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Numerical Methods and Algebra

Assignment 1

1. Let $x = \sqrt{7 + \sqrt{7 + \sqrt{7 + \dots}}}$

$$\Rightarrow x^2 = 7 + \sqrt{7 + \sqrt{7 + \dots}}$$

$$\Rightarrow x^2 = 7 + x.$$

$$\Rightarrow x^2 - x - 7 = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = 3.1925 \quad \text{or} \quad x = -2.192.$$

Since $\sqrt{x} \Rightarrow x > 0$; $x = \underline{\underline{3.1925}}$

2. $7x^2 - 6y^2 = -11x^2 - 4y^2$

$$\Rightarrow 7x^2 + 11x^2 = -4y^2 + 6y^2$$

$$\Rightarrow 18x^2 = 2y^2$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{1}{9}$$

$$\Rightarrow \frac{x}{y} = \underline{\underline{\frac{1}{3}}}$$

3. $x + 3y = 4 \Rightarrow x = 4 - 3y.$
 $x^2 + y^2 - 4y = 12$

$$\Rightarrow (4 - 3y)^2 + y^2 - 4y = 12.$$

$$\Rightarrow 16 + 9y^2 - 24y + y^2 - 4y - 12 = 0.$$

$$\Rightarrow 10y^2 - 28y + 4 = 0.$$

$$\Rightarrow y = 2.6489 \text{ or } y = 0.151$$

Solution: $(-3.9467, 2.6489)$ or $(3.547, 0.151)$

A. $d = 2r^2 - 16r + 34$

a) $r = 3.5$

$\Rightarrow d = 2.5$ million particles/cubic foot

b) $d' = \frac{d}{dr}(2r^2 - 16r + 34) = 4r - 16 = 0.$

$\Rightarrow r = \underline{\underline{4}}$

$d'' = 4 > 0 \Rightarrow \text{minimum.}$

$\therefore$ Minimum no. of revolutions is 400/minute.

c) $d = 100$

$\Rightarrow 100 = 2r^2 - 16r + 34.$

$\Rightarrow 2r^2 - 16r - 66 = 0.$

$r = 11 \text{ or } r = -3.$

$\therefore$ Speed of engine is 1100 rpm.

5.

a) $C_1 = 1500 + 25n.$

b) $C_2 = 2100 + 22n$

c) $C_2 < C_1$

$\Rightarrow 2100 + 22n < 1500 + 25n$

d) $\Rightarrow 600 < 3n$

$\Rightarrow 200 < n \Rightarrow \underline{\underline{n > 200}}$

6.

a) $x^3 - 7x^2 + 14x - 6 = 0.$

$f(0) = -6 < 0.$

$f(1) = 2 > 0.$

$\frac{a+b}{2} = 0.5 = p_0.$

$f(0.5) = -0.625$

n	p_n	$f(p_n)$
0	0.5	-0.625
1	0.75	0.984375
2	0.625	0.2597
3	0.5625	-0.1617
4	0.59375	0.0540
5	0.578125	-0.0540
6	0.5859375	0.00103.

$\therefore \text{Ans} \approx \underline{\underline{0.5859375}}$

b) $f(x) = x^3 - 7x^2 + 14x - 6.$

n	p_n	$f(p_n)$
0	2.1	1.791
1	2.65	0.5521
2	2.925	0.0858
3	3.0625	-0.054
4	2.99375	0.00632
5	3.0813	-0.0265
6	3.01094	-0.016969
7	3.00235	

$\text{Ans} = 3.00235$

c)	n	P_n	$f(P_n)$
	0	3.6	0.336
	1	3.4	-0.016
	2	3.5	0.125
	3	3.45	0.0461
	4	3.425	0.013
	5	3.4125	-0.0099
	6	3.41875	-
	7	-	-

$$\text{Ans} \approx \underline{\underline{3.41875}}$$

7. $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9.$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 920x^3 + 54x^2 + 18x - 221.$$

$$\text{let } x_0 = 0.4.$$

$$\therefore x_1 = -0.0927$$

$$x_2 = -0.0408$$

$$x_3 = \underline{\underline{-0.04066.}}$$

$$\text{if } x_0 = 0.7.$$

$$x_1 = 1.4326$$

$$x_2 = 1.1599$$

$$x_3 = 1.0138.$$

$$x_4 = 0.9671$$

$$x_5 = 0.9624$$

$$x_6 = \underline{\underline{0.9624.}}$$

8) a) Arithmetic mean = $\frac{70+86+81+83+x}{5}$
 $= \frac{320+x}{5}$

b) $80 \leq \frac{320+x}{5} \leq 90$
 $x \leq$

c) $\Rightarrow 400 - 320 \leq 450 - 320$

$\Rightarrow 80 \leq x \leq 130$

9. NPV = 8.54 @ 4%.

NPV = -11.81 @ 20%.

$\therefore @ 0$

$y = \frac{-11.81 - 8.54}{0.2 - 0.04} (0 - 0.04) + 8.54$

$= 0.136275$

10. $R(x) = 32x - 0.21x^2$

$C(x) = 195 + 12x$

Profit = revenue - cost.

$\Rightarrow P(x) = 20x - 0.21x^2 - 195$

For profit $P(x) > 0$.

$\Rightarrow 20x - 0.21x^2 - 195 > 0$

$$\Rightarrow 84.21 \cdot (x - 84.21)(x - 11.026) > 0.$$

$$\Rightarrow \underline{\underline{x > 84}}$$