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NMA Assigment

2.

x	$f(x)$
4	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.7781513

(a) log(5) let log(5) be y

$y = 0.60206 + \frac{1}{2}(0.7781513 - 0.60206)$

Hence, log(5) = 0.690105

(b) log 5 if log 5 is z

$z = 0.6532125 + \frac{1}{2}\left[0.7403627 - 0.6532125\right]$

log(5) = 0.6967876

1. Given that
wrong radius = 12.65
actual radius = 12.5

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(i) absolute error

$\Rightarrow$ actual value - measured value

$|12.5 - 12.65|$

$= \underline{\underline{0.15}}$

(ii) Relative error

$\Rightarrow \dfrac{\text{Measured} - \text{actual}}{\text{Actual}}$

$\Rightarrow \dfrac{12.65 - 12.5}{12.5}$

$= \underline{\underline{0.012}}$

(iii) Percentage error

$= \underline{\underline{1.2\%}}$

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3.

Let $x^4 - x - 10 = 0$

$a = 1.5$

$b = 2$

Now $f(a) = f(1.5) = (1.5)^4 - 1.5 - 10 = \cancel{0} = -6.4375$

$f(b) = f(2) = (2)^4 - 2 - 10 = \cancel{0} = 4$

Now the roots lie between 1.5 & 2

Hence $\frac{1.5+2}{2} = 1.75$

We will find the equation

$f(1.75) = -2.37109375$

$f(2) = 4$

The roots lie between 1.75 & 2

Hence $\frac{1.75+2}{2} = 1.875$

$f(1.875) = 0.4846191406$

$f(1.75) = -2.37109375$

The roots lie between 1.875 & 1.75

$= \frac{1.875+1.75}{2} = 1.8125$

$f(1.8125) = -1.020248413$

$f(1.875) = 0.4846191406$

The roots lie between 1.875 & 1.8125

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Hence, $\frac{1.875 + 1.8125}{2} = 1.84375$

$f(1.84375) = -0.2877340317$

$f(1.875) = 0.4846191406$

The roots lie between 1.84375 & 1.875

$\frac{1.84375 + 1.875}{2} = 1.859375$

$f(1.859375) = 0.09337812662$

$f(1.84375) = -0.2877340317$

The roots lie between 1.859375 & 1.84375

$\frac{1.859375 + 1.84375}{2} = 1.8515625$

$f(1.8515625) = -0.0984334312$

$f(1.859375) = 0.09337812662$

" " $\frac{1.8515625 + 1.859375}{2}$

$= 1.85546875$

$f(\cancel{1.85546875})$

The roots of the equation are

~~0.09~~ 1.859375 & 1.8515625

4. find the roots $f(x) = x^3 - x - 1$ using newton raphson,

x	$f(x)$
0	-1
1	-1
2	5

Now,

$f'(x) = 3x^2 - 1$

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.5$

$y_1 = 0.875$

$f'(x_1) = 5.75$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5 - \frac{0.875}{5.75} = 1.347826$

$y_2 = 0.100681$

$f'(x_2) = 4.449904$

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$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 1.325200$$

$y_3 = 0.0020566$
$f'(x_3) = 4.268465$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.325200 - \frac{0.0020566}{4.268465}$$
$$= 1.3247181$$
$y_4 = 0.00000060874$

5. Given that,
$A^2 = A$
To find $7A - (I+A)^3$

$\Rightarrow 7A - (I^3 + A^3 + 3IA + 3A^2)$
$= 7A - (I + AA + 3A + 3A)$
$= 7A - (I + A + 6A)$
$= \cancel{7A} - I - \cancel{7A}$
$= -I$

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6. $$X = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

Find the inverse of ~~A~~

$|X| = 1[-1-6] + 1[-4-0] + 2[4]$

$= -2$

As $|X| \neq 0$ ~~&~~ A^{-1} escists

Adjoint of X,

$$\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 1 \\ 4 & -1 & 4 \end{bmatrix}$$

Now $\cancel{A}^{-1} = \frac{\text{Adjo } X}{|X|}$

$$= \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & -2 \end{bmatrix}$$

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7.

