Financial Mathematics - Assignment 1

1. $d^{(4)} = 8\%$

(i) Equivalent force of interest (δ) $= -\ln(1-d)$

$$\Rightarrow (1-d) = \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$\Rightarrow 1-d = (1-0.02)^4$$

$$\Rightarrow d = 7.763184\%$$

$$\Rightarrow \delta = -\ln(1-0.0776318)$$

Ans $\Rightarrow \delta = 8.081078\%$

ii Equivalent eff. rate of interest/annum (i)

$$\Rightarrow 1+i = e^{\delta}$$

$$\Rightarrow i = e^{\delta} - 1$$

Ans $\rightarrow i = 8.416573\%$

iii $d^{(12)} = ?$

$$\rightarrow \left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1-d)$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1 - 0.0776318$$

Ans $\Rightarrow d^{(12)} = 8.053929\%$

2. $\delta(t) = 0.004t + 0.0002t^2$

i. $PV_0 = \cancel{1000\, e^{-\int_0^{10}}}\ 1000\,[v(0,10)]$

$$= 1000\, e^{-\int_0^{10}(0.004t + 0.0002t^2)\,dt}$$

$$= 1000\, e^{-\left[0.002t^2 + \frac{0.0002t^3}{3}\right]_0^{10}}$$

$$= 1000\, e^{-\left[0.2 + \frac{0.2}{3}\right]}$$

$$= \frac{1000}{e^{[0.26667]}}$$

Ans $PV_0 = \underline{\underline{765.925}}$

ii $i = ?$

$\Rightarrow$ ~~$1+i = e^{\int_0^{10} 0.002t^2 + \frac{0.0002}{3}}$~~

$\Rightarrow (1+i) = e^{\int_0^{10}(0.004t + 0.0002t^2)dt}$

$\Rightarrow i = e^{[0.26667]} - 1$

$i = \underline{\underline{30.56095\%}}$

3 (i) Suppose a person borrows 1 at discount rate $= d$.

$\Rightarrow$ Present value $= (1-d)$

Also 'i' is the effective annual interest rate & is the ratio of amount of interest to the principle. ie

$$i = \frac{d}{(1-d)} \quad [d \to \text{amount of interest}]$$

$\to$ Interest earned on an investment of 1 paid at the beginning is d, while the earned at the end of the period is i.

$\to$ Thus if i is discounted from the end to the beginning of the period with discounting factor 'v'; we obtain 'd' ie

$$\boxed{d = iv}$$

ii $d^{(2)} = ?$, when

(a) $\frac{d^{(4)}}{4} = 2.25\%$

$$\Rightarrow \left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$\Rightarrow \left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - 0.0225\right)^4$$

$$d^{(2)} = \underline{\underline{8.89875\%}}$$

(b) $d^{(\frac{1}{2})} = 5\%$

$$\Rightarrow \left(1 - \frac{d^{(\frac{1}{2})}}{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$\Rightarrow \left(1 - 0.1\right)^{\frac{1}{2}} = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$\Rightarrow d^{(2)} = \underline{\underline{5.199250\%}}$$

(iii) $i = 5\%$

Annual simple discount rate $(d) = ?$

$$\Rightarrow (1+i)^{\frac{91}{365}} = \left(\frac{1}{1-nd}\right)$$

$$\Rightarrow (1+i)^{\frac{91}{365}} = \frac{1}{\left(1 - \frac{91}{365}d\right)}$$

$$\Rightarrow d = \underline{\underline{4.84946\%}}$$

4. $i = 0.08$

a. $d^{(12)} - ?$

$$(1+i) = \frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{12}}$$

Scanned with CamScanner

$d^{(12)} = \cancel{7.7346}\ 7.671477\%$

b. $i^{(365)} = ?$

$$(1+i) = \left(1 + \frac{i^{(365)}}{365}\right)^{365}$$

$i^{(365)} = 7.696915\%$

c. $\delta = ?$

$\Rightarrow 1+i = e^{\delta}$

$\Rightarrow \delta = \ln(1+i)$

$\delta = 7.696104\%$

d. $i^{(0.5)} = ?$

$$\Rightarrow 1+i = \left(1 + \frac{i^{(\frac{1}{2})}}{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$i^{(0.5)} = 8.32\%$

5. (i) $d^{(4)} = 6\%$

(a) $i^{(2)} = ?$

$$\left(1 + \frac{i^{(2)}}{2}\right)^{2} = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^{4}}$$

$i^{(2)} = 6.137751\%$

(b) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \left(1 - \frac{d^{(4)}}{4}\right)^{4}$$

