Probability & Statistic - Assignment 1

1. i Data: 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

Ans Mean $(\bar{x}) = \frac{\sum x}{n} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$

$= \boxed{13.4}$

Ans Median : $n = 10$ (even)

$= \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}\right]$

$= \frac{1}{2}\left[5^{th} \text{term} + 6^{th} \text{term}\right]$

$= \frac{1}{2}\left[11 + 15\right] = \boxed{13}$

Ans Mode $= 18$ [occurs the most no. of times]

Ans Standard deviation $= \sqrt{\frac{1}{N}\sum(x_i - \bar{x})^2}$

$= \sqrt{\frac{88.36 + 40.96 + 19.36 + 19.36 + 5.76 + 2.56 + 21.16 + 21.16 + 21.16 + 134.56}{10}}$

$= \underline{6.1188}$

Inter Quartile range $= Q_3 - Q_1$

$Q_1 = \frac{1}{4}(n+1)^{th} \text{term} = \frac{10}{4} = 2.5^{th} \text{term}$

$= 7 + 0.5(9-7)$

$= 8$

$Q_3 = \frac{3}{4}(n+1)^{th} \text{term} = 7.5^{th} \text{term}$

$= 18 + 0.5(18-18) = 18$

Ans $\Rightarrow IQR = 18 - 8 = \boxed{10}$

ii

No. of households (x)	No. of people (f)	fx	cf	$(x-\bar{x})^2$
0	5	0	5	4
1	11	11	16	1
2	26	52	42	0
3	15	45	57	1
4	3	12	60	4
	= 60	= 120		= 10

Ans Mean $(\bar{x}) = \frac{\Sigma fx}{\Sigma f} = \frac{120}{60} = \boxed{2}$

Median : N = 60 (even)

$\cancel{\text{Median} = \frac{1}{2}\left[\left(\frac{N}{2}\right)^{th} \text{term} + \left(\frac{N}{2}+1\right)^{th} \text{term}\right]}$

$\cancel{= \frac{1}{2}\left[30^{th} \text{term} + 31^{st} \text{term}\right]}$

$\cancel{= \boxed{2}}$

Median term $= \frac{N+1}{2} = \frac{61}{2} = 30.5^{th}$ term

Ans Median $= \boxed{2}$

Ans Mode $= \boxed{2}$

Standard deviation (σ) : $\sqrt{\frac{1}{N}\Sigma(x_i-\bar{x})^2}$

$: \sqrt{\frac{1}{60}(10)}$

Ans $\sigma = \underline{\underline{0.4082}}$.

$IQR = Q_3 - Q_1$

$Q_1 = \frac{1}{4}(N+1)^{th}$ term $= 15.25^{th}$ term

$= 1$

$Q_3 = \frac{3}{4}(N+1)^{th}$ term $= 45.75^{th}$ term

$= 3$

Ans $IQR = \boxed{2}$

2. $X \sim Poisson(0.5)$

(i) Prob. that no claims arise in a year $= P(X=0) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$= \frac{e^{-(0.5)}[(0.5)^0]}{0!}$

$= e^{-(0.5)}$

$= 0.60653$

ii. Prob. of one or more claim in one year & no claim in other 2 years $=$ ~~$X \sim Bin(3, \frac{1}{2})$~~

~~$\Rightarrow \lambda = 0.5 \times 3 = 1.5$~~ ~~$P(X \geq 1)$~~

$\Rightarrow \quad = P(X \geq 1) \times P(X=0) \times P(X=0)$

$= P(X>0) \times P(X=0) \times P(X=0)$

$= [1 - P(X=0)][P(X=0)]^2$

$= 0.14474$

iii. ~~$X \sim G$~~ $X \sim Exponential(\lambda)$

$\Rightarrow \lambda$: 1 yr $\longrightarrow$ 0.5 claim

2 yr $\longrightarrow$ 1 claim

$\Rightarrow \quad \lambda = 1$

$X \sim Exp(1)$

$P(X>2) = 1 - (1 - e^{-\lambda x})$

$= e^{-\lambda x}$

$= e^{-2}$

$= 0.135335$

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3. $X \sim Gamma(2, \frac{1}{2})$; $0 < x < \infty$

(i) Mean $= E(X) = \frac{\alpha}{\lambda} = \frac{2}{\frac{1}{2}} = \boxed{4}$

Standard deviation $= \sqrt{V(X)}$

$= \sqrt{\frac{\alpha}{\lambda^2}} = \sqrt{\frac{2}{\frac{1}{4}}} = 2\sqrt{2}$

$= 2 \times 1.414$

$= 2.828$

(ii) The shape of the Gamma distribution is such that it is positively skewed or right skewed.