(a) $A = 8\hat{i} - 9\hat{j}$

$B = 7\hat{i} - 9\hat{j}$

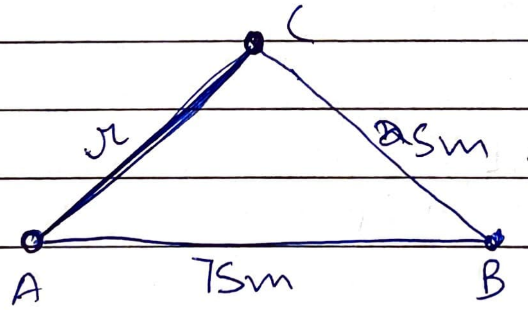
now $|\vec{a}| = \sqrt{64+72} = 2\sqrt{34}$

& $|\vec{b}| = \sqrt{49+81} = \sqrt{130}$

$$= \frac{(8\hat{i}-9\hat{j}).(7\hat{i}-9\hat{j})}{2\sqrt{39} \times \sqrt{130}}$$

$$\cos\theta = \frac{137}{2\sqrt{4420}}$$

8.



Sabrina walked A to B for 75m then turned 30° and walked 25m to C

We have to find the distance between A & C

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$\vec{r} = \vec{a} + \vec{b}$

$r^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\bar{a}\bar{b}$

$= |\bar{a}|^2 + |\bar{b}|^2 + 2|\bar{a}||\bar{b}| \cos 30$

$= 75^2 + 25^2 + 2(75)(25)\frac{\sqrt{3}}{2}$

$r^2 = 9497.596 \, m$

$\vec{R} = 97.46 \, m$

9. We know that

$d = 3$ &

$a = 101$

$a_n = 998$

Now $a_n = a + (n-1)d$

Hence

$998 = 101(n-1)3$

$300 = n$

$S_n = \frac{n}{2}(a + a_n)$

$= 150(101 + 998)$

$= 164850$

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11.

Here, $(4+3x)^5$

Expand using binomial theorem

$\Rightarrow$ $5C_0(3x)^5 + 5C_1(3x)^4(4) + 5C_2(3x)^3(4)^2 + 5C_3(3x)^2(4)^3 + 5C_4(3x)(4)^4 + 5C_5(4)^5$

$\Rightarrow$ $243x^5 + 1620x^4 + 4320x^3 + 7560x^2 + 3840x + 1024$

12 Now

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Observing the matrix, we get to know that $(A - \lambda I) = 0$

$\Rightarrow$ $-(2-\lambda)(5+\lambda)(3-\lambda) - (-3)[(3-\lambda)(2) - 0] = 0$

$\Rightarrow$ $[-30 + 2\lambda^2 + 4\lambda + 15\lambda - \lambda^3 - 2\lambda^2] + [18 - 6\lambda] = 0$

$\Rightarrow$ $\lambda^3 - 13\lambda + 12 = 0$

If $\lambda = 1$

$\Rightarrow$ $(1)^3 - 13 + 12$

$\Rightarrow$ 0

Hence $\lambda = 1$ is definitely one of the roots

When we use synthetic division

$\lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$\lambda(\lambda + 4) - 3(\lambda + 4) = 0$

So

$(\lambda + 4)$ & $(\lambda - 3)$

The eighen values are $-4, 3$

13. $f(x) = x^3 - 7x^2 + 8x - 3$

given that $x_0 = 5$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(x_0) = -13$

$f'(x_0) = 13$

So $x_1 = 5 - \frac{(-13)}{13} = 6$

$f(x_1) = 9$

$f'(x_1) = 32$

$x_2 = 6 - \frac{9}{32} = \underline{\underline{5.71875}}$

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14. To expand,
$(1-x+x^2)^4$

Now,
we let $(1-x) = y$
So the equation becomes $(y+x^2)^4$

Then we use the binomial theorem

$\to$ ${}^4C_0 y^4 + {}^4C_1 y^3(x^2) + {}^4C_2 y^2(x^2)^2 + {}^4C_3 (x^2)^3 y$
$+ {}^4C_4 (x^2)^4$

$\to$ $(1-x)^4 + 4(1-x)^3 x^2 + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$

$\to$ $1-4x+10x^4-16x^3+14x^4-16x^5+10x^6-4x^7+x^8$