

## Assignment

① Area of a wire =  $\pi r^2$

Area calculated =  $r = 12.65$

$$= \pi \times (12.65)^2 = 502.92785$$

Correct value =  $r = 12.5$

$$= \pi \times (12.5)^2 = 491.071$$

i) Absolute error =  $|502.92785 - 491.071|$   
 $= 11.85685$

ii) Relative error =  $\frac{11.85685}{491.071} = 0.024144879$

iii) Percentage error =  $\frac{11.85685}{491.071} \times 100 = 2.4144\%$

②

| $x$ | $P(x)$    |
|-----|-----------|
| 4.0 | 0.60206   |
| 4.5 | 0.6532125 |
| 5.5 | 0.7403627 |
| 6.0 | 0.7781513 |

a)  $n = 4, n = 6$



| $n$ | $F(n)$    |
|-----|-----------|
| 4   | 0.60206   |
| 6   | 0.7781513 |

$n=2$

$$h = \frac{n - n_0}{n} = \frac{5 - 4}{2} = 0.5$$

$$F(n) = F(n_0) + h \Delta F(n_0)$$

$$= 0.60206 + \frac{1}{2} (0.7781513 - 0.60206)$$

$$= 0.69010565$$



b)

| $x$ | $P(x)$    |
|-----|-----------|
| 4.5 | 0.6532125 |
| 5.5 | 0.7403627 |

$$h = 1$$

$$u_2 = \frac{5.5 - 4.5}{2} = \frac{1}{2} = 0.5$$

$$P(u) = 0.6532125 + \frac{1}{2} (0.7403627 - 0.6532125)$$

$$= 0.6967876$$

3)  $x^4 - x - 10 = 0$

| $x$ | $y$     |
|-----|---------|
| 1.5 | -6.4375 |
| 2   | 4       |

$$x_3 = \frac{x_1 + x_2}{2} = \frac{1.5 + 2}{2} = 1.75$$

$$y_3 = (1.75)^4 - 1.75 - 10 = -2.37109375$$

| $x$  | $y$         |
|------|-------------|
| 1.75 | -2.37109375 |
| 2    | 4           |



$$x_4 = \frac{1.75 + 2}{2} = 1.875$$

$$y_4 = (1.875)^4 - 1.875 - 10 = 0.484619$$

| $x$   | $y$         |
|-------|-------------|
| 1.75  | -2.37109375 |
| 1.875 | 0.484619    |

$$x_5 = 1.8125$$

$$y_5 = (1.8125)^4 - 1.8125 - 10 = -1.020248$$

| $x$    | $y$       |
|--------|-----------|
| 1.8125 | -1.020248 |
| 1.875  | 0.484619  |

$$x_6 = 1.84375$$

$$y_6 = (1.84375)^4 - 1.84375 - 10 = -0.287734$$

| $x$     | $y$       |
|---------|-----------|
| 1.84375 | -0.287735 |
| 1.875   | 0.484619  |

$$x_7 = 1.859375$$

$$y_7 = 0.093378126$$



$$④ \quad P(x) = x^3 - x - 1$$

$$P'(x) = 3x^2 - 1$$

$$x_1 = 1$$

$$P(x_1) = -1$$

$$P'(x_1) = 2$$

$$x_2 = x_1 - \frac{P(x_1)}{P'(x_1)} = 1 - \frac{(-1)}{2} = \frac{1+1}{2} = 1.5$$

$$P(x_2) = 0.875$$

$$P'(x_2) = 5.75$$

$$x_3 = 1.5 - \frac{0.875}{5.75} = 0.84782$$

$$P(x_3) = -1.2384$$

$$P'(x_3) = 1.1564$$

$$x_4 = 0.84782 - \frac{(-1.2384)}{1.1564} = 1.9187$$

$$P(x_4) = 4.1448$$

$$P'(x_4) = 10.044$$

$$x_5 = 1.9187 - \frac{4.1448}{10.044} = 1.5060$$

$$P(x_5) = 0.9096$$



$$f'(x_5) = 5.8041$$

$$x_6 = 1.5060 - \frac{0.9096}{5.8041} = 1.3492$$

$$P(x_6) = 0.10680$$

$$P'(x_6) = 4.46102$$

$$x_7 = 1.3492 - \frac{0.10680}{4.46102} = \underline{\underline{1.3252}}$$

$$5) \quad A^2 = A$$

$$= 7A - (1+A)^3$$

$$= 7A - (1^3 + A^3 + 3A^2 \cdot 1 + 3A \cdot 1^2)$$

$$= 7A - (1^3 + A^3 + 3A^2 + 3A)$$

$$= 7A - (1 + A^3 + 3A + 3A^2)$$

$$= 7A - (1 + A^2 \cdot A + 3A + 3A^2)$$

$$= 7A - (1 + A \cdot A + 3A + 3A) \quad [A^2 = A]$$

$$= 7A - (1 + A^2 + 6A)$$

$$= 7A - (1 + 7A)$$

$$= -I$$



$$b) \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{adj } A$$

$$|A| = 1(0 \times 1 - 6) - (-1)(4 \times 1 - 0 \times 6) + 2[4 \times 1 - 0]$$

$$= -6 - 4 + 8 = -2$$

$$\text{adj } A = \begin{bmatrix} 1(0 \times 1 - 6) & -1(4 \times 1 - 0 \times 6) & 2(4 \times 1 - 0) \\ -1(0 \times 1 - 6) & 1(4 \times 1 - 0 \times 6) & -2(4 \times 1 - 0) \\ 2(0 \times 1 - 6) & -2(4 \times 1 - 0 \times 6) & 1(4 \times 1 - 0 \times 6) \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -4 & 4 \\ 6 & 4 & -8 \\ -12 & 8 & 4 \end{bmatrix}$$