$d^{(12)} = 6.030252\%$

ii $C = ₹2,00,000$

$AV(0,1) = ₹2,20,000$

$\Rightarrow i = 10\%$

from 15 Apr → 17 Jul ⇒ No. of days = 93 days

⇒ 365 days → $i = 10\%$

⇒ 93 days → $i = \left(\frac{93 \times 10}{365}\right)\% = 2.547945\%$

→ $AV(0, 93\ days) = 200000\,(1+i)^{\frac{93}{365}}$

Ans $AV(93\ days) = ₹2,01,286.2577$

6. Let the retail price of the toy be 100

⇒ when cash payment is done = 30% below retail price

⇒ $PV_0 = 100 - 30\%\ of\ 100$
$= 70$

⇒ On six months credit = 25% below retail price
$= 75$

70	75
0	6 months

⇒ $PV_0 = 75(1-d)^{\frac{1}{2}}$ [d → effective annual discount rate]

$70 = 75(1-d)^{\frac{1}{2}}$

Ans $d = 12.8889\%$

* $d^{(12)} = ?$

$(1-d) = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

Ans $\boxed{d^{(12)} = 13.71944\%}$

* $i^{(2)} = ?$ $\Rightarrow \left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{(1-d)}$

Ans $\Rightarrow \boxed{i^{(2)} = 14.285604\%}$

* $\delta = ?$ $\Rightarrow (1-d) = e^{-\delta}$

$\Rightarrow \delta = -\ln(1-d)$

Ans $\boxed{\delta = 13.798472\%}$

7. $AV(0, 17 \text{ months}) = ₹\,26000$

$C = ₹\,20000$

$\Rightarrow AV(17 \text{ months}) = 20000\,(1+i)^{\left(\frac{17}{12}\right)}$

$i = 20.345706\%$

(i) $i^{(4)} = ?$

$(1+i) = \left(1 + \frac{i^{(4)}}{4}\right)^4$

Ans $i^{(4)} = 18.95525\%$

(ii) $d^{(2)} = ?$

$(1+i) = \frac{1}{\left(1 - \frac{d^{(2)}}{2}\right)^2}$

Ans $\underline{d^{(2)} = 17.688234\%}$

iii. ~~##~~ Simple rate of interest p.a

$\Rightarrow AV = 20000\,(1+ni)$

$\Rightarrow 26000 = 20000\left(1 + \frac{17i}{12}\right)$

$1.3 = 1 + \frac{17i}{12}$ $\Rightarrow i = \frac{0.3 \times 12}{17} = \underline{21.17647\%}$

Ans $i = 21.17647\%$

iv. As the compounding frequency increases interest rate decrease.

Also; $d^{(2)} < i^{(4)} < i$

$\Rightarrow$ Simple interest is greater than the corresponding annual effective rate & nominal rates.

8. $i = 8\%$ p.a [Effective interest rate]

Relationship b/w $i^{(p)}$ and p:

where $i^{(p)} \rightarrow$ nominal rate of interest convertible pthly.

We know that: $(1+i) = \left[1 + \frac{i^{(p)}}{p}\right]^p$

$\Rightarrow$ For eg: $i^{(2)} = 7.846096\%$

$i^{(10)} = 7.7257952\%$

$i^{(100)} = 7.699066\%$

$\cancel{i^{(500)}}$ $i^{(365)} = 7.6969155\%$

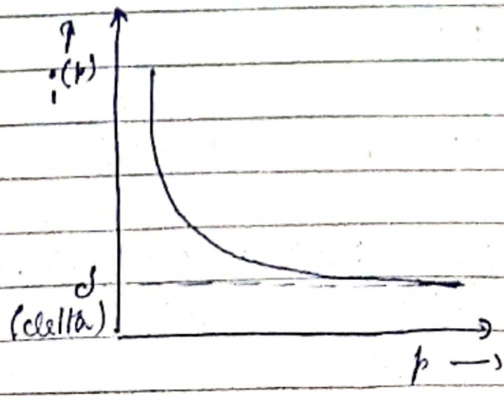
$i^{(1000)} = 7.696400\%$

$\Rightarrow$ From the above example: as 'p' increases value of $i^{(p)}$ decreases.

$\Rightarrow$ As p ~~tea~~ tends to infinity; $i^{(p)}$ reaches a limiting value such that

$$\lim_{p \to \infty} i^{(p)} = \delta$$

[$\delta \rightarrow$ nominal rate of interest per unit time convertible continuously]



7.

9. i Force of interest is the nominal rate of interest per unit time convertible continuously ie it is the measure of interest at individual moments of time.

