classmate

Date

Page

FM- I

1. Given that

(i) $d^{(4)} = 8\%$

$$d = \left[1 - \frac{d^{(4)}}{4}\right]^4$$

$= 7.7632\%$

Now force of interest,

$= \cancel{\log(1-d)}$

$$\delta = -\ln(1-d)$$

$= -\ln(1 - 0.077632)$

$= \underline{\underline{8.081\%}}$

(ii) $$1+i = \frac{1}{[1 - d^{(4)}/4]^4}$$

$$i = \frac{1}{[1 - d^4/4]^4} - 1$$

$(i) = 8.417\%$

(iii) $d^{(12)} = [1 - (1-d)^{1/12}]\,12$

$= 7.913\%$

2. Given that

$\delta(t) = 0.004t + 0.0002t^2$ for all t

(i) $PV_0 = 1000 \times v(0,10)$

$= 1000 \times e^{-\int_0^{10} 0.004t + 0.0002t^2 dt}$

$= 1000 \times \exp[-(0.002t^2 + 0.0002t^3)_0^{10}]$

$= 1000 \times \exp[-4/15]$

$= 765.928$

(ii) $AV_{10} = PV_0 \times (1+i)^{10}$

$1000 = 765.928 \times (1+i)^{10}$

$i = 2.703\%$ p.a.

3. We know that

$d = iv$

$\therefore \quad v = \dfrac{1}{1+i}$

So we can say that

$d = i \cdot \dfrac{1}{1+i} = \dfrac{i}{1+i}$ — (i)

Now also $1-d = \dfrac{1}{1+i}$

$d = \dfrac{1}{1+i} - 1 = \dfrac{\cancel{1} - \cancel{1} + i}{1+i}$ — (ii)

Hence we can say that they are equal

(ii)

(a) $\frac{d^{(4)}}{4} = 2.25\%$

Here we can say that,

$$\left(1-\frac{d^{(4)}}{4}\right)^4 = \left(1-\frac{d^{(2)}}{2}\right)^2$$

$$\left(1-0.025\right)^4 = \left(1-\frac{d^{(2)}}{2}\right)^2$$

$$0.91299 = \left[1-\frac{d^{(2)}}{2}\right]^2$$

$$d^{(2)} = 8.899\%$$

(b) Given that $\frac{d^{(1/2)}}{(1/2)} = 5\%$

So

$$\left[1-\frac{d^{(2)}}{2}\right]^2 = \left[1-\frac{d^{(1/2)}}{1/2}\right]^{1/2}$$

$$\left[1-\frac{d^{(2)}}{2}\right]^2 = \left[1-0.05\right]^{1/2}$$

$$d^{(2)} = 5.199\%$$

(iii) If the bill is 100,

$AV = 100 \times 1.05 = \underline{\underline{105}}$

$PV = AV(1-d)$

$100 = 105((1-d)$

$d = 4.762\%$

4.

(ii) Given $i = 0.08$

(a) $d^{(12)} = ?$

$$\left[1 - \frac{d^{(12)}}{12}\right]^{12} = \frac{1}{1+i}$$

$$\left[1 - \frac{d^{(12)}}{12}\right]^{12} = \left[\frac{1}{1+0.08}\right]$$

$$d^{(12)} = 7.671\%$$

(b) $i^{(365)} = ?$

$$(1+i) = \left[1 + \frac{i^{(365)}}{365}\right]^{365}$$

$$(1.08)^{1/365} = \left[1 + \frac{i^{(365)}}{365}\right]$$

$$i^{(365)} = 7.696\%$$

(c) δ

$$(1+i) = e^{\delta}$$

$$\ln(1.08) = \delta$$

$$\delta = 7.6961\%$$

(d) $i^{(0.5)}$

$$(1+i) = \left[1 + \frac{i^{(0.5)}}{0.5}\right]^{0.5}$$

$$[1.08]^{1/0.5} = \cancel{0.5}\left[1 + \frac{i^{(0.5)}}{(0.5)}\right]$$

$$i^{(0.5)} = 8.32\%$$

(i)

5. $d^{(4)} = 6\%$

(a) $$\frac{1}{\left(1-\frac{d^{(4)}}{4}\right)^4} = \left[1+\frac{i^{(2)}}{2}\right]^2$$

$$\frac{1}{0.9413} = \left[1+\frac{i^{(2)}}{2}\right]^2$$

$$= 1.06236^{(1/2)} = \left[1+\frac{i^{(2)}}{2}\right]$$

$= \cancel{1.0307}$

$i^{(2)} = 6.1378\%$

(b) $$\cancel{\frac{1}{\left[1-d^{(4)}/4\right]^4} = \left[1- \right.}$$

(b) $$\left(1-\frac{d^{(4)}}{4}\right)^4 = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$

$d^{(12)} = 6.03\%$

(ii) Timeline

(a)

200000 — 15/4/05

17/07/05

15/4/06 — 2,20,000

Actual rate for one year:

$2,20,000 = 200000(1+i)^1$

$i = 0.1 = 10\%$

classmate

Date

Page

$\therefore$ AV on 17/7/05: $200000 \times \left(1 + \frac{i^{365}}{365}\right)^{93}$

$= \underline{\underline{204916.356}}$

6.

