Financial Mathematics
Assignment - 2.

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Q1 i] Loan amount = 20,000
Monthly payments = 427.9
Time = 5 years.

$$20{,}000 = 427.9(12)\, a_{\overline{5}|}^{(12)} \ @\ i\%.$$

$$20{,}000 = 427.9(12)\left(\frac{1-(1+i)^{-5}}{\left((1+i)^{1/12}-1\right)12}\right).$$

At $i = 10\%$. RHS = 20,341
$i = 11\%$. RHS = 19916.

By interpolating $i = 10.8\%$.
$\therefore$ APR = 10.8%.

ii] $$PV_1 = 427.9(12)\, a_{\overline{4}|}^{(12)} \ @\ i = 10.8\%.$$

$$427.9(\cancel{12})\left(\frac{1-(1.108)^{-4}}{\left((1.108)^{1/12}-1\right)\cancel{12}}\right)$$

$$427.9(39.205369) = 16775.97728.$$

$$16775.97728 = 244.49(12)\, a_{\overline{t}|}^{(12)} \ @\ 10.8\%.$$

$$\frac{16775.97728}{274.49} = \frac{\left(1-(1.108)^{-t}\right)(\cancel{12})}{\left((1.108)^{1/12}-1\right)\cancel{12}}$$

$\therefore 1-(1.108)^{-t} = 0.52456676.$

$-t = -7.2499.$

$\therefore t = 7.2499$

iii. Total interest of initial loan $\Rightarrow 427.9 \times 48 - 16775.97$

$= 3763.22.$

Total interest on restructured loan $\Rightarrow 7.2499 \times 274.49 \times 12$

$- 16775.977$

$= 7104.3233$

$\therefore$ They will pay 3341.43 more interest.

Q2 i. Payback period

It is the number of years necessary to recover funds invested in the project without considering the interest, both ont the borrowing and investing.

Discounted payback period.

It is the number of years after which the cumulative discounted cash flows cover the initial investment.

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ii. Unlike the NPV, neither the DPP nor the PP give any indication of how profitable a project is, as the ignore cashflows after the accumulated value of zero is reached.

The PP can give misleading results as it does not take into account the time value of money.

Hence, NPV is a superior measure as compared to DPP or PP.

iii a. Project A.

PV of outlay $= -170 - 20v - 10v^2$

PV of inlay $= 20v + 20v^2 + 200v^3$

NPV $= -170 - 20v - 10v^2 + 20v + 20v^2 + 200v^3$

$0 = -170 + 10v^2 + 200v^3$

IRR $\Rightarrow$ $170 = 10(1+i)^{-2} + 200(1+i)^{-3}$

At $i = 0.07$ LHS $= 171.99$

$i = 0.08$ LHS $= 167.33$.

By interpolating $i = 7.4\%$.

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Project B

PV of outlaay $= -200$
PV of inlay $= 14\, a_{\overline{6}|} + 200\, v^6$

$$NPV = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$$

$$IRR \Rightarrow 200 = 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200\,(1+i)^{-6}.$$

At $i = 0.07$ LHS $= 200$.

$\therefore i = 7\%$.

b. $NPV_A = -170 + 10v^2 + 200v^3$
At $d = 6\%$.

$NPV_A = 4.9528$

$NPV_B = -200 + 14\, a_{\overline{6}|} + 200\, v^6$

$$= -200 + 14\left(\frac{1-(0.94)^6}{0.06}(0.94)\right) + 200(0.94)^6$$

$NPV_B = 5.9958509.$

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c. For Project A, the interest rate should be below 7.4, if above this then the project in unprofitable.

For Project B, the interest rate should be below 7%. If above this then the project is unprofitable.

Other factors :-
- Project A (No net income during FY)
- Low IRR for Project B for a long period of both projects.

Q3. PV of annuity 1 $= 200\, \ddot{a}_{\overline{22}|}\, v^{3/12}$

$$= (1.08)^{-1/4} \frac{(200)(10.2007)(0.08)}{0.074074}$$

$$= 2161.365534$$

PV of annuity 2 $= 320\, \ddot{a}^{(4)}_{\overline{16.25}|}\, v^{2/12}$

$$= 320 \left(\frac{1-(1.08)^{-16.25}}{0.08} \right) (1.049519)(1.08)^{-1/6}$$

$$= 2996.048870$$

PV of annuity 3 $= 15\, \ddot{a}^{(12)}_{\overline{226}|}$ @ $\frac{d^{(12)}}{12}$

$$= 15 \left(\frac{1-(1.08)^{-226/12}}{0.006392917} \right)$$

$$= 1795.651419$$

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Total PV = 6953.065823.