(ii) $i^{(2)} = 0.0775$

(a) $i = ?$

$\Rightarrow 1+i = \left(1 + \frac{i^{(2)}}{2}\right)^2$

$i = \underline{7.900156\%}$

(b) $d^{(12)} = ?$

$\Rightarrow (1+i) = \frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{12}}$

$\Rightarrow d^{(12)} = \underline{7.579574\%}$

(c) $i^{(\frac{1}{2})} = ?$

$\Rightarrow (1+i) = \left(1 + \frac{i^{(\frac{1}{2})}}{\frac{1}{2}}\right)^{\frac{1}{2}}$

$i^{(\frac{1}{2})} = \underline{\underline{8.212218\%}}$

10. $\delta(t) = \begin{cases} 0.08 & ; 0 \le t \le 5 \\ 0.06 & ; 5 \le t \le 10 \\ 0.04 & ; t \ge 10 \end{cases}$

$V(t)\ \cancel{PV_0} = 1 \times V(0,5) \times V(5,10) \times V(10,t)$

$V(t)\ \cancel{PV_0} = e^{-\int_0^5 0.08\, dt} \times e^{-\int_5^{10} (0.06) dt} \times e^{-\int_{10}^{t} (0.04) dt}$

Scanned with CamScanner

$$= e^{-0.4} \times e^{-[0.6-0.3]} \times e^{-[0.04t-0.4]}$$

$$= e^{[0.4-0.3+0.4-0.04t]}$$

Ans $\rightarrow$ $EV(t) = e^{-[0.3+0.04t]}$

11. a $PV_0 = 50000\ v(0, 9\text{ months})$

$$= 50000\,(1-18\%)^{\frac{9}{12}}$$

$$PV_0 = \underline{43085.43}$$

b(i) $\delta = 6\%$

$d^{(12)} = ?$

$$\Rightarrow e^{-\delta} = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

Ans $d^{(12)} = \underline{5.985024\%}$

(ii) $d^{(4)} = 6\%$

$i^{(12)} = ?$

$$\Rightarrow \left(1 + \frac{i^{(12)}}{12}\right)^{12} = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^4}$$

Ans $\rightarrow$ $i^{(12)} = \underline{6.060708\%}$

c. $d < \delta < i^{(4)} < i$

As we know $i^{(p)} = p\,[(1+i)^{\frac{1}{p}} - 1]$

$\Rightarrow$ As compounding frequency increases ie as p increases i smaller is the equivalent nominal annual rate.

Since 'd' is the discount rate in which interest is paid at the outset; the corresponding rate is the least

d. Consider a borrowing of 1 at discount rate d
$\rightarrow PV_0 = (1-d)$

Let i be the effective annual interest rate such that; $i = \frac{d}{(1-d)}$ $\left[i = \frac{\text{interest amount}}{\text{principle}} \right]$

→ Interest earned on 1 paid at the beginning is 'd' while that ~~at~~ paid at the end of the term is 'i'.

→ Thus if 'i' is discounted from the end to the beginning of the term with discounting factor 'v', we get 'd' ie
$\boxed{d = iv}$

12. Timeline: C=7049

$0 \quad i^{(12)} = 6 \quad 2 \quad \frac{i^{(4)}}{4} = 1.5 \quad 4.5 \quad d^{(12)} = 6\% \quad 6$

$AV(0,6) = 7049 \ A(0,2) \ A(2,4.5) \ V(4.5,6)$

$= 7049 \left[\left(1 + \frac{i^{(12)}}{12}\right)^{24} \right] \left[\left(1 + \frac{i^{(4)}}{4}\right)^{10} \right] \left[\frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{18}} \right]$

$= 7049 \times (1.127159)(1.16054)(1.094421)$

Ans $AV(0,6) = \underline{10091.534}$

13. ~~AV(0,t) =~~ Let initial amount be C
$\Rightarrow AV(0,t) = 2C$

(i) $i = 10\%$
$\Rightarrow 2C = C(1+i)^t$
$\Rightarrow 2 = (1+i)^t$
$\Rightarrow \ln 2 = t \ln(1+0.1)$
$t = \underline{\underline{7.2725 \text{ yrs}}}$

(ii) $d^{(12)} = 10\%$

$\Rightarrow 2C = C\,A(0,t)$

$2C = \dfrac{C}{v(0,t)}$

$\cancel{2 = \dfrac{1}{\left(1-\frac{d^{(12)}}{12}\right)}}$ $\quad 2 = \dfrac{1}{(1-d)^t}$

$\Rightarrow d = 1 - \left(1 - \dfrac{d^{(12)}}{12}\right)^{12}$

$d = 9.5541625\%$

$\Rightarrow 2 = (1-d)^{-t}$

$\Rightarrow \ln 2 = -t \ln(1-0.095541625)$

Ans $\Rightarrow t = \underline{6.90255 \text{ yrs}}$

Scanned with CamScanner