① X = claim size of on a home insurance

$X \sim N(800, 100^2)$

Y = Claim size on a car insurance

$Y \sim N(1200, 300^2)$

$P(y_1 + y_2 + y > x_1 + x_2 + x_3 + x_4 + 800)$

$P(y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 > 800)$

$y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 \sim N(3(1200) - 4(800),$

$3 \times 300^2 + 4 \times 100$

$\therefore \sim N(400, 310000)$

$P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right) = P(Z > 0.718)$

$= 1 - P(Z < 0.718)$

$= 0.23576$

② N = number of claims

$N \sim$ negative binomial distribution $(3, 0.9)$

X = claim amount

$X \sim$ Gamma $(6, 2)$

Y = total claim amount

$M_y(t) = M_N \ln(M_X(t))$

$M_X(t) = (1 - t/2)^{-6}$

$$M_y(t) = M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$\left(M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^K\right)$$

$$M_y(t) = \left(\frac{pe^{\log(1-t/2)^{-6}}}{1-q(e^{\log(1-t/2)^{-6}}}\right)^3$$

$$M_y(t) = \left(\frac{(0.9)\left(1-\frac{t}{2}\right)^{-6}}{1-(0.1)\left(1-\frac{t}{2}\right)^6}\right)^3$$

$$E(N) = \frac{0.3}{0.9} = \frac{10}{3} = 3.33$$

$$V(N) = \frac{K(0.1)}{(0.9)^2} = 3.33 \times \frac{1}{9} = 0.37$$

$$E(X) = \frac{\alpha}{\lambda} = \frac{6}{2} = 3 \qquad V(X)\ \frac{\alpha}{\lambda^2} = \frac{6}{4} = 1.5$$

$$V(Y) = E(N)\,V(X) + E(X)^2\,V(N)$$

$$= 5 + \frac{10}{3} = \frac{25}{3} = 8.333$$

$$SD(Y) = \sqrt{\frac{25}{3}} = 2.88$$

(3) $f_X(x) = \lambda e^{-\lambda(x-\alpha)}$

$$M_X(t) = E(e^{tx})$$

$$= \int_\alpha^\infty e^{tx} \lambda e^{-\lambda(x-\alpha)} \, dx$$

$$= \lambda e^{\lambda\alpha} \int_\alpha^\infty e^{x(t-\lambda)} \, dx$$

$$= \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left(e^{(t-\lambda)x} \right)_0^\infty = \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left[e^{(\lambda-t)x} \right]_0^\infty$$

$$M_X(t) = \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left(0 - e^{-\lambda\alpha} e^{tx} \right)$$

$$= \frac{\lambda e^{t\alpha}}{\lambda - t}$$

ii) $M'_X(t) = \lambda (e^{t\alpha})(\lambda - t)^{-2}(-1)(-1) + \lambda(\lambda - t)^{-1}(e^{t\alpha})(\alpha)$

$$= \frac{\lambda e^{t\alpha}}{(\lambda - t)^2} + \frac{\lambda e^{t\alpha}\alpha}{\lambda - t}$$

$E(X) = M'_X(0) =$

$$\frac{\lambda(\alpha)}{\lambda^2} + \lambda\frac{\alpha}{\lambda} = \frac{1+\lambda\alpha}{\lambda}$$

$E(x^2) = M_x''(0) = \frac{2\gamma(1)}{\lambda^2} + \frac{\lambda\alpha}{\lambda^2} + \frac{\lambda\alpha^2}{\lambda}$

$= \frac{2}{\lambda} + \frac{2\alpha}{\lambda} + \alpha^2$

$V(x) = E(x^2) - (E(x))^2$

$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \left(\frac{1 + \lambda^2\alpha^2 + 2\alpha\lambda}{\lambda^2}\right)$

$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \frac{1}{\lambda^2} - \alpha^2 - \frac{2\alpha}{\lambda}$