(i) Annual effective discount rate can be given as

$= 70 = 100(1-d)$

$= d = 0.3 = \underline{\underline{30\%}}$

(ii) $d = 30\%$

$(1-d) = \left[1 - \frac{d^{12}}{12}\right]^{12}$

$d^{(12)} = 35.142\%$

$i^{(2)} = ?$

$\frac{1}{(1-d)} = \left[1 + \frac{i^{2}}{2}\right]^{2}$

$\left[\frac{1}{0.7}\right]^{1/2} = \left[1 + \frac{i}{2}\right]$

$i^2 = 39.04\%$

$\delta = ?$

$(1-d) = e^{-\delta}$

$\ln(1-d) = -\delta$

$\delta = -\ln(1-d)$

$= \underline{\underline{35.667\%}}$

7. $26000 = 20000\left[1 + \frac{i^{(4)}}{4}\right]^{17/3}$

$1.3 = \left[1 + \frac{i^{(4)}}{4}\right]^{17/3}$

$1.3^{(1/(17/3))} = \left[1 + \frac{i^{(4)}}{4}\right]$

$\cancel{1.06778 = \left[1 + \frac{i^{(4)}}{4}\right]}$ $\qquad i^{(4)} = 18.955\%$

$i^{(4)} =$ [illegible]

(ii) $20000 = 26000\left[1 - \frac{d^{(2)}}{2}\right]^{17/6}$

$d^{(2)} = 17.688\%$

(iii) $20000 = 26000 \times \left[1 - \frac{17}{2}\right].d$

$d = 16.289\%$

9.
(i) Force of interest can be described as the ratio which is continuously compounded. It can be called as the nominal rate of interest convertible continuously which is interest paid in every moment of time.

(ii) $i^{(2)} = 7.75\%$

classmate

Date

Page

$(1+i) = \left(1+\frac{i^2}{2}\right)^2$

$(1+i) = \left[1+\frac{0.0775}{2}\right]^2$

$i = 7.90\%$

$\delta = \ln(1+i)$

$= 7.603\%$

$d^{(12)} = ?$

$\Rightarrow \left(1-\frac{d^{12}}{12}\right)^{12} = \frac{1}{(1+i^2/2)^2}$

$d^{12} = 7.57\%$

$i^{(1/2)}$

$\Rightarrow \left[1+\frac{i^2}{2}\right]^2 = \left[1+\frac{i^{1/2}}{1/2}\right]^{1/2}$

$i^{1/2} = 8.21\%$

10. We know that

$PV = C \cdot v(t)$

$= \exp\left[-\left(\int_0^5 0.08\,dt + \int_5^{10} 0.06\,dt + \int_{10}^{t} 0.04\,dt\right)\right]$

$= \exp\left[-(0.08t)_0^5 + (0.06t)_5^{10} + (0.04t)_{10}^{t}\right]$

$= \exp -[0.04t + 0.3]$

$= v(t) = e^{-0.94t - 0.3}$

11.

9 months — 50000

(a) 18% discount

$PV_0 = C \cdot v(0, 9)$

$PVO = 50000 \cdot (1 - 0.18)^{3/4}$

$= 43085.426$

(b) (i) $\delta = 6\%$ $\quad d^{12}$ $\qquad$ (ii) $d = 6\%$

$\left[1 - \frac{d^{12}}{12}\right]^{12} = e^{-\delta}$ $\qquad$ $\left[1 - \frac{d^4}{4}\right]^{1/4} = \left[1 + \frac{i^{12}}{12}\right]^{12}$

$d^{12} = 5.985\%$ $\qquad$ $i^{12} = 6.0607\%$

(c) $d < \delta < i^{4} < i$

d being the lowest value and i being the largest value. Force of interest is limit that comes from both d and i when they are compounded continuously.

$i^{(4)}$ is paid per quarter hence ~~is~~ lower then i

d) D is considered to be equal to discounting the interest which is earned from 0 to 1, that is the same amount of accumulation.

12. timeline

0

7049 $i^{(12)} = 6\%$ 2 $i^{(10)} = 1.5$ 4.5 $d^{(18)} = 6\%$ 6

$AU_6 = C \cdot A(0,6)$

$= 7049 \left[\left[1 + \frac{i^{(12)}}{12}\right]^{24} + \left[1 + 10i\right] + \left[1 / \left[1 - \frac{d^{12}}{12}\right]^{18}\right] \right]$

$= 7049 \times (1.127159) \times (1.15) \times (1.094421)$

$= 9999.8938$

13. $AU = c(1+i)^n$

$2 = d(1+0.1)^n$

$\ln 2 = \ln(1.1)^n$

$\ln 2 = n \ln(1.1$

$n = 7.27$ yers

$AU = C \cdot \frac{1}{\left[1 - \frac{d^{12}}{12}\right]^{12n}}$

$2 = 1 \left[\frac{1}{(1 - \frac{d^{12}}{12})^{12n}} \right]$

$\left(1 - \frac{d^{12}}{12}\right)^{12n} = \frac{1}{2}$

$12n \ln(0.991661) = \ln\left(\frac{1}{2}\right)$

$n = 6.9$ years