Assignment - 2

A1) $$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$$

Let $u = \frac{2}{w}$

$$\Rightarrow \frac{du}{dw} = \frac{-2}{w^2} \quad \Rightarrow dw = \frac{-w^2}{2} du$$

$$\Rightarrow \frac{-1}{2}\int e^u\, du$$

Using exponential rule,

$$\int e^u\, du = e^u$$

$$= \frac{-e^u}{2}$$

$$= \frac{-e^{2/w}}{2}$$

$$\Rightarrow \int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw = \left[\frac{-e^{2/w}}{2}\right]_{1/50}^{2} \cancel{+K} = \frac{e^{100} - e}{2}$$

or

$$= 1.344058571 \times 10^{43}$$

A3) $f'(x) = x^4 + 3x - 9$.

$$f(x) = \int f'(x)\,dx$$

$$= \int [x^4 + 3x - 9]$$

$$= \cancel{\frac{x^5}{5} + 3\frac{x^2}{2} -}$$

$$= \int x^4 dx + 3\int x\,dx - 9\int 1\,dx.$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} \cancel{+x} - 9x + K$$

$$= \frac{2x^5 + 15x^2 - 90x}{10} + K$$

$$f(x) = \frac{x(2x^4 + 15x - 90)}{10} + K$$

Ans

A4) $f(x) = \begin{cases} 6 & , x > 1 \\ 3x^2 & , x \le 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx.$$

$$= 3\left[\frac{x^3}{3}\right]_{-2}^{1} + 6[x]_1^3$$

$$= 9 + 12$$

$$\int_{-2}^{3} f(x)\,dx = 21$$

Ans

A5) $\int 3(8y-1)\, e^{4y^2-y}\, dy$.

Let $\mu = 4y^2 - y$.

$\Rightarrow \frac{du}{dy} = 8y-1 \Rightarrow dy = \frac{1}{8y-1} du$.

$= 3\int e^{u}\, du$

$= 3e^{u}$

$= 3e^{4y^2-y}$

$\Rightarrow \int 3(8y-1)\, e^{4y^2-y}\, dy$

$= 3e^{4y^2 \cancel{4} -y} + K$

$= 3e^{y(4y-1)} + K$

A6). $f''(x) = 15\sqrt{x} + 5x^3 + 6$,

$f'(x) = \int 15\sqrt{x} + 5x^3 + 6 \, dx$

$\cancel{= 3\int 5\sqrt{x} +}$

$= 15\int \sqrt{x}\, dx + 5\int x^3 dx + 6\int 1 dx$

$= 15 \frac{2x^{3/2}}{3} + \frac{5x^4}{4} + 6x + K$

$= 10x^{3/2} + \frac{5x^4}{4} + 6x + K$

$f(x) = \int f'(x)\, dx$

$= \int \frac{5x^4}{4} + 10x^{3/2} + 6x + K dx$.

$= \frac{5}{4}\int x^4 dx + 10\int x^{3/2} dx + 6\int x\, dx + \int K dx$

$$= \frac{5}{4}\left(\frac{x^5}{5}\right) + 10\left(\frac{2x^{5/2}}{5}\right) + 6\left(\frac{x^2}{2}\right) + Kx^1 + C$$

$$f(x) = \frac{x^5}{4} + 4x^{5/2} + 3x^2 + Kx^1 + C$$

Ans.

$$f(1) = \frac{(1)^5}{4} + 4(1)^{5/2} + 3(1)^2 + K(1) + C = \frac{-5}{4}$$

$$\Rightarrow \frac{1}{4} + 4 + 3 + K + C = \frac{-5}{4}$$

$$\Rightarrow \frac{29}{4} + K + C = \frac{-5}{4}$$

$$\Rightarrow K + C = \frac{-5}{4} - \frac{29}{4}$$

$$\Rightarrow K + C = \frac{-34}{4} \Rightarrow 4K + 4C = -34 \rightarrow \text{I}$$

$$f(4) = \frac{(4)^5}{4} + 4(4)^{5/2} + 3(4)^2 + K(4) + C = 404$$

$$\Rightarrow 432 + 4K + C = 404$$

$$4K + C = -28 \rightarrow \text{II}$$

I − II

$$3C = -6 \Rightarrow \underline{C = -2}$$

putting in II,

$$4K = -28 + 2 \Rightarrow K = \frac{-26}{4} \Rightarrow \cancel{K = -6.5}$$

$$\Rightarrow K = \frac{-13}{2}$$

$$\Rightarrow f(x) = \frac{x^5}{4} + 4(x)^{5/2} + 3x^2 + \frac{-13x}{2} - 2$$

Ans

A7). $f(x+iy) + g(\cancel{x}\ x - iy) = z \longrightarrow \mathrm{I}$

Diff. I wrt x,

$$\frac{dz}{dx} = f'(x+iy) + g'(x-iy)$$

Diff. I wrt x,

$$\frac{d^2 x}{dx^2} = f''(x+iy) + g''(x-iy)$$

Diff. I wrt y.

