

# Numerical Methods & Algebra.

5 Assignment  
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10 Q1. Original radius of circle =  $12.5 \text{ cm} = r_1$ .

$$\text{Area} = \pi r^2$$

$$= 491.0714286.$$

15 radius of circle with error radius =  $12.65 = r_2$

$$\text{Area} = \pi r^2$$

$$= 502.9278571.$$

20 Absolute error =  $502.927 - 491.071$

$$= 11.85642850.$$

25 Relative error :  $\frac{\text{Absolute error}}{\text{Original area}}$

$$= 0.024144.$$

30 Percentage error : Relative error  $\times 100$ .

$$= 2.41439985.$$

|    |    |     |                 |
|----|----|-----|-----------------|
| 2. | a) | $n$ | $F(n) - \log n$ |
|    |    | 4   | 0.60206         |
|    |    | 6   | 0.7781513       |

Interpolation.

$$\frac{(6-4)}{2} = \frac{(0.7781513 - 0.60206)}{2}$$

$$2 = 0.1760913$$

$\uparrow$  causes change of:  $\frac{0.176093}{2}$

$$\uparrow = 0.088045650$$

Thus,  $n=5$

$$\log(5) = \log(4) + \log(1)$$

$$= 0.60206 + 0.8804565$$

$$= 0.690105650$$

|    |     |           |
|----|-----|-----------|
| b) | $n$ | $\log(n)$ |
|    | 4.5 | 0.6532125 |
|    | 5.5 | 0.7403627 |

$$5.5 - 4.5 = 0.7403627 - 0.6532125$$

$$1 = 0.0871502$$

0.5 causes change of =  $\frac{0.0871502}{2}$

$$= 0.0435751$$

$$n = 5$$

$$\log(5) = \log(4.5) + 0.043575100$$

$$= 0.6967876$$

$$n^4 - n - 10 = 0$$

$$n_0 = 1.5 \quad y = -6.437$$

$$n_1 = 2 \quad 4$$

$$n_2 = \frac{n_0 + n_1}{2}$$

$$= 1.75$$

$$y_2 = -2.371093750$$

$$n_3 = \frac{n_1 + n_2}{2}$$

$$= 1.875$$

$$y_3 = 0.4846194$$

$$n_4 = \frac{n_2 + n_3}{2} = 1.8125 \quad y_4 = -1.0202787$$

$$n_5 = \frac{n_3 + n_4}{2}$$

$$y_5 = -0.287737$$

$$= 1.84375$$

$$n_6 = \frac{n_4 + n_5}{2}$$

$$y_6 = -0.7668625$$

$$= 1.8235$$

Q9)  $f(x) = x^3 - x - 10$   
roots using Newton Raphson.

$$f'(x) = 3x^2 - 1$$

|     |    |    |   |
|-----|----|----|---|
| $x$ | 0  | 1  | 2 |
| $y$ | -1 | -1 | 5 |

$$x_0 = 1$$

$$f(x_0) = -1$$

$$f'(x_0) = 2.$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \left( \frac{-1}{2} \right)$$

$$= 1.5$$

$$y = 0.875$$

$$f'(x_1) = 5.75$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5 - \left( \frac{0.875}{5.75} \right)$$

$$= 1.37783$$

$$f(x_2) = 0.100699$$

$$f'(x_2) = 7.4494$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.3252$$

$$y_3 = 0.00205$$

$$f'(x_3) = 7.26846$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.32472$$

$$y_4 = 0.0008712$$

Q5)  $A^2 = A$ .

$$7A - (1+A)^3$$

$$= 7A - (I^3 + A^3 + 3A^2I + 3AI^2)$$

$$= 7A - I + 7A$$

$$= -I$$

Q6

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$|A| = 0 - 1(6-8) + (-1)(0+4)$$

$$= 2-4$$

$$= -2$$

$$\text{adj}(A) = \begin{bmatrix} -1 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{adj}(A)$$

$$= \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ -4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

$$\begin{aligned}\vec{a} &= 8\hat{i} - 9\hat{j} \\ \vec{b} &= 7\hat{i} - 9\hat{j}\end{aligned}$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= ab \cos \theta \\ &= (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j}) \\ &= 56 + 81 \\ &= 137\end{aligned}$$

$$\begin{aligned}a &= \sqrt{64 + 81} \\ &= \sqrt{145}\end{aligned}$$

$$\begin{aligned}b &= \sqrt{49 + 81} \\ &= \sqrt{130}\end{aligned}$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= ab \cos \theta \\ \cos \theta &= \frac{137}{\sqrt{130 \times 145}}\end{aligned}$$

$$\cos \theta = 0.99787$$

$$\theta = 3.75856^\circ$$

Q8. Displacement vector =  $\vec{R}$

$$\vec{R} = \vec{a} + \vec{b}$$

$$R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$\theta = 30^\circ$$

$$R^2 = 75^2 + 25^2 + 2(75)(25) \cos 30^\circ$$

$$R^2 = 9997.515264$$

$$R = 97.4556 \text{ metres}$$

$$= \text{Displacement}$$

Q9). Common difference,  $d = 3$ .