Revised annuity ⇒

$$6953.065823 = X \ddot{a}^{(2)}_{\overline{21.5|}} \; @ \; d^{(2)}.$$

$$= X \left(\frac{1-(1.08)^{-21.5}}{0.08} \right) (1.059615)$$

$$\therefore X = 647.0135657.$$

Q4. To derive $I\ddot{a}_{\overline{n|}} = \dfrac{\ddot{a}_{\overline{n|}} - nv^n}{d}.$

Proof:

$$PV = 1 + 2v + 3v^2 + \dots \; nv^{n-1} \quad -(1)$$

$$(1-d)\,PV = v + 2v^2 + 3v^3 \dots\dots \; nv^n \quad -(2).$$

(1) − (2).

$$PV - (1-d)PV = 1 + v + v^2 + v^3 + \dots\dots v^{n-1} - nv^n.$$

$$d(PV) = \ddot{a}_{\overline{n|}} - nv^n$$

$$\boxed{\therefore I\ddot{a}_{\overline{n|}} = \frac{\ddot{a}_{\overline{n|}} - nv^n}{d}.}$$

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Q5 i) For Property Fund.

$$1+i = \frac{\cancel{13.1}}{12.4} \times \frac{\cancel{14.8}}{\cancel{13.1}} \times \frac{\cancel{15.8}}{\cancel{14.8}} \times \frac{16.4}{\cancel{15.8}}$$

$$TWRR = i = 32.258064\%$$

For Equity Fund.

$$1+i = \frac{\cancel{9.2}}{12.1} \times \frac{\cancel{10.3}}{\cancel{9.2}} \times \frac{\cancel{13.1}}{\cancel{10.3}} \times \frac{15.5}{\cancel{13.1}}$$

$$TWRR = i = 28.0991736\%.$$

ii) $12.4(1+i) + 13.1(1+i)^{3/4} + 14.8(1+i)^{1/2} + 15.8(1+i)^{1/4}$
$= 4(16.4)$

$i = 29.32\%.$

iii) $(12.1)(1+i) + 9.2(1+i)^{3/4} + 10.3(1+i)^{1/2}$
$+ 13.1(1+i)^{1/4} + 4 \times 15.5.$
$\therefore i = 67.31\%.$

Q6. i. $160000 = X\, a_{\overline{10}|}$ @ 8%.

$\therefore X = \frac{160000}{6.7101} = 23844.65201$

ii. $PV_4 = 23844.65201\, a_{\overline{6}|}$ @ 8%.
$= 23844.65201 \times 4.6229$
$= 110231.4422$

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$$110231.4422 = x' \, a_{\overline{6}|} \text{ @ } 10\%.$$

$$x' = \frac{110231.4422}{4.3553} = 25309.72428.$$

iii] $PV_7 = 25309.72488 \, a_{\overline{3}|} \text{ @ } 10\%.$

$$= 25309.72488 \, (2.4869)$$
$$= 62942.75331$$

$$62943.75331 = x'' \, a_{\overline{3}|} \text{ @ } 9\%.$$

$$x'' = \frac{62943.75331}{2.5313}$$
$$= 24865.78174$$

Q8. i PV of outlay $= 2.7 + 0.2 \, a_{\overline{3}|}.$

PV of inlay $= \bar{a}_{\overline{10}|} (0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3$

$0 = NPV = \bar{a}_{\overline{10}|} (0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2 \, a_{\overline{3}|}$

$i = 9.3\%$

ii. $NPV = 0.1v^3 + 0.85\,\bar{a}_{\overline{10}|}\,v^3 - 2.7 - 0.2\,a_{\overline{3}|}$ @ 10%

$$= 0.1(1.1)^{-3} + 0.85(1.1)^{-3}\left(\frac{1-(1.1)^{-10}}{\ln(1.1)}\right) - 2.7 - 0.2\left(\frac{1-(1.1)^{-3}}{0.1}\right)$$

$$= 1.0089.$$

Q9. i] $$(1+i)^3 = \frac{33}{30.5} \times \frac{38.5}{33+6} \times \frac{41.05 - 4.5}{38.5} \times \frac{45}{41.05} \times \frac{45.6}{45} \times \frac{47}{45.6-2.5}$$

$$(1+i)^3 = 1.228313.$$

$$i = 7.0951\% = \text{TWRR}$$

$$30.5(1+i)^3 + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} = 47.$$

At $i = 7\%$ RHS $= 46.74$

$i = 7.5\%$ RHS $= 47.37$

By interpolation, $i = 7.208\%$ $= \text{MWRR}.$

ii] Linked rate of return.