$$\frac{dz}{dy} = f'(x+iy)(i) + g'(x-iy)(-i)$$

diff. wrt. y,

$$\frac{d^2 z}{dy^2} = f''(x+iy)[i^2] + g''(x-iy)(-i^2)$$

$$= i^2 [f''(x+iy) + g''(x-iy)]$$

$$\frac{d^2 z}{dy^2} = i^2 \left(\frac{d^2 z}{dx^2}\right)$$

Ans

A8). $\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}$, $r(1) = 2$.

$\Rightarrow \cancel{\frac{dr}{2}} \int_{\cancel{2}}^{\cancel{r}} \frac{1}{r}\, dr = \int_{\cancel{1}}^{\cancel{\theta}} \frac{1}{\theta}\, d\theta$

~~$\ln|r| + c_r = \ln|\theta| + K$.~~

~~$\ln|1| + c(1) = 2$.~~
~~$0 + c = 2$~~
~~$c = 2$~~

$\Rightarrow \int_2^r u^{-2}\, du = \int_1^\theta v^{-1}\, dv$

$\left[\frac{1}{u}\right]_2^r = -\ln u \Big|_1^\theta$

$\frac{1}{r} - \frac{1}{2} = -\left[\ln\theta - \ln(1)\right]$

$\frac{1}{r} - \frac{1}{2} = -\ln\theta$.

$\Rightarrow \frac{1}{r} = -\ln\theta + \frac{1}{2}$

$\frac{1}{r} = \frac{1 - 2\ln(\theta)}{2}$

$\Rightarrow r = \frac{2}{1 - 2\ln(\theta)}$

A9).

$$\frac{dv}{dt} = 9.8 - 0.196v$$

$$\Rightarrow \frac{dv}{dt} + 0.196v = 9.8$$

$$e^{\int 0.196\,dt} = e^{0.196t}$$

$$\Rightarrow e^{0.196t}\frac{dv}{dt} + 0.196e^{0.196t}v = 9.8e^{0.196t}$$

$$\Rightarrow \int (ve^{0.196t})'\,dt = \int 9.8e^{0.196t}\,dt$$

$$\Rightarrow ve^{0.196t} + k = 50e^{0.196t} + c$$

$$\Rightarrow \underline{ve^{0.196t} = 50e^{0.196t} + c - k}$$

A10).

$$ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}$$

$$\cancel{\int ty' + 2y\,dy = \int t^2 - t + 1\,dt}$$

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

$$\frac{dy}{dt} + \frac{2y}{t} = t - 1 + \frac{1}{t}$$

$$\mu\left(\frac{dy}{dt}\right) + \frac{2\mu y}{t} = \mu\left(t - 1 + \frac{1}{t}\right)$$

$$\text{let } \frac{d\mu}{dt} = \frac{2\mu}{t}. \quad \cancel{\text{Also} \int \frac{d\mu}{\mu} = \int \frac{2dt}{t}}$$

$$\mu\left(\frac{dy}{dt}\right) + y\left(\frac{d\mu}{dt}\right) = \mu\left(t - 1 + \frac{1}{t}\right)$$

$$\Rightarrow \mu y = \int \mu \left(t - 1 + \frac{1}{t}\right) dt$$

$$y = \frac{1}{\mu}\int \mu \left(t - 1 + \frac{1}{t}\right) dt$$

Also, $\int \frac{d\mu}{\mu} = \int \frac{2\,dt}{t} \Rightarrow \ln|\mu| = 2\ln|t| + c$

$$\cancel{\Rightarrow} \quad = \ln t^2 + c$$

$$\Rightarrow \mu = t^2.$$

$$y = \frac{1}{t^2}\int (t^3 - t^2 + t)\, dt.$$

$$y = \cancel{\frac{t^4}{4}}\frac{1}{t^2}\left(\frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c\right)$$

$$y = \left(\frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + c\right)\frac{1}{t^2}$$

$y = \frac{1}{2}$ & putting $t = 1$.

$$\frac{1}{2} = \left(\frac{1^2}{4} - \frac{1}{3} + \frac{1}{2} + c\right)$$

$$\Rightarrow \quad c = \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$$

$$\Rightarrow \quad c = \frac{6 - 3 + 4 - 6}{12}.$$

$$\Rightarrow \quad c = \frac{1}{12}$$

$\therefore$ The eq. is $\rightarrow$ $\underline{y = \frac{1}{t^2}\left(\frac{t^4}{4} - \frac{t}{3} + \frac{1}{2} + \frac{1}{12}\right)}$, A11). $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $y(0) = -1$.