$$A.P. = 101, 104, 107.$$

$$a = 101$$

$$a_n = a + (n-1)d$$

$$998 = 101 + (n-1)3$$

$$n = 300.$$

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$= 150 (1099)$$

$$= 164850.$$

Q10.  $a_6 = 32$ .

$$ar^5 = 32$$

$$a_8 = 128$$

$$ar^7 = 128$$

$$\frac{ar^7}{ar^5} = \frac{128}{32}$$

$$r^2 = 4$$

$$\underline{r = 2}$$

Q12  $(3n + 4)^5$

$$\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & & & 1 & & 1 & \\
 & & & & & & 1 & & 1 \\
 5 & & & 1 & 2 & 2 & 1 & & \\
 & & 1 & 3 & 3 & 1 & & & \\
 & 1 & 4 & 6 & 4 & 1 & & & \\
 & & 1 & 5 & 10 & 10 & 5 & 1 & 
 \end{array}$$

10  $(3n + 4)^5 = (3n)^5 + 5 \times (3n)^4 \times 4 + 10(3n)^3 \times 4^2 + 10(3n)^2 \times 4^3 + 5(3n) \times 4^4 + 4^5$   
 $= 243n^5 + 1620n^4 + 4320n^3 + 5760n^2 + 3840n + 1024$

15 Q12  $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

20  $A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

25  $= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

30  $|A - \lambda I| = 0 - 0 + (3-\lambda) [(2-\lambda)(-5-\lambda) + 6]$   
 $= 9\lambda - 12 - \lambda^3$   
 $= -\lambda^3 + 9\lambda - 12$

$a + b + c + d = 1 - 13 + 12 = 0$

Thus, only 1 root.

$$\begin{array}{c|ccc} 1 & 1 & 0 & -13 & 12 \\ & & 1 & 1 & -12 \\ \hline & & 1 & -12 & 0 \end{array}$$

$$\lambda^2 + \lambda - 12 = 0.$$

$$\lambda = 3 \quad \text{or} \quad \lambda = -4$$

Eigen values = -4, 1, 3

Q13)  $f(x) = x^3 - 7x^2 + 8x - 3$   
 $f'(x) = 3x^2 - 14x + 8$

$$x_0 = 5$$

$$f(5) = -13$$

$$f'(5) = 13$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \left( \frac{-13}{13} \right)$$

$$= 6$$

$$f(x_1) = 9$$

$$f'(x_1) = 32$$

$$x_2 = x_1 - \left( \frac{f(x_1)}{f'(x_1)} \right)$$

$$x_2 = 6 - \frac{9}{32}$$

$$= 5.71875$$

$$f(x_2) = 0.84787$$

Q14.  $(n^2 - n + 1)^7 = ((n-1)^2 + n)^7$

Using pascal's triangle,

$$\begin{array}{cccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
 1 & 3 & & 3 & & 1 & \\
 1 & 4 & 6 & 4 & 1 & & 
 \end{array}$$

$$\begin{aligned}
 ((n-1)^2 + n)^7 &= (n-1)^8 + 4n(n-1)^6 + 6n^2(n-1)^4 \\
 &\quad + 4n^3(n-1)^2 + n^7 \\
 &= (n^2 - n + 1)^7
 \end{aligned}$$

Q15.  $e^{-n} = 3 \log(n)$   
 $3 \log(n) - e^{-n} = 0$

| $n$ | $y$      |
|-----|----------|
| 0.1 | -2.07516 |
| 0.2 | -1.27818 |
| 0.3 | -0.82782 |
| 0.4 | -0.5235  |
| 0.5 | -0.29658 |
| 0.6 | -0.11673 |
| 0.7 | 0.3185   |
| 0.8 |          |

$$x_0 = 0.6$$

$$x_1 = 0.7$$

$$y_0 = -0.11673$$

$$y_1 = 0.03188$$

$$x_2 = \frac{x_0 + x_1}{2}$$

$$= 0.65$$

$$y_2 = -0.03921$$

$$x_3 = \frac{x_2 + x_1}{2}$$

$$= 0.675$$

$$y_3 = -0.00293$$

$$x_4 = \frac{x_3 + x_1}{2}$$

$$= 0.6875$$

$$= 0.01485$$

$$y_4$$

$$x_5 = \frac{x_4 + x_3}{2}$$

$$= 0.68125$$

$$y_5 = 0.0059$$