

Assignment 1

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Numerical Methods & Algebra

1) Let $x = \sqrt{7 + \sqrt{7 + \sqrt{7 + \dots}}}$

$$\therefore x = \sqrt{7 + x}$$

$$\therefore x^2 = 7 + x$$

$$\therefore x^2 - x - 7 = 0$$

$$\therefore x = \frac{1 \pm \sqrt{1 + 28}}{2}$$

$$\therefore x = \frac{1 \pm \sqrt{29}}{2}$$

x can't be negative

$$\therefore x \neq \frac{1 - \sqrt{29}}{2}$$

$$\therefore x = \frac{1 + \sqrt{29}}{2}$$

2)

$$7x^2 - 6y^2 = -11x^2 - 4y^2$$

$$18x^2 = 2y^2$$

$$\therefore \frac{x^2}{y^2} = \frac{2}{18} = \frac{1}{9}$$

$$\therefore \frac{x}{y} = \pm \frac{1}{3}$$

3)

$$x + 3y = 4 \dots (i)$$

$$x^2 + y^2 - 4y = 12 \dots (ii)$$

From (i), $x = 4 - 3y$

Subs $x = 4 - 3y$ in eq (ii), we get;

$$(4 - 3y)^2 + y^2 - 4y = 12$$

$$10y^2 - 28y + 16 = 12$$

$$10y^2 - 28y + 4 = 0$$

$$\therefore y = \frac{28 \pm \sqrt{784 - 160}}{20}$$

$$\therefore y = \frac{28 \pm \sqrt{624}}{20}$$

$$\therefore y = \frac{28 \pm \sqrt{39 \times 16}}{20} = \frac{28 \pm 4\sqrt{39}}{20}$$

$$\therefore y = \frac{7 \pm \sqrt{39}}{5}$$

$$\therefore x = \frac{-1 - 3\sqrt{39}}{5} \text{ and } x = \frac{3\sqrt{39} - 1}{5}$$

4)

$$d = 2x^2 - 16x + 34$$

a. when $x = 3.5$,

$$d = 2(3.5)^2 - 16(3.5) + 34$$

$$\therefore d = 2.5$$

$$b. \frac{d}{dx} = 4x - 16 = 0$$

$$\therefore 4x = 16 \quad \therefore x = 4$$

When $x = 4$, $d = 2$

$$c. \quad 100 = 2x^2 - 16x + 34$$

$$2x^2 - 16x - 66 = 0$$

$$x = \frac{16 \pm \sqrt{256 + 528}}{4}$$

$$x = 11 \text{ or } x = -3$$

$$5) a. \text{Company 1: } C \leq 1500 + 25x$$

$$b. \text{Company 2: } C \leq 2100 + 22x$$

$$c. \quad 2100 + 22x > 1500 + 25x$$

$$600 > 3x$$

$$x < 200$$

$$105) f(x) = x^3 - 7x^2 + 14x - 6 = 0$$

a. $[0, 1]$

Iterations	a	b	$\frac{a+b}{2} = c$	$f(a)$	$f(b)$	$f(c)$
1	0	1	0.5	-6	2	-0.625
2	0.5	1	0.75	-0.625	2	0.984375
3	0.5	0.75	0.625	-0.625	0.984375	0.259766
4	0.5	0.625	0.5625	-0.625	0.259766	-0.16187
5	0.5625	0.625	0.59375	-0.16187	0.259766	0.054047
6	0.5625	0.59375	0.578125	-0.16187	0.054047	-0.05262
7	0.578125	0.59375	0.585938	-0.05262	0.054047	0.001031
8	0.581031	0.585938	0.582031	-0.05262	0.001031	-0.02572
9	0.582031	0.585938	0.583984	-0.02572	0.001031	-0.01232

b. $[1, 3.2]$

$$c = \frac{a+b}{2}$$

Iterations	a	b	$\frac{a+b}{2} = c$	$f(a)$	$f(b)$	$f(c)$
1	1	3.2	2.1	2	-0.112	1.791
2	2.1	3.2	2.65	1.791	-0.112	0.55213
3	2.65	3.2	2.925	0.55213	-0.112	0.08583
4	2.925	3.2	3.0625	0.8583	-0.112	-0.05444
5	2.925	3.0625	2.99375	0.8583	-0.05444	0.00633
6	2.99375	3.0625	3.02813	0.00633	-0.05444	-0.0652
7	2.99375	3.02813	3.01094	0.00633	-0.02652	-0.0107
8	2.99375	3.01094	3.00234	0.00633	-0.0107	-0.00233
9	2.99375	3.00234	2.99805	0.00633	-0.00233	0.00196

$$\epsilon c [3.2, 4]$$

Iterations	a	b	$c = \frac{(a+b)}{2}$	$f(a)$	$f(b)$	$f(c)$
1	3.2	4	3.6	-0.112	2	0.336
2	3.2	3.6	3.4	-0.112	0.336	-0.016
3	3.4	3.6	3.5	-0.016	0.336	0.125
4	3.4	3.5	3.45	-0.016	0.125	0.046125
5	3.4	3.45	3.425	-0.016	0.046125	0.013016
6	3.4	3.4245	3.4125	-0.016	0.013016	-0.001998
7	3.4125	3.4245	3.41875	-0.001998	0.013016	0.005382
8	3.4125	3.41875	3.415625	-0.001998	0.005382	0.001660
9	3.4125	3.41563	3.414065	-0.001998	0.001666	-0.000174

$$7) f(x) = 230x^4 + 18x^3 + 9x^2 - 22x - 9$$

Intervals $[-1, 0]$ $[0, 1]$
 Midpoint of $[-1, 0]$ $p_0 = -\frac{1}{2}$

" $[0, 1]$ $p_0 = \frac{1}{2}$

$$p_4 = -0.040659288 \text{ for the interval } [-1, 0]$$

$$p_6 = -0.040650288 \text{ for the interval } [0, 1]$$

for $p_0 = -0.5$, we have $p_5 = -0.04$
 and for $p_0 = 0.5$, we have $p_2 = 0.06$

$$8) a. \text{ Average} = \frac{70+86+81+83+x}{5} = \frac{320+x}{5}$$

$$b. \frac{320+x}{5} \geq 80 \text{ and } \frac{320+x}{5} < 90$$

$$c. 320+x \geq 400 \text{ and } 320+x < 450$$

$$\therefore x \geq 80 \text{ and } x < 130$$

$$80 \leq x < 130$$

$$\therefore 80 \leq x < 100 \text{ as paper is of 100 marks.}$$

9) $x_1 = 4, y_1 = 8.54, y_2 = 0, x_3 = 20, y_3 = -11.81$

$$x_2 = \frac{(y_2 - y_1)(x_3 - x_1)}{(y_3 - y_1)} + x_1$$

$$= \frac{(0 - 8.54)(20 - 4)}{(-11.81 - 8.54)} + 4$$

$$= \frac{-136.64}{-20.35} + 4$$

$$= 10.7145$$

10) Revenue $R(x) = 32x - 0.21x^2$
 Cost $C(x) = 195 + 12x$

For Profit,

$$R(x) > C(x)$$

$$32x - 0.21x^2 > 195 + 12x$$

$$-0.21x^2 + 20x - 195 > 0$$

$$\therefore 10.398812 < x < 156.267855$$