Coeff of skewness $= \frac{2}{\sqrt{\alpha}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

(iii) Cumulative distribution func:-

$$F_X(x) = \int_0^x f_X(x).$$

$$= \int_0^x \frac{\lambda^\alpha e^{-\lambda x} x^{\alpha-1}}{\Gamma(\alpha)} dx$$

$$= \int_0^x \frac{(\frac{1}{2})^2 e^{-\frac{1}{2}x} x}{(2-1)!}$$

$$= \frac{1}{4}\int_0^x x e^{-\frac{1}{2}x} dx$$

$$= \frac{1}{4}\left[-2xe^{-\frac{1}{2}x} + 2\times(-2)e^{-\frac{1}{2}x}\right]_0^x$$

$$= -\frac{1}{4}\left[4e^{-\frac{1}{2}x} + 2xe^{-\frac{1}{2}x}\right]_0^x$$

$$= -\frac{1}{4}\left[4e^{-\frac{1}{2}x} - 4 + 2xe^{-\frac{1}{2}x}\right]$$

$$= -\frac{1}{2}e^{-\frac{1}{2}x}\left[2+x\right] + 1$$

4. No. of male = 10
No. of female = 8
No. of qualified male = 6
" " " female = 7

Let Event A → woman chosen
Event B → man chosen
Q → qualified worker chose

$P(A) = \frac{8}{18}$ $P(Q \cap A)$ $P(\cancel{Q/A}) = \frac{7}{8}$

$P(B) = \frac{10}{18}$ $P(Q \cap B)$ $P(Q/B) = \frac{6}{10}$

Total no. of qualified activities = 13

~~Let Prob that atleast 1 woman is chosen = P(C)~~

$\cancel{\Rightarrow P(C) = P(A)P(A) + P(A)P(B) + P(B)P(A)}$

$\cancel{= \frac{8}{18} \times \frac{8}{18} + \frac{8}{18} \times \frac{10}{18} + \frac{8}{18} \times \frac{10}{18}}$

$\cancel{= \frac{8}{18}\left[\frac{28}{18}\right] = \frac{224}{324} = \frac{56}{81}}$

$\cancel{\Rightarrow P(\text{2 qualified women}/C) = \frac{\frac{7}{8} \times \frac{6}{7}}{\frac{56}{81}}}$

~~=~~

Prob. that atleast 1 woman is chosen = P(E) = P(2 women) + P(male)P(female) + P(female)P(male)

$= \frac{8}{18} \times \frac{7}{17} + \frac{10}{18} \times \frac{8}{17} + \frac{8}{18} \times \frac{10}{17}$

$= \frac{216}{306} = \boxed{\frac{12}{17}}$

Ans $\Rightarrow P(\text{2 qualified female}/\text{atleast one female}) = \frac{\frac{7}{13} \times \frac{6}{12}}{\frac{12}{17}} = \boxed{\frac{119}{312}}$

Q5. $P(L_1) = 0.4$

$P(L_2) = 0.5$

$P(L_3) = 0.1$

$P(L/L_1) = 0.2$

$P(L/L_2) = 0.4$

$P(L/L_3) = 0.1$

$$\therefore P(L_2/L) = \frac{P(L_2)P(L/L_2)}{P(L_1)P(L/L_1) + P(L_2)P(L/L_2) + P(L_3)P(L/L_3)}$$

$$= \frac{0.5 \times 0.4}{(0.4)(0.2) + (0.5)(0.4) + (0.1)(0.1)}$$

$$= \frac{0.02}{0.08 + 0.02 + 0.01} = \boxed{\frac{2}{11}}$$

Q6. $X \sim U(1, 2)$; $1 < x < 2$; $a = 1$, $b = 2$

$$Y = \frac{3}{6 - X}$$

$$\Rightarrow Y \sim U\left(\frac{3}{6-a}, \frac{3}{6-b}\right)$$

$$\Rightarrow Y \sim U\left(\frac{3}{5}, \frac{3}{4}\right)$$

$$\Rightarrow \text{PDF of } Y = f_Y(y) = \frac{1}{b' - a'} \quad \left[b' = \frac{3}{4}, a' = \frac{3}{5}\right]$$

$$= \frac{1}{\frac{3}{4} - \frac{3}{5}} = \boxed{\frac{20}{3}}$$

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7.

