Financial Mathematics

1. $d^{4} = 8\%$

i) $\delta = ?$

$\cancel{\delta} \; d = 1 - \left(1 - \frac{d^{p}}{p}\right)^{p}$

$= 1 - \left(1 - \frac{0.08}{4}\right)^{4} = 0.07763184$

$\delta = \cancel{\frac{\log}{1-d}} \; \log \frac{1}{1-d} = 0.0808108 = 8.08108\%$

ii) $i = \frac{d}{1-d} = 0.0841657 = 8.41657$

iii) $d^{12} = ?$

$d^{12} = \cancel{1}\,p\,(1-(1-d)^{1/p}) = 0.0805393$

$= 8.05393$

2. $\delta(t) = 0.004t + 0.0002t^{2}$

i) $PV = 1000 \cdot e^{-\delta t}$

$= 1000 \cdot \cancel{e^{-\int_0^{10}(0.004\times 10 + 0.0002\times 10^{2})}} \; e^{-\int_0^{10} \delta(t)}$

$= \cancel{941.764533} \; 765.92833$

ii) $\cancel{941.764533 \times (1+i)^{10} = 1000}$

$\cancel{(1+i)^{10} = \frac{1000}{941.764533}}$

$\therefore \cancel{i = 0.00601803}$

ii) $\cancel{765}$ $PV(1+i)^{10} = 1000$

$$(1+i)^{10} = \frac{1000}{PV}$$

$$i = 0.027025\cancel{41}$$

$$= 2.70254\%$$

3] i) Interest earned on a investment of 1 paid at the beginning of the period is d. Interest earned on an investment of 1 paid at the end of the period is i. Therefore if we discount i from the end of the period to the beginning of the period to the disc. factor v, we get d.

ii) a) $d^{2} = ?$

$$\frac{d^{4}}{4} = 2.25\%$$

$$d^{4} = 9\%$$

$$d = 1 - \left(1 - \frac{d^{4}}{4}\right)^{4}$$

$$= 0.087007$$

$$d^{2} = 2\left(1 - (1-d)^{1/n}\right) = 0.0889875$$

b) $d^{1/2} = 5\%$

$$d = 1 - \left(1 - \frac{d^{1/2}}{1/2}\right)^{1/2} = 0.0513267$$

$$d^{2} = 2\left(1 - (1-d)^{1/n}\right) = 0.0519925$$

iii) $i = 5\%$, $n = \frac{91}{365}$

$d = \frac{i}{1+i} = 0.047619$

$(1-d)^n = 1 - nd$

$(1-d)^{\frac{91}{365}} = 1 - \frac{91}{365} d$

$\frac{91}{365} d = 1 - 0.9870909$

$= 0.0484946 1$

4] i) Already Done in 3(i)

ii) $i = 0.0\cancel{8}8$

a) $d^{(12)} = ?$

$d = \frac{i}{1+i} = 0.0740740$

$d^{(12)} = 12(1 - (1-d)^{1/12})$

$= 0.0767147$

b) $i^{(365)} = \cancel{365(1-(1-d)^{1/365})}\ 365((1-d)^{1/365} - 1)$

$= 0.0769691$

c) $\delta = ?$

$= \ln(1+i) = 0.076961$

d) $i^{(0.5)} = 0.5((1+i)^{1/0.5} - 1) = 0.0832$

5] i) $d^{(4)} = 6\%$

$i^{(2)} = 9\%$

$$d = 1 - \left(1 - \frac{d^{4}}{4}\right)^{4}$$

$$= 0.0586634$$

$$i = \frac{d}{1-d} = 0.062319315$$

a) $$i^{(2)} = 2\left((1+i)^{1/2} - 1\right)$$
$$= 0.0613775$$

b) $$d^{(12)} = 12\left(1 - (1-d)^{1/12}\right)$$
$$= 0.0603025$$

(ii) $$2{,}00{,}000(1+i)^{1} = 2{,}20{,}000$$
$$1+i = 1.1$$
$$i = 0.1$$

15/4/05   17/7/05   15/4/06

$$2{,}00{,}000(1+i)^{\frac{93}{365}} = 2{,}04{,}916.3563$$

OR

Original term of the loan : 365 day

O/s term of the loan = 272 days

Rebate allowed = $\frac{272}{365} \times 2{,}00{,}000$

$$= 14904.1096$$

Sam paid to settle the loan = 220000 - 14904.1096

$$= 205095.89$$

7] $26{,}000 = 20{,}000\,(1+i)^{\frac{17}{12}}$

$\therefore i = 20.3457067\%$

i) $i^{(4)} = 4\left((1+i)^{1/4} - 1\right)$

$= 18.955 25\%$

ii) $d^{(2)} = ?$

$d = \frac{i}{1-i} = \cancel{25.54254} \; 16.9060511$

$d^{(2)} = 2\left(1 - (1-d)^{1/2}\right)$

$= \cancel{27.422...} \; 17.0882352$

iii) $26{,}000 = 20{,}000\left(1 + \frac{17}{12} i\right)$

$\therefore i = 21.17647$

iv) The simple interest is greater than compound interest while $i^{(4)} < i$.