$\Rightarrow \frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$\Rightarrow \frac{dy}{y^3} = \frac{x}{\sqrt{1+x^2}} dx$.

$\Rightarrow \int \frac{1}{y^3} dy = \int \frac{x}{\sqrt{1+x^2}} dx$.

$\Rightarrow \frac{-1}{2y^2} = \sqrt{1+x^2} + c$

$\Rightarrow \frac{1}{y^2} = -2\left(\sqrt{1+x^2} + c\right)$

$\Rightarrow y^2 = \frac{1}{-2\left(\sqrt{1+x^2} + c\right)}$

$\Rightarrow (-1)^2 = \frac{1}{-2\left(\sqrt{1+0} + c\right)}$

$\Rightarrow 1 = \frac{1}{-2(1+c)}$.

$\Rightarrow \frac{-1}{2} = 1+c$.

$\Rightarrow c = -\left(1 + \frac{1}{2}\right) \Rightarrow c = \frac{-3}{2}$.

$\Rightarrow$ Req. eq $\rightarrow y^2 = \frac{1}{-2\left(\sqrt{1+x^2} - \frac{3}{2}\right)}$

A12). $f(x) = x^3 - 10x^2 + 6$ at $x = 3$.

~~at a = 1~~.

~~f(a) =~~
~~f'(a) =~~
~~f''(a) =~~
~~f'''(a) =~~

$f(x=3) = -57$

$f'(x) = 3x^2 - 20x = -33$

$f''(x) = 6x - 20 = \cancel{8}\, -2$

$f'''(x) = 6$

$f''''(x) = 0$

$$f(x) = -57 + \frac{(x-3)}{1!}(-33) + \frac{(x-3)^2}{2!}(-2) + \frac{(x-3)^3}{3!}(6)$$

$$f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

Ans

A13). $(1+x)^\mu = f(x)$ $\quad f(0) = 1^\mu$, $f'(0) = \mu$, $f''(0) = 0$.

$$f(x) = 1^\mu + \frac{(1+x)^\mu(\mu)}{1!}$$

~~$= 1^\mu + (1+x)^\mu$~~ $\Rightarrow f(x) = 1 + \mu(1+x)^\mu$

Ans

A14). $f(x) = \sqrt{1+x}$.

$f(0) = 1$ $\quad f'(0) = \frac{1}{2}$

$f''(0) = -\frac{1}{4}$ $\quad f'''(0) = \frac{3}{8}$

$$f(x) = 1 + \frac{\sqrt{1+x}}{1!}\left(\frac{1}{2}\right) + \frac{(\sqrt{1+x})^2}{2!}\left(\frac{-1}{4}\right) + \frac{(\sqrt{1+x})^3}{3!}\left(\frac{3}{8}\right) \dots$$

$$f(x) = 1 + \frac{\sqrt{1+x}}{2} - \frac{(1+x)}{8} + \frac{(\sqrt{1+x})^3}{16} \dots$$

Ans

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A15) $f(x) = e^{-6x}$. $\quad x = -4$

$f(-4) = e^{24}$

$f'(-4) = -6e^{24}$

$f''(-4) = 36e^{24}$

$f'''(-4) = -216e^{24}$

$$f(x) = e^{24} + \frac{(x+4)(-6e^{24})}{1!} + \frac{(x+4)^2(36e^{24})}{2!} + \frac{(x+4)^3(-216e^{24})}{3!} \dots$$

$$f(x) = e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4)^2 - 36e^{24}(x+4)^3 \dots$$

A16) $f(x) = \ln(3+4x)$ $\quad x = 0$.

$f(0) = \ln(3)$, $\quad f'(0) = \frac{4}{3}$

$f''(0) = \frac{-16}{9}$ $\quad f'''(0) = \frac{128}{27}$

$$f(x) = \ln(3) + \frac{(x-0)}{1!}\left(\frac{4}{3}\right) + \frac{(x-0)^2}{2!}\left(\frac{-16}{9}\right) + \frac{(x-0)^3}{3!}\left(\frac{128}{27}\right) \dots$$

$$f(x) = \overset{1.09861}{\cancel{\ln(3)}} + \frac{4x}{3} - \frac{8x^2}{9} + \frac{64x^3}{81}$$