2011 ⇒

$$30.5(1+i) + 6(1+i)^{0.25} = 38.5$$

$$i = 6.266\%.$$

2012 ⇒

$$38.5(1+i) + 4.5(1+i)^{0.5} = 45$$

$$i = 4.92\%.$$

2013 ⇒

$$45(1+i) + 2.5(1+i)^{0.5} = 47.$$

$$i = 10.276\%.$$

$$(1+i)^3 = 1.06266 \times 1.0492 \times 1.10276$$

$$i = 7.13\%.$$

iii TWRR eliminates the effect of the cashflow amount and timing and therefore give a fairer basis for assessing the investment performance for the fund. MWRR is sensitive to the amount and timing of the net cashflows.

Q11 i] TWRR → $(1+i)^2 = \frac{500}{460} \times \frac{550-50}{500+40} \times \frac{\cancel{600}}{550} \times \frac{x}{\cancel{600}}$

$1.44 = \frac{500 \times 500 \times x}{460 \times 540 \times 550}$

$x = 786.9312$

ii] MWRR ⇒ $460(1+i)^2 + 40(1+i)^{21/12} - 50(1+i)$
$= 786.9312$

$i = 30.909\%$

Q12 i. Discounted Payback period is the number of years after which the cumulative discounted cash inflows cover the initial investment.

ii a] $-800(1+i)^t - 50(1+i)^{t-1} + 100\,\bar{s}_{\overline{t-2}|} = 0$ @ $i = 7\%$

$-800(1.07)^t - 50(1.07)^{t-1} + 100\left(\frac{(1.07)^{t-2}-1}{\ln(1.07)}\right) = 0$

$t = 17.7676$

b] $100\,\bar{s}_{\overline{4.233}|}$ @ 6%

$= 100\left(\frac{(1.06)^{4.233}-1}{\ln(1.06)}\right) = 480$

iii. $-800(1+i)^{t} - 50(1+i)^{t-1} + 102.971\, s_{\overline{t-2}|}$ @ 7%

$= 0$.

Taking $t = 18$.

$-800(1+i)^{18} - 50(1+i)^{17} + 102.971\, s_{\overline{16}|}$ @ 7%

$= 9.7714$.

$\Rightarrow 9.7714(1+i)^{4} + 100\, \ddot{s}_{\overline{4}|}$ @ 6%.

$= 462.79$.

The accumulated amount after 22 years is 462.79.

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Q13.

i) $988000 = X^{(12)} a_{\overline{25}|}^{(12)}$ @ 7%

$= 12X \left(\frac{1-(1.07)^{-25}}{((1.07)^{1/12}-1)\, 12} \right)$

$X = 6848.$

$\therefore$ Monthly repayment = 6848

ii. $X a_{\overline{34/3}|}^{(12)} = 6848 \left(a_{\overline{34/3}|}^{(12)} @ 7\% \right)$

$= 6,48,600$

iii. PV after 10th Sept payment $= X a_{\overline{83/6}|}^{(12)}$ @ 7%

PV after 10th October's payment $= X a_{\overline{13.75}|}^{(12)}$ @ 7%

Capital repaid $\Rightarrow X a_{\overline{13.75}|}^{(12)} - X a_{\overline{83/6}|}^{(12)}$

$= X \left(\frac{v^{13.75} - v^{83/6}}{i^{(12)}} \right)$ @ 7%

$= 26.86$

iv. Capital repayment in these 12 installments $\Rightarrow$

$X \left(a_{\overline{22/3}|}^{(12)} - a_{\overline{19/3}|}^{(12)} \right)$ @ 7%

$= 516.2$

$\therefore$ Total interest in these 12 installments $\Rightarrow$

$12X - 51620 = \underline{\underline{30556}}.$