x	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

(i) $F(3) = \sum_{i=1}^{3} P(X=x)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

(ii) $E(x) = \frac{k+1}{2}$; [k = no. of outcomes = 5]

$= \frac{5+1}{2} = \boxed{3}$

(iii) $V(x) = \frac{k^2-1}{12} = \frac{25-1}{12} = \boxed{2}$

(iv) Coeff. of Skewness $= \frac{\mu_3}{\sigma^3} = \frac{E(x-\bar{x})^3}{\sigma^3}$

$= \frac{E(x^3 - \bar{x}^3 - 3x^2\bar{x} + 3x\bar{x}^2)}{\sigma^3}$

$= \frac{E(x^3) - 3\bar{x}E(x^2) + 3\bar{x}^2E(x)}{\sigma^3}$

$= \frac{E(x^3) - 3\bar{x}[V(x) + [E(x)]^2] + 3\bar{x}^3}{\sigma^3}$

$= \frac{E(x^3) - 3E(x)V(x) - \cancel{3[E(x)]^3} + \cancel{3[E(x)]^3}}{\sigma^3}$

$= \frac{E(x^3) - 3E(x)V(x)}{\sigma^3}$

$= \frac{\sum_{i=1}^{5} x^3 P(X=x) - 3\times3\times2}{2\sqrt{2}}$

$= \frac{[0.05 + 1.2 + 9.45 + 25.6 + 6.25] - 18}{2\sqrt{2}}$

Coeff. of skewness = $\underline{8.679}$.

8. $X \sim Bin(15, 0.2)$

$P(X<3) = P(X\leq 2) = P(X=0)+P(X=1)+P(X=2)$

$= {}^{15}C_0 (0.2)^0 (0.8)^{15} +$

${}^{15}C_1 (0.2)(0.8)^{14} +$

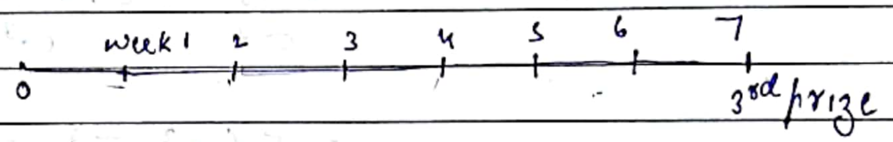
${}^{15}C_2 (0.2)^2 (0.8)^{13}$

$= (0.8)^{15} + 15(0.2)(0.8)^{14} + 15\times 7(0.2)^2(0.8)^{13}$

$= 0.0352 + 0.1319 + 0.2308$

$= \underline{\underline{0.3979}}$

9. Probability of winning $= 0.01$



$X \sim$ Negative Bin $(7, 0.01)$

$\Rightarrow P(X=3) = {}^{n-1}C_{r-1}\, p^r q^{n-r}$; $[\, n=7,\ r=3\,]$

$= {}^{6}C_2 (0.01)^3 (0.99)^4$

$= \cancel{3\times5}\ \underline{0.00001455}$

10. Avg No. of claims in a working week (5 days) $= 2$

$\Rightarrow$ Avg / day $= \frac{2}{5} = \cancel{0.00}\ 0.4$

$\Rightarrow X \sim Poi(\lambda = 0.4)$

$\Rightarrow P(X>2) = 1 - P(X\leq 2)$

$= 1 - 0.99207$

$= \underline{\underline{0.00793}}$

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11. Probability of having male ~~chat~~ child = Prob. of female = $\frac{1}{2}$

Prob. of having 18 boys after 5 boys & 2 girls ~~follow~~ is such that :

$X \sim Geo(1, 0.5)$

Prob. of 3 consecutive ~~boob~~ boys = ~~$P(X=$~~ ~~$[pq^{n-1}]^3$~~

~~$= [(0.5)(0.5)^2]^3$~~

~~$= (pq^{8-1})(pq^{9-1})(pq^{10-1})$~~

~~$= p^3 q^{24}$~~

~~$= (0.5)^{27}$~~

12. ~~$X \sim Poi(10)$~~

~~Prob. that time b/w 2 claims is less than 0.1 days:~~
~~In this case $X \sim$ Exponential (λ)~~

