Calculus

Assignment - II

1) $\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$

<u>Sol</u> let $e^{2/w} = t$ $\qquad$ $w = 1/50 \Rightarrow t = 5$

$-2/w^2 = dt/dw$ $\qquad$ $w = 2 \Rightarrow t = 5$

$$\frac{dw}{w} = \frac{-dt}{2}$$

$$\int_{100}^{1} -e^{t}\, \frac{dt}{2}$$

$$= \frac{-1}{2}\left[e^{1} - e^{100}\right]$$

$$= \frac{1}{2}\left[e^{100} - e^{1}\right]$$

2) $\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

$t^4 + 2t = x$

$4t^2 + 2 = dx/dt$

$(2t^3+1)\,dt = \frac{dx}{2}$

$$= \int \frac{dx}{2x^3} = \frac{1}{2}\left[\frac{x^{-2}}{-2}\right]$$

$$= \frac{-1}{4(t^4+2t)^2}$$

3) $f(x) = \int f''(x)\, dx$

<u>Sol</u> $f(x) = -\int (x^4 + 3x - 9)\, dx$

$$f(x) = \frac{x^5}{5} + \frac{3}{2}x^2 - 9x + 6$$

4) $f(x) = \begin{cases} 6, & \text{if } x > 1 \\ 3x^2, & \text{if } x \le 1 \end{cases}$

<u>Sol</u> $\int_{-2}^{3} f(x)\, dx = \int_{-2}^{1} f(x)\, dx + \int_{1}^{3} f(x)\, dx$

$$= \int_{-2}^{1} 3x^2\, dx + \int_{1}^{3} 6\, dx$$

$$= \left[x^3\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= 9 + 12$$

$$= 21$$

5) $\int 3(8y-1)e^{4y^2-y}\,dy$

Sol let $4y^2 - y = t$

$(8y-1)\,dy = dt$

$= 3\int e^t\,dt$

$= 3e^{4y-y^2} + C$

6) $f(x) = 15\sqrt{x} + 5x^3 + 6$

Sol $f(x) = \int\int (15\sqrt{x} + 5x^3 + 6)\,dx\,dx$

$= \int \left(\frac{15x^{3/2}}{3/2} + \frac{5}{4}x^4 + 6x \right) dx$

$= \int \left(10x^{3/2} + \frac{5}{4}x^4 + 6x \right) dx$

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + 6$

8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$, $r = 1$

$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$

$\frac{-1}{r} = \log\theta + 6$

$r = 1$, $\theta = 2$

$$\frac{-1}{(1)} = \log(2) + 6$$

$$C = -3.329928$$

$$\frac{-1}{r} = \log\theta - 3.321928$$

9) Sol $$\frac{dv}{dt} = 9.8 - 0.196v$$

$$\int \frac{dv}{9.8-0.196v} = \int dt$$

$$\int \frac{1}{(9\sqrt{5}/1)^2 - (1/10\sqrt{10})^2}\, dv = t + c$$

$$\frac{1}{2+\sqrt{5}/\sqrt{5}} \ln\left[\frac{9\sqrt{5}/5 + 7\sqrt{104}/50}{7\sqrt{5}/5 - 7\sqrt{104}/50}\right] = t + c$$

$$\log\left[\frac{\sqrt{50}+\sqrt{v}}{\sqrt{50}-\sqrt{v}}\right] = \frac{14}{\sqrt{5}}\, t + c$$

10) $$ty + 2y = t^2 - t + 1$$

$$y(1) = 1/2$$

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

$$\frac{dy}{dt} + \frac{2y}{t} = t - 1 + \frac{1}{t}$$

$$\frac{dy}{dt} = t - 1 + \frac{1}{t} - \frac{2t}{t}$$

11) $\frac{dy}{dx} = \frac{x y^3}{\sqrt{1+x^2}}$

$$\int \frac{dy}{y^3} = \int \frac{x\,dx}{\sqrt{1+x^2}}$$, let $1+x^2 = t$

$x\,dx = dt$

$$\frac{-1}{2y^2} = \frac{1}{2}\int \frac{t}{t^{1/2}}$$

$$\frac{-1}{2y^2} = \sqrt{1+x^2} + C$$

$$\frac{-1}{2(-1)^2} = 1 + C$$ $y(0) = -1$

$C = -3/2$ $\therefore \frac{-1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$

12) $f(x) = x^3 - 10x^2 + 6$, For $x = 3$

Sol $F(x) = x^3 - 10x^2 + 6$

$F(3) = 27 - 90 + 6 = -57$

$f'(x) = 3x^2 - 20x$ $\qquad$ $f'(3) = 3(3)^2 - 70(3) = -33$

$f''(x) = 6x - 20$ $\qquad$ $f''(3) = 18 - 20 = -2$

$f'''(x) = 6$ $\qquad$ $f'''(3) = 6$

$$x^3 - 10x^2 + 6 = (-57) + (x-3)(-33) - (x-3)^2(2) + (x-3)^3$$

13) Maclaurin Series for $(1+x)^{\mu}$

Sol $F(x) = (1+x)^{\mu}$ $\qquad$ $f(0) = 1$

$F''(x) = \mu(1+x)^{\mu-1}$ $\qquad$ $f'(0) = \mu$

$F''(x) = \mu(\mu-1)(1-x)^{\mu-2}$ $\qquad$ $f''(0) = \mu(\mu+1)$

$F'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$ $\qquad$ $f'''(0) = \mu(\mu-1)(\mu-2)$

$$f(x) = 1 - \sum_{1=1}^{n} \frac{\mu x^n(\mu-n)}{n!}$$

14) $f(x) = \sqrt{1+x}$ ... Maclaurian Series

Sol $f(x) = \sqrt{1+x}$ $\qquad$ $f(0) = 1$

$f'(x) = \dfrac{1}{2\sqrt{1+x}}$ $\qquad$ $f'(0) = \dfrac{1}{2}$

$f''(x) = \dfrac{-1}{4(1+x)^{3/2}}$ $\qquad$ $f''(0) = \dfrac{-1}{4}$

$$f'''(x) = \frac{3}{8(1+x)^{5/2}} \qquad f'''(0) = \frac{3}{8}$$

$$f(x) = 1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{16} + \cdots$$

15.) Taylor series

$F(x) = e^{-6x}$ $\qquad F(4) = e^{24}$

$F'(x) = (-6)e^{-6x}$ $\qquad F'(x) = -6e^{24}$

$F''(x) = 36e^{-6x}$ $\qquad F''(x) = 36e^{24}$

$$f(x) = e^{24} - (x+4)\,6e^{24} + (x+4)^2 \times 18e^{24} + \cdots$$

16) Taylor Series

Sol $f(x) = \ln(3+4x)$ $\qquad f(0) = \ln(3)$

$f'(x) = \frac{1}{3+4x}$ $\qquad f'(x) = \frac{1}{3}$

$f''(x) = \frac{-4}{(3+4x)^2}$ $\qquad f''(0) = \frac{-4}{9}$

$f'''(x) = \frac{8}{(3+4x)^3}$ $\qquad f'''(0) = \frac{8}{27}$

$$f(x) = \ln(3) + \frac{x}{3} - \frac{2x^2}{9} + \frac{4x^3}{81} + \cdots$$