Financial Mathematics.

Q. 1) Given : $d^{4} = 8\%$.

(i) $\delta = ?$

$$\delta = -\ln(1-d)$$

$$= -\ln\left(1-\frac{d^{4}}{4}\right)^{4}$$

$$= -4\ln\left(1-\frac{8\%}{4}\right)$$

$$= 8.081083\%$$

(ii) $i = ?$

$$1+i = (1-d)^{-1} = \left(1-\frac{d^{4}}{4}\right)^{-4}$$

$$i = (1-2\%)^{-4} - 1$$

$$= 8.4165784\%$$

(iii) $d^{12} = ?$

$$d^{12} = 12\left(1-(1-d)^{1/12}\right)$$

$$= 12\left(1-\left(1-\frac{d^{4}}{4}\right)^{1/3}\right)$$

$$= 12\left(1-(1-2\%)^{1/3}\right)$$

$$d^{12} = 8.053934\%$$

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Q.2 $\delta(t) = 0.004t + 0.0002t^2$

(i) $A_{10} = 1000$

$PV = 1000\, v(0,10)$

$= 1000\, e^{-\int_0^{10} 0.004t + 0.0002t^2\, dt}$

$= 1000\, e^{-\left(\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right)_0^{10}}$

$= 1000\, e^{-(0.2 + 0.067)}$

$= 1000\, e^{-0.26667}$

$= 765.928$

(ii) $i = ?$

$AV_{10} = PV(1+i)^{10}$

$1000 = 765.928(1+i)^{10}$

$i = \left(\frac{1000}{765.928}\right)^{1/10} - 1$

$= 2.7025404\%$

3)

(i) let $A(1)$ = accumulated value at $t=1$ &

$A(0)$ = value at $t=0$

$$A(1) = A(0)(1+i) \quad (1)$$

$$A(1)(1-d) = A(0) \quad (2)$$

on multiplying (1) & (2)

$$(1+i)(1-d) = 1.$$

$$1-d = \frac{1}{1+i} \Rightarrow d = \frac{i}{1+i} = iv$$

(ii) $d^{(2)} = ?$

(a) $d = 2.25\%$.

$$d^{2} = 2(1-(1-2.25\%)^{1/2})$$

$$= 2.26280067\%.$$

(b) $d^{0.5} = 5\%$.

$$d^{2} = 2\left(1-\left(1-\frac{5\%}{0.5}\right)^{1/4}\right)$$

$$= 5.199251\%.$$

(iii) $i = 5\%$.

$$\left(1+\frac{91}{365}i\right)^{91/365} = \frac{1}{1-91/365\, d}.$$

$$1-\frac{91}{365}d = \frac{7300}{7209}$$

$$d = \frac{365}{91}\left(1-(1+5\%)^{91/365}\right)$$

$$= 4.849462\%.$$

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4) $A(1)(1-d) = A(0)$

(i) $A(0)(1+i) = A(1)$

The rates i & d correspond the same movement but in opp directions

on multiplying the above we get:

$(1-d)(1+i) = 1.$

$d = \frac{i}{1+i} \Rightarrow$ &v

(ii) $i = 0.08$

a. $d^{(12)} = 12(1-(1-d)^{1/12})$

$= 12(1-(1+i)^{-1/12})$

$= 12(1-(1+0.08)^{-1/12})$

$= 7.6714776\%.$

b. $i^{(365)} = 365((1+0.08)^{1/365} - 1)$

$= 7.6969155\%.$

c. $\delta = \ln(1+i)$

$= \ln(1+8\%)$

$= 7.6961041\%.$

d. $i^{(0.5)} = 0.5((1+i)^{1/2} - 1)$

$= \frac{1}{2}(1+0.08)^{1/2} - 1)$

$= 8.32\%.$

5) i) $d^{4} = 6\%$.

(a) $i^{2} = ?$

$i^{2} = 2((1+i)^{1/2} - 1)$

$= 2((1 - \frac{d^{4}}{4})^{-4/2} - 1)$

$= 2((1 - \frac{6\%}{4})^{-2} - 1)$

$= 6.137752\%$

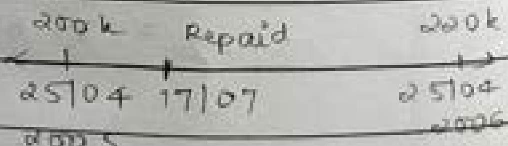
(b) $d^{12} = ?$

$d^{12} = 12(1 - (1 - \frac{d^{4}}{4})^{4/12})$

$= 12(1 - (1 - \frac{6\%}{4})^{1/3})$

$= 6.030253\%$

(ii) ~~Int~~



$AV_{t0} = 200k(1+i)$

$i = \frac{220000}{200000} - 1$

$= 10\%$

Due to early repayment amt paid by Amit will be:

$AV = 200000(1 + 10\%)^{93/365}$

$= 204916.3564$

6) ~~a)~~

1) a) cash payment : 30% below MRP.

b) 3 month credit : 25% below MRP

A) ~~Inter~~ Discount for ~~to~~ if price of toy is 100.

cash price = 70 & or credit : 75.

$$70 = 75(1-d)^{1/2}$$

$$d = 1 - \left(\frac{70}{75}\right)^2 = 12.8889\%.$$

B)

\# $$d^{(12)} = 12\left(1-(1-d)^{1/12}\right)$$

$$= 12\left(1-(1-12.8889\%)^{1/12}\right)$$

$$= 13.719544\%.$$

\# $$i^{(2)} = 2\left(\left(1-\frac{d^{12}}{12}\right)^{-12/2} - 1\right)$$

$$= 2\left(\left(1-\frac{13.719544\%}{12}\right)^{-6} - 1\right)$$

$$= 14.2857143\%.$$

\# $$\delta = \ln(1+i)$$

$$= \ln\left(1+\frac{i2}{2}\right)^2$$

$$= 2\ln\left(1+\frac{14.2857143\%}{2}\right)$$

$$= 13.798574\%.$$

7) PV = 20000 , AV = 26000 , t = 17 mont

(i) $i^{(4)} = ?$

$$\frac{26000}{20000} = \left(1 + \frac{i^{(4)}}{4}\right)^{17/12 \times 4}$$

$$i^{(4)} = 4\left(\left(\frac{26000}{20000}\right)^{3/17} - 1\right)$$

$$= 18.955254\%$$

(ii) $d^{(2)} = ?$

$$26000 = 20000\left(1 - \frac{d^{(2)}}{2}\right)^{-2 \times 17/12}$$

$$d^{2} = 2\left(1 - \left(\frac{26000}{20000}\right)^{-6/17}\right)$$

$$= 17.688235\%$$

(iii) $i = ?$

$$i = \left(1 + \frac{i^{(4)}}{4}\right)^{4} - 1$$

$$= 20.3457061\%$$

(iv) We know that

$$i > i^{(P)} > \delta > d^{(P)} > d.$$

Here Also

$$i > i^{(4)} > d^{(2)}$$

$\Rightarrow$ so we know that interest rate gets lower with higher frequency of compounding

8) $i = 8\%$, $i^{(p)} = p((1+i)^{1/p} - 1)$

$i^{(5)} = 7.7556\%$.

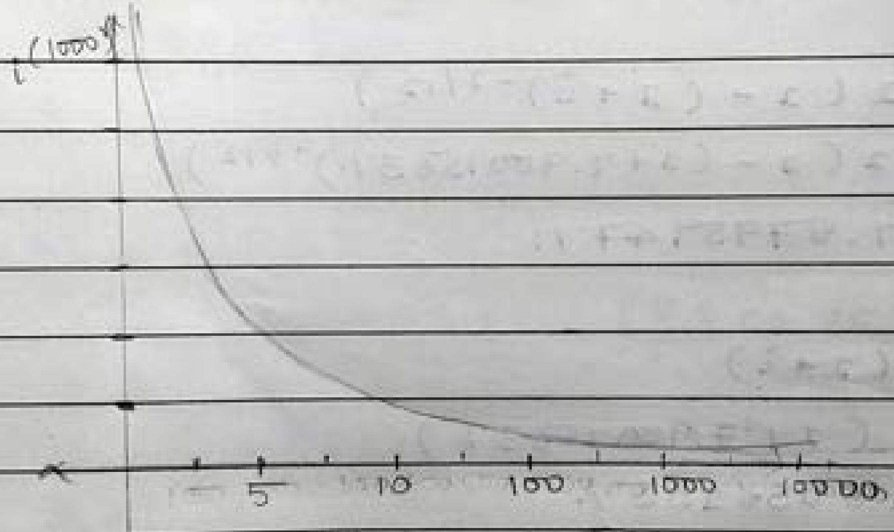
$i^{(10)} = 7.72579$

$i^{(100)} = 7.699066$

$i^{(1000)} = 7.6964002$.

$i^{(10000)} = 7.69613372$

We can see as value of p is increasing $i^{(p)}$ is decreasing & then becoming constant as the values of p get higher to 7.69.



* More frequency compounded lower equivalent nominal interest rate
* This is to limit benefit of compounding so after a amt of compounding value gets constant
* So as P approaches infinity:

$$\lim_{p \to \infty} (1+i) = \lim_{p \to \infty} \left(1 + \frac{i^{(p)}}{p}\right)^p = e^{\delta} \text{ (constant)}$$

9)

(i) Force of interest is the instantaneous change in fund value expressed as an annualised percentage of current fund value.