~~Here $\lambda = 0.1 \times 10 = 1$ [using unitary method]~~

~~as 10 claims $\to$ 1 day~~
$\Rightarrow$ 0.1 days = 1 claim

~~$\Rightarrow P(X<2) = \int_0^2 \lambda e^{-\lambda x} = 1 - e^{-\lambda}$~~

12. $X \sim Exp(\lambda)$

$\Rightarrow \lambda = $ 10 claims $\rightarrow$ 1 day

1 claim $\rightarrow$ 0.1 days

$\Rightarrow \lambda = 0.1$

$\Rightarrow P(X<0.1) = F_X(0.1)$

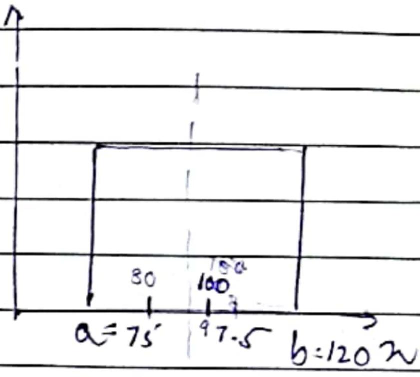
$= 1 - e^{-(0.1)(0.1)}$

$= 0.0099501$

13. $X \sim U(75, 120)$; $a = 75, b = 120$

To find : $P(80 < X < 100)$

$E(X) = \frac{a+b}{2} = 97.5$



$P(80 < X < 100) = P(X<100) - P(X<80)$

$= F_X(100) - F_X(80)$

$= \frac{100-75}{120-75} - \frac{80-75}{120-75}$ $\left[F_X = \frac{x-a}{b-a}\right]$

$= \frac{\cancel{20}\,4}{\cancel{45}\,9} = \boxed{\frac{4}{9}}$

14. $X \sim Gamma(5, 0.4)$; $\alpha = 5, \lambda = 0.4$

$P(X < 22.89) = ?$

$\Rightarrow$ ~~$\pm$~~ $X \sim Gamma(5, 0.4)$

Converting Gamma $\rightarrow$ Chi Square distribution

$$2\lambda X \sim \chi^2_{\nu = 2\alpha}$$

$$\Rightarrow \quad 0.8X \sim \chi^2_{10}$$

$$P(X < 22.89) = P(0.8X < 18.312)$$

$$= P(\chi^2_{10} < 18.312)$$

Using table book:

$$P(\chi^2_{10} < 18) = 0.9450 \quad \& \quad P(\chi^2_{10} < 18.5) = 0.9529$$

$$\Rightarrow \text{Change in } 0.5 \Rightarrow \Delta P = 0.0079$$

$$\Rightarrow \text{Change in } 0.312 \Rightarrow \frac{0.312 \times 0.0079}{0.5} = \Delta P$$

$$= 0.0049296$$

~~$\Rightarrow P$~~

Ans $$P(X < 22.89) = P(\chi^2_{10} < 18.312) = 0.9450 + 0.0049296$$

$$= \underline{\underline{0.9499296}}$$

15 $$f_X(x) = \frac{k}{(x+5)^4}, \quad x > 0$$

(i) K :

$$\Rightarrow \int_0^\infty f_X(x) = 1$$

$$\Rightarrow k \int_0^\infty \frac{1}{(x+5)^4} dx = 1$$

$$\Rightarrow \frac{-K}{3[(x+5)^3]_0^\infty} = 1$$

$$\Rightarrow -k\left[0 - \frac{1}{5^3}\right] = 3$$

$$\frac{K}{125} = 3$$

Ans $\boxed{k = 375}$

(ii) $$F(10) = \int_0^{10} \frac{375}{(x+5)^4}\,dx$$

$$= \frac{-375}{3}\left[\frac{1}{(x+5)^3}\right]_0^{10}$$

$$= -125\left[\frac{1}{15^3} - \frac{1}{5^3}\right]$$

$$= -125\left[\frac{1-27}{3375}\right]$$

$$= \boxed{\frac{26}{27}}$$

iii $$E(x) = 375\int_0^{\infty} \frac{x}{(x+5)^4}\,dx$$

$$= 375\int_0^{\infty} \frac{1}{(x+5)^3} - \frac{5}{(x+5)^4}\,dx$$

$$= \left[\frac{-375}{2(x+5)^2} + \frac{\overset{125}{\cancel{375}}\times 5}{\cancel{3}\times(x+5)^3}\right]_0^{\infty}$$

$$= \frac{-375}{2}\left[0 - \frac{1}{25}\right] + 625\left[0 - \frac{1}{125}\right]$$

$$= 7.5 + 5$$

$$= \boxed{12.5}$$