9] i) ~~Force of Int. as~~ is Measure of interest at individual moment of times is called FOI.

ii) $i^{(2)} = 0.0775$

$\therefore$ EL ER $(1+i)^{r} = \left(1 + \frac{i^{(p)}}{p}\right)^{p}$

$i = \left(1 + \frac{i^{(p)}}{p}\right)^{p} - 1 = 7.9001\%$

$\delta = \ln(1+i)$
$= 7.603613$

$d^{a} = \frac{i}{1+i} = 0.07321728$

$d^{(12)} = \cancel{1-(1-d} \ 12(1-(1-d)^{n})$
$= 7.5795$

$i^{(1/2)} = \frac{1}{2}((1+i)^{2} - 1)$

$= 8.2122\%$

10] $\delta(t) = \begin{cases} 0.08 & 0 \le t \le 5 \\ 0.06 & 5 \le t \le 10 \\ 0.04 & t \ge 10 \end{cases}$

Case I : $0 \le t \le 5$

$v(t) = e^{-\int_0^t 0.08 \, dt}$
$= e^{-[0.08t]_0^t} = e^{-0.08t}$

Case II : $5 \le t \le 10$

$v(t) = e^{-\int_5^t 0.06 \, dt}$
$= e$

11.] a) $d = 18\%$ p.a

$PV = 30{,}000\,(1-d)^{9/12}$
$= 43085.42622$

b) i) $\delta = 6\%$
$d = 1 - e^{-\delta}$
$= 0.0582354$

$d^{(12)} = 12(1-(1-d)^{1/12})$
≈ 0.0598502

ii) $d^{(4)} = 6\%$
$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^{4}$

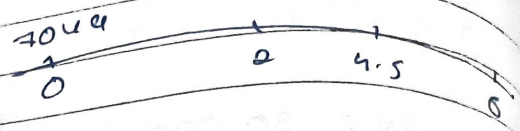
≈ 0.058669

$i = \frac{d}{1-d} = 0.0623193$

$i^{(12)} = 12\left((1+i)^{1/12} - 1\right)$
$\approx 0.06060708 8$

$i^{(4)} = 4\left((1+i)^{1/4} - 1\right)$
$\approx 0.06091340 591$

c) $d^{(4)}, \delta, i^{(4)}, i$



12.

$C = 7049$

$AV_6 = C \cdot A(0,6)$

$= C \cdot A(0,2) \cdot A(2,4.5) \cdot A(4.5,6)$

$\Rightarrow$ For $A(0,2)$

$i^{(12)} = 6\%$. $\quad \frac{i^{(12)}}{12} = 0.5\%$.

$A(0,2) = (1.005)^{24}$

$\Rightarrow$ For $A(2,4.5)$

$i^{(4)} = 1.5\%$.

$A(2,4.5) = (1 + 10(0.015)) = 1.15$

$\Rightarrow$ For $A(4.5,6)$

$d^{(12)} = 6\%$. $\quad \frac{d^{(12)}}{12} = 0.5\%$

$A(4.5,6) = \frac{1}{(1-0.005)^{18}}$

$\therefore AV_6 = 7049 \times (1.005)^{24} \times 1.15 \times \frac{1}{(1-0.005}$

$= 9999.896$

13. Amt invested : $C = x$

$AV_n = 2x$

Time $= n$

(i) $i : 10\%$

$AV_n = C(1+i)^n$

$2x = x(1+10\%)^n$

$2 = 1.1^n$

$\ln 2 = n \ln 1.1$

$n = 7.27$ years

ii) $d^{(12)} = 10\%$ $\quad \frac{d^{(12)}}{12} = 0.83333\%$

$$AV_n = \frac{C}{\left(1 - \frac{d^{(12)}}{12}\right)^{n \cdot 12}}$$

$$2x = \frac{x}{(1 - 0.8333)^{n \cdot 12}}$$

$(1 - 0.833333\%)^{12n} = 1/2$

$12n \ln(1 - 0.83333) = \ln 1/2$

$n = 6.902$ years