Calculus Assignment 1

1. $\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h}$

$= \lim_{h\to 0} \frac{\cancel{18} - 12h + 2h^2 - \cancel{18}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2h-12)}{\cancel{h}} = 2h - 12 = \underline{\underline{-12}}$

2. $g(x) = \begin{cases} 2x & x < 6 \\ x - 1 & x \geq 6 \end{cases}$

a. L.H.L $\lim_{x\to 4^-} g(x)$ R.H.L $\lim_{x\to 4^+} g(x)$ $g(4) = 8$

$= \lim_{x\to 4^-} (2x) = 8$ $= \lim_{x\to 4^+} (2x) = 8$

$\therefore$ Function is continuous at $x = 4$

b. L.H.L $\lim_{x\to 6^-} g(x)$ R.H.L $\lim_{x\to 6^+} g(x)$ L.H.L $\neq$ R.H.L

$= \lim_{x\to 6^-} (2x) = 12$ $= \lim_{x\to 6^+} (x-1) = 5$

$\therefore$ Function is not continuous at $x = 6$.

3. $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} = \lim_{x\to 9} \frac{(\sqrt{x})^2 - 3^2}{(3-\sqrt{x})}$

$= \lim_{x\to 9} \frac{\cancel{(\sqrt{x}-3)}(\sqrt{x}+3)}{-\cancel{(\sqrt{x}-3)}} = -(\sqrt{9}+3)$

$= -6$;

4. $f(x) = \dfrac{4x+5}{9-3x}$

a. $\lim\limits_{x\to -1} f(x) = \lim\limits_{x\to -1} \dfrac{4x+5}{9-3x} = \dfrac{1}{12}$ $\therefore$ continuous

b. $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{4x+5}{9-3x} = \dfrac{5}{9}$ $\therefore$ continuous

c. $\lim\limits_{x\to 3} f(x) = \lim\limits_{x\to 3} \dfrac{4x+5}{9-3x} = \dfrac{17}{0}$. not defined

$\therefore$ infinite discontinuous

5. $\lim\limits_{x\to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$= \lim\limits_{x\to \infty} \left(\dfrac{6e^{4x} - e^{-2x}}{e^{4x}}\right)\left(\dfrac{e^{4x}}{8e^{4x} - e^{2x} + 3e^{-x}}\right)$

$= \lim\limits_{x\to \infty} \dfrac{(6 - e^{-6x})}{8 - e^{-2x} + 3e^{-5x}} = \dfrac{6}{8} = \dfrac{3}{4}$

6. $g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$

a. $\lim\limits_{y\to 6} g(y) = \lim\limits_{y\to 6} (1-3y) = -17$

b. L.H.L

$\lim\limits_{y\to 2^-} g(y) = \lim\limits_{y\to 2^-} (y^2+5)$

$= 9$

R.H.L

$\lim\limits_{y\to 2^+} g(y) = \lim\limits_{y\to 2^+} (1-3y)$

$= -5$

$g(2)$

$= 1-3(2)$

$= -5$

7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$$\frac{d\,f(t)}{dt} = (4t^2 - t)\frac{d}{dt}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12)\frac{d}{dt}(4t^2 - t)$$

$$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$$
$$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12$$
$$= 20t^4 - 132t^3 + 24t^2 - 12 + 96t$$

8. $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

$$\frac{d(R(w))}{dw} = \frac{(2w^2+1)(3+3w^3) - (3w+w^4)(4w+1)}{(2w^2+1)^2}$$

9. ~~10.~~ $f(x) = 2e^x - 8^x$

$$f'(x) = 2e^x - 8^x \log_e 8$$

10. $y = x^5 - e^x \ln(x)$

$$\frac{dy}{dx} = 5x^4 - \left[\frac{e^x}{x} + \ln(x)\, e^x\right]$$

$$= 5x^4 - \frac{e^x}{x} + e^x \ln(x)$$

11.