(ii) $i^{(2)} = 0.0775 = 7.75\%$

(a) $i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1$

$= 7.9001563\%$

(b) $d^{12} = 12\left(1 - (1+i)^{-1/12}\right)$

$= 12\left(1 - (1+7.9001563\%)^{-1/12}\right)$

$= 7.5795747\%$

(c) $\delta = \ln(1+i)$

$= \ln(1 + 7.9001563\%)$

$= 7.6036135\%$

(d) $i^{(1/2)} = \frac{1}{2}\left((1+i)^{1/2} - 1\right)$

$= \frac{1}{2}\left((1+7.9001563\%)^{1/2} - 1\right)$

$= 8.0112219\%$

10) $$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$$

$$v(t) = e^{-\int_0^t \delta(t)\,dt}$$

1) case 1. : $0 \le t \le 5$

$$= e^{-\int_0^t 0.08\,dt}$$

$$= e^{-0.08t}$$

2) case 2: $5 \le t \le 10$.

$$v(t) = e^{-\left(\int_0^5 0.08\,dt + \int_5^t 0.06\,dt\right)}$$

$$= e^{-\left((0.08t)_0^5 + (0.06t)_5^t\right.}$$

$$= e^{-(0.4 + 0.06t - 0.3)} = e^{-(0.06t + 0.1)}$$

3) case 3 : $t \ge 10$

$$v(t) = e^{-\left(\int_0^5 0.08t\,dt + \int_5^{10} 0.06t\,dt + \int_{10}^t 0.04\,dt\right)}$$

$$= e^{-(0.4 + 0.6 - 0.3 + 0.04t - 0.4)}$$

$$= e^{-(0.04t + 0.3)}$$

11)

9M

0 → 9M

$(1-\frac{18}{100})x$; 50000

a) $50000\left(1-\frac{18}{100}\times\frac{9}{12}\right) = 43250.$

(b) $\delta = 6\%$.

$d^{(12)} = ?$

$\left(1-\frac{d^{(12)}}{12}\right)^{12} = e^{-\delta}$.

$d^{(12)} = (1-e^{-\delta/12})12$

$= 5.985025\%$.

(ii) $d^{(4)} = 6\%$.

$i^{(12)} = 12\left(\left(1-\frac{d^{4}}{4}\right)^{-\frac{4}{12}} - 1\right)$

$= 6.060709\%$.

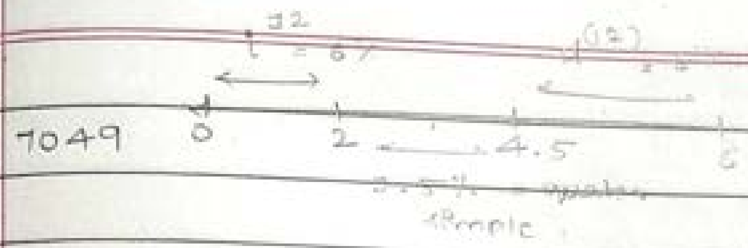
c) The order is based on when interest is paid so d is interest paid immediately so smallest amt & and i is paid at the end so max value & δ corresponds to payment throughout

$i > i^{(4)} > \delta > d$.

d) If we invest 1 unit of initial capital i will be interest payable at the end of term & d will be interest paid in the beginning so d will be equal to interest i payable at the P.V at beg at the end of term

$d = iv$.

12)



1) $i^{(12)} = 6\%$.

$i = \left(1 + \frac{i^{(12)}}{12}\right)^{12} - 1 = 6.1677812\%$.

$d^{(12)} = 12(1-(1)$

$\rightarrow d \quad i = \left(1 - \frac{d^{12}}{12}\right)^{-12} - 1 = 6.1996367\%$.

$\Rightarrow AV_6 = 7049\, A(0,2)\, A(2,4.5)\, A(4.5,6)$

$= 7049(1 + 6.1677812\%)^2 (1 + 2.5 \times 4 \times 1.5\%)$
$(1 + 6.1996367\%)^{1.5}$.

$= \cancel{7049}(7945.349(1 + 15\%)(1 + 6.1996367\%)$

$= 9137.15654(1 + 6.996367\%)^{1.5}$

$= \cancel{9999.8936}\ 10000.0556$

* $A(0,2) = 1.1272$.

$A(2,4.5) = 1.15$

$A(4.5,6) = 1.094421$

13)

$\Rightarrow$ $PV = x$

$AV = 2x$

(i) $i = 10\%$.

$$2x = x(1+10\%)^t$$

$$2 = (1+10\%)^t.$$

$$t = \log_{(1.1)}(2)$$

$$t = 7.27 \text{ years}$$

(ii) $d^{(12)} = 10\%$.

$$i = \left(1 - \frac{d^{(12)}}{12}\right)^{-12} - 1.$$

$$= 10.53340773\%.$$

$$2x = x(1+10.53340773\%)^t$$

$$t = \log_{(1+10.53\%)}(2)$$

$$= 6.902 \text{ years}.$$