //

$S_j(t) = [S_i(t)]^{0.8607}$ so $S_i(t) > S_j(t)$

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(c) $\frac{\text{male, smoker, 30}}{\text{male, smoker, 40}} = \frac{\exp(0+0-0.05)}{\exp(10(0.01) - 0.05)} = 0.9048$

$\therefore S_j(t) = [S_i(t)]^{0.9048}$ so $S_j(t) > S_i(t)$

Q7) (i) using policy year rate interval starting on policy anniversary where lives are aged x next birthday. the avg age at start of rate interval is $x - 1/2$ assuming birthdays are uniformly distributed.

(ii) $P_x(t) = \frac{1}{2}(P_{x,t} + P_{x+1,t})$

$E^c_x = \int_0^{10} P_x(t)\, dt = \frac{1}{2} \sum_{t=0}^{9} P_x(t) P_x(t+1)$

$E_x = E^c_x + \frac{1}{2} \sum_{t=0}^{10} v_{x,t}$.

Q8) (i) right censoring - patient may die of disease after end of investigation.

type I censoring - the investigation time is restricted to max 45 days.

type II censoring - the no of patients checked is 13.

random censoring - death may be due to other reasons

(ii) the patients discharged may be recovering well than patients who died.

(iii)

at risk	time	death	c	d / risk : 1−(d/n)	S(t)
13	0	–	–	–	1
13	5	1	0	0.9231	0.92
12	7	1	0	0.9167	0.85
11	14	1	2	0.9091	0.77
8	28	1	2	0.8750	0.67
5	35	1	–	0.8	0.54

(iv) no, as per the survival function, 77% people recover.

Q9) i)(a) $\int_0^1 {}_tp_x \, dt = \int_0^1 (1 - t\,q_x)\,dt = \left[t - 0.5t^2 q_x\right]_0^1$

$= 1 - 0.5 q_x$,

$m_x = \frac{0.3}{1-0.15} = 0.353$.

(b) $q_x = 1 - e^{-\mu}$ $\quad \int_0^1 {}_tp_x \, dt = \int_0^1 e^{-\mu t} dt$

$\Rightarrow \frac{1}{\mu}(1 - e^{-\mu}) = \frac{q_x}{\mu}$.

$m_x = \mu = -\ln(1-q_x) = -\ln(0.7) = \underline{\underline{0.357}}$

(i) as it is proportional to baseline hazard & constant of proportionality depending on covariates

(ii) baseline hazard is annual policy taken through online channel & where premiums are paid by debit.

(ii)

t	S(t)	$\lambda(t)$	n	d	c
0	1	0	12	0	
1	0.9167	0.0833	12	1	2
3	0.713	0.22	9	2	2
6	0.4278	0.4	5	2	3

2 died each at duration 3 & 6 weeks.

(iii) 7 were censored.

(i) $\lambda_x = B \cdot C^x$ $\quad \lambda_x = \mu_x$ (force of mortality)

(ii) $\lambda_x = e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2} \cdot C^x$, C^x depends on time & $\exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2)$ on covariates.

so its propotional hazard model.

(iii) non smoker female

(iv) $\lambda_x = e^{(-4 + 0.65)} \times 1.05^4 = \underline{0.04266}$

(v) $\lambda_{25} = e^{(\beta_0 + \beta_1 x_1)} \cdot C^5$.

$\lambda_5 = e^{\beta_0 + \beta_1 x_1 + 0.65} \cdot C^t$

$\lambda_{ns} = \lambda_s \Rightarrow e^{(\beta_0 + \beta_1 x_1)} c^{s} = e^{(\beta_0 + \beta_1 x_1 + 0.65)} c^{t}$

$c^{s-t} = e^{0.65} \Rightarrow 1.05^{s-t} = e^{0.65}$

$s - t = 13.3$

non-smoker is 13 years older than smoker when hazard is equal.

Q.13) (i) $E^c_x = \int_0^2 P'(x)\, dt$

$E^c_{56} = \frac{1}{2}\left(P'_{56}(2015) + P'_{56}(2016)\right) + \frac{1}{2}\left(P'_{56}(2016) + P'_{56}(2017)\right)$

$= P'_{56}(2016) + \frac{1}{2}\left(P'_{56}(2015) + P'_{56}(2017)\right)$

$P'_{56}(2015) = \frac{1}{2}\left(P_{55}(2015) + P_{56}(2015)\right)$

$= 20050.$

$P'_{56}(2016) = \frac{1}{2}\left(P_{55}(2016) + P_{56}(2016)\right)$

$= 20800.$

$P'_{56}(2017) = \frac{1}{2}\left(P_{55}(2017) + P_{56}(2017)\right)$

$= 19250.$

$E^c_{56} = 40450 \quad \therefore \mu_{56} = 1380/40450$

$= \underline{0.0341}$

(ii) $q_{55.5} = 1 - e^{-\mu_{56}} = 0.0335$

$q_{56.5} = 1 - e^{-\mu_{57}} = 0.0352$

Q14) (i) $\lambda(t,z) = \lambda_0(t) e^{\{0.3z_1 + 0.2z_2 + 0.3z_3 + 0.5z_4 + 0.1z_5 + 0.7z_6 + 0.15z_7 - 0.428\}}$.

Q15) (i) • easier to calculate.

• diff to model exposed risk in terms of underlying process being modelled.

(ii) rita (3 months), sita (1 year), nita (9 months), gita (0).

(iii) central => 0.25 + 1 + 0.25 = 2 yrs.

initial => 2.75 yrs.