a. $f(x) = (6x^2 + 7x)^4$
$$= 4(6x^2 + 7x)^3(12x + 7)$$

b. $f(t) = 5 + e^{4t+t^7}$

$$f'(t) = e^{4t+t^7}(4+7t^6)$$

12. a. $z = \ln(7-x^3)$

$$\frac{dz}{dx} = \frac{-1}{7-x^3}(3x^2)$$

$$\frac{d^2z}{dx^2} = -\left[\frac{(7-x^3)(6x)-(3x^2)(-3x^2)}{(7-x^3)^2}\right]$$

$$\frac{d^2z}{dx^2} = -\left[\frac{42x-6x^4+9x^4}{(7-x^3)^2}\right] = \left[\frac{-42x-3x^4}{(7-x^3)^2}\right]$$

b. $Q(v) = \frac{2}{(6+(2v)-v^2)^4} = y$

$$\frac{dy}{dv} = \frac{2\,(-8)\,(2-2v)}{(6+2v-v^2)^5} = \frac{-16(1-v)}{(6+2v-v^2)^5}$$

$$\frac{d^2y}{dv^2} = -16\left[\frac{(6+2v-v^2)^5(-1)-(1-v)(6+2v-v^2)^4(5)(2-2v)}{(6+2v-v^2)^{10}}\right]$$

$$= -16\frac{\cancel{(6+2v-v^2)^4}}{\cancel{(6+2v+v^2)^4}}\left[\frac{(v^2-2v-6)-(1-v)(2-2v)}{(6+2v-v^2)^6}\right]$$

$$= -16\left[\frac{(v^2-2v-6)-2(1-v)^2}{(6+2v-v^2)^6}\right]$$

$$= \frac{(2(1-v)^2-(v^2)+2v+6)\,16}{(6+2v-v^2)^6}$$

c. $f(t) = \ln(1+t^2)$

$f'(t) = \frac{1}{1+t^2}(2t)$

$f''(t) = \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$

$= \frac{2+2t^2-4t^2}{(1+t^2)^2} = \frac{2-2t^2}{(1+t^2)^2}$

13. $f(x) = y = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$0 = 3x^2 - 6x - 9$
$0 = x^2 - 2x - 3$
$0 = x^2 - 3x + x - 3$
$0 = x(x-3) + 1(x-3)$
$0 = (x+1)(x-3)$
$x = -1 \quad ; \quad x = 3$

$f''(x) = 6x - 6$

$f''(-1) = -6 - 6$
$= -12$

$f''(3) = 18 - 6 = 12$

$f(-1) = -1 - 3 + 9 + 12$
$= 17$ (maximum value)

$f(3) = \cancel{27} - \cancel{27} - 27 + 12$
$= -15$ (minimum value)

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$
$f''(x) = 24x - 36$

$x^2 - 3x + 2 = 0$
$= x^2 - x - 2x + 2$
$= x(x-1) - 2(x-1)$
$= (x-2)(x-1)$
$x = 2 \quad ; \quad x = 1$

$f''(2) = 48 - 36$
$= 12$

$f''(1) = 24 - 36$
$= -12$

$f(1) = 4 - 18 + 24 - 7$
$= 3$
(maximum value)

$f(2) = 32 \overset{-72}{\cancel{-112}} + 48 - 7$
$= 80 - 119$
$= -39 + 40$ (minimum value)
$= 1$

15) $f(x) = x^2(x+3) = x^3 + 3x^2$

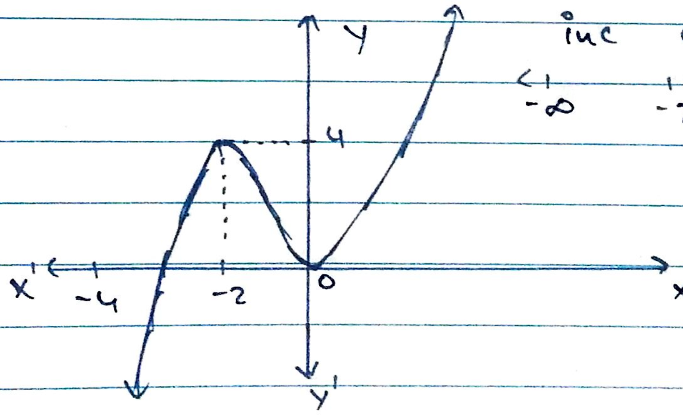
$f'(x) = 3x^2 + 6x = 0$
$x^2 + 2x = 0$
$x(x+2) = 0$
$x = 0 ; x = -2$

$f''(x) = 6x + 6$
$f''(0) = 6$
$\therefore$ (At 0, $f(x)$ has minimum value)

$f''(-2) = -6$
$\therefore$ (At -2, $f(x)$ has maximum value).

$f(0) = 0$ (minimum value)
$f(-2) = -8 + 12 = 4$ (maximum value)

$f'(1) = 9$
$f(-3) = 9$



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16) $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$
$(x = 1)$

$f''(x) = 6$
... At $x = 1$, $f(x)$ attains minimum value.

$f(1) = -3$

$f'(0) = -6$, $f'(2) = 6$

dec inc
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