$V(x) = \frac{1}{\lambda^2}$

Q4) $\phi_x(t) = \left(\frac{p}{1-qt}\right)^K$, $p+q=1$

$\therefore$ Distribution is -ve binomial

$P(x=4) = \frac{\sqrt{K+4}}{\sqrt{5}\sqrt{K}} p^4 q^4$

Q5) $f(x,y) = ax^2 + axy$ $\quad 0 < x < 5$

$10 < y < 20$

$1 = \int_{10}^{20} \int_0^5 ax^2 + axy \, dx\, dy$

$1 = \int_{10}^{20} \frac{1250}{3} + \frac{25}{2} ay \, dy$

$a = \frac{3}{6875}$

$a = \frac{3}{6875}$

$f_X(x) = a \int_{10}^{20} x^2 + xy \, dy$

$= \frac{3}{6875} \left(10x^2 + 150x\right)$

6) $X \rightarrow \text{Poi}(\lambda) \quad Y \sim \text{Poi}(-\mu)$

$M_X(t) = e^{\lambda(e^t - 1)}$

$M_Y(t) = e^{\mu(e^t - 1)}$

$M_{X-Y}(t) = E\left(e^{t(x-y)}\right)$

$= E\left(e^{tx} - ty\right)$

$= \frac{E(e^{tx})}{E(e^{ty})} = \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$

$M_{X+Y}(t) = E\left(e^{t(x+y)}\right)$

$= E(e^{tx})\, e^{ty})$

$= e^{\lambda+\mu(e^t-1)}$

Q7) i) $Var(X|Y=Z) = E(X^2|Y=Z) - (E(X|Y=Z))^2$

$= \frac{1^2(0)+2^2(0.3)+3^2(0.1)+4^2(0.2)}{0.6} - \left(\frac{1(0)+2(0.3)+3(0.1)\ldots}{0.6}\right)^2$

$= \frac{29}{36}$

ii) $f(u,v) = \frac{48}{67}(2uv - v^2)$

$E(U|V=v) = \int_0^1 \frac{u\, f(U, V=v)\, du}{f(V=v)}$ — ①

$f_V(V=v) = \int_0^1 \frac{48}{67}(2vu - v^2)\, dv$

$= \frac{48}{67}\left(v - \frac{1}{3}\right)$

$= \frac{48}{67}\left(\frac{3v-1}{(3)}\right)$

Q8) $\alpha = 3$ $\lambda = 2$

$X \sim \text{Gamma}(3, 2)$

$E(Y/X = x) = 3x + 1$

$E(Y) = E(E(Y/X = x))$

$= E(3x + 1)$

$= 3E(X) + 1$

$= 3(3/2) + 1 = 11/2$

$V(Y) = E(2x^2 + 5) + V(3x + 1)$

$= 2(3) + 5 + 9(3/4)$

$= 11.75$ Ans

Q9) $z = x + y$

$E(x) = 3$ $E(Y) = 4$

$\sigma X = \sqrt{V(X)} = 2$ $\sigma y = \sqrt{V(y)} = 1$

$\text{cov}(x, z) = \text{cov}(x, x+y)$

$= \text{cov}(x, X) + \text{cov}(x, y)$

$= V(x) + (-0.6)$

$= 3.4$

$$\text{Var}(Z) = \text{Var}(X+Y)$$

$$= \text{Var}(X) + V(Y) + \text{Cov}(X,Y)$$

$$= 3.8 \text{ Ans}$$

Q10) $f(x,y) = K x^{-\alpha} e^{-y/\beta}$

$$1 = K \int_1^\infty \int_1^\infty x^{-\infty} e^{-y/\beta} \, dx \, dy$$

$$1 = K \left(\frac{1}{\infty + 1}\right)\left(-\beta e^{-2/\beta}\right)$$

$$K = \frac{-1+\alpha}{\beta e^{-1/\beta}} \quad \text{Ans}$$