$$a_{11} = (-1 \times 0 + 6) = -6$$

$$a_{12} = (-1 \times 4 - 0) = -4$$

$$a_{13} = (4 \times 1) = 4$$

$$a_{21} = (-1 \times 1 - 2) = 0$$

$$a_{22} = (1 \times 1 - 0) = 1$$

$$a_{23} = (1 \times 1 - 0 \times 1) = 1$$

$$a_{31} = (-1 \times 6 - 0) = -6$$

$$a_{32} = (1 \times 6 - 8) = -2$$

$$a_{33} = (1 \times 0 - (-1 \times 4)) = 4$$

$$\begin{bmatrix} -6 & -4 & 4 \\ 0 & -1 & 1 \\ -6 & -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 0 & -6 \\ -4 & -1 & -2 \\ 4 & 1 & 4 \end{bmatrix}$$



$$A^{-1} = \frac{1}{-2} \begin{bmatrix} -6 & 0 & -6 \\ -4 & -1 & -2 \\ 4 & 1 & 4 \end{bmatrix}$$

$$\begin{aligned} \vec{A} &= 8\hat{i} - 9\hat{j} \\ \vec{B} &= 7\hat{i} - 9\hat{j} \end{aligned}$$

$$\begin{aligned} \vec{A} \cdot \vec{B} &= (8)(7) + (-9)(-9) \\ &= 56 + 81 = 137 \end{aligned}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$|\vec{a}| = \sqrt{8^2 + 9^2} = \sqrt{81 + 64} = \sqrt{145}$$

$$|\vec{b}| = \sqrt{7^2 + 9^2} = \sqrt{49 + 81} = \sqrt{130}$$

$$137 = \sqrt{145} \cdot \sqrt{130} \cos \theta$$

$$\cos \theta = \frac{137}{\sqrt{145} \cdot \sqrt{130}}$$



$$9) \quad 101, 104, 107, \dots, 998$$

$$a = 101$$

$$d = 3$$

$$n = ?$$

$$a_n = a + (n-1)d$$

$$998 = 101 + (n-1)3$$

$$\frac{897}{3} = n-1$$

$$n = 300$$

$$\text{Sum} = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{300}{2} [2 \times 101 + (299)3]$$

$$= \underline{\underline{164850}}$$

$$10) \quad \begin{array}{l} 6\text{th term} = a r^5 = 32 \\ 8\text{th term} = a r^7 = 128 \end{array}$$

$$\frac{a r^7}{a r^5} = \frac{128}{32} = 4$$

$$r = 2$$



$$1) (4+3n)^5$$

$$= {}^5C_0 4^5 + {}^5C_1 4^4 (3n)^1 + {}^5C_2 4^3 (3n)^2 + {}^5C_3 4^2 (3n)^3 + {}^5C_4 4 (3n)^4 + {}^5C_5 (3n)^5$$

$$= 1024 + 5(256)(3n) + (10)(64)(9n^2) + (10)(16)(27n^3) + (5)(4)(81n^4) + 243n^5$$

$$= 1024 + 3840n + 5760n^2 + 4320n^3 + 1620n^4 + 243n^5$$

$$= 243n^5 + 1620n^4 + 4320n^3 + 5760n^2 + 3840n + 1024$$

2)

$$\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$|A| = 2(-15) + 3(6) - 0 = 18 - 30 = -12$$

$$= A^3 - (a + e + j)A^2 + (M_{11} + M_{22} + M_{33})A - |A|I = 0$$

$$= \lambda^3 - (2 - 5 + 3)A^2 + \left[ \begin{vmatrix} -5 & 0 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 2 & -3 \\ 2 & -5 \end{vmatrix} \right] A - |A|I = 0$$

$$= (\lambda^3 - (2(-15) + 3(6) + 0)) = 0$$



$$\lambda^3 + (-15+6-4)\lambda - (-12) = 0$$

$$\lambda^3 - 14\lambda + 12 = 0$$

13)

$$f(x), x^3 - 7x^2 + 8x - 3 \quad \text{if } x_0 = 5$$

$$f'(x) = 3x^2 - 14x + 8$$

$$x_0 = 5$$

$$f(x_0) = -13$$

$$f'(x_0) = 13$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \frac{(-13)}{13} = 5 + 1 = 6$$

$$x_1 = 6$$

$$f(x_1) = 9$$

$$f'(x_1) = 32$$

$$x_2 = 6 - \frac{(9)}{32} = 5.71875$$







| $x$ | $y$          |
|-----|--------------|
| 1   | 0.3678       |
| 1.5 | -0.305143777 |

$$x_{42} = \frac{1+1.5}{2} = 1.25$$

$$y_{42} = -0.004225$$

| $x$  | $y$       |
|------|-----------|
| 1    | 0.3678    |
| 1.25 | -0.004225 |

$$x_5 = \frac{1+1.25}{2} = 1.125$$

$$y_5 = 0.17119$$

| $x$   | $y$       |
|-------|-----------|
| 1.125 | 0.17119   |
| 1.25  | -0.004225 |

$$x_6 = \frac{1.125+1.25}{2} = 1.1875$$

$$y_6 = 0.08108$$

| $x$    | $y$       |
|--------|-----------|
| 1.1875 | 0.08108   |
| 1.25   | -0.004225 |



$$u_1 = \frac{1.1975 + 1.25}{2} = 1.21875$$

$$y_1 = e^{-1.21875} - 3 \log(-1.21875)$$

$$= 0.038659$$