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Calculus Assignment - 1

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① $$\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h} = \lim_{h\to 0} \frac{2(h-3)^2 - 2(9)}{h}$$

$$= \lim_{h\to 0} \frac{2[(h-3)^2 - (3)^2]}{h}$$

$$= \lim_{h\to 0} \frac{2}{h}[(h-3+3)(h-3-3)]$$

$$= \lim_{h\to 0} \frac{2}{h} \times h \times (h-6)$$

$$= \lim_{h\to 0} 2(h-6)$$

$$= 2(-6) = \boxed{-12}$$

② $$g(x) = \begin{cases} 2x & ; x < 6 \\ x-1 & ; x \geq 6 \end{cases}$$

(a) $x = 4$

$g(4) = 2x = 2(4) = 8$

$\therefore g(x)$ exists

$\lim_{x\to 4} g(x) = 2x$

$= 8$

$\therefore \lim_{x\to 4} g(x) = g(4)$

$\therefore g(x)$ is continuous at $x=4$

(b) $x = 6$

$g(6) = x - 1 = 6 - 1 = 5$

$\therefore g(x)$ exists

$\lim_{x\to 6^-} g(x) = 2x$

$= 12$

$\lim_{x\to 6^+} g(x) = x - 1$

$= 5$

$\therefore \lim_{x\to 6^-} g(x) \neq \lim_{x\to 6^+} g(x)$

$\therefore g(x)$ is discontinuous at $x=6$

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③ $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} = \lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$

$= \lim_{x\to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$

$= \lim_{x\to 9} \frac{-(9-x)(3+\sqrt{x})}{(9-x)}$

$= \lim_{x\to 9} -3-\sqrt{x}$

$= -3-\sqrt{9} = -3-3 = \boxed{-6}$

④ $f(x) = \frac{4x+5}{9-3x}$

(a) $x = -1$

$\therefore f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{1}{12}$

$\therefore f(x)$ exists

$\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{4x+5}{9-3x} = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12}$

$\because \lim_{x\to -1} f(x) = f(-1)$ $\therefore f(x)$ is continuous at $x=-1$

(b) $x = 0$

$\therefore f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$\therefore f(x)$ exists

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{4x+5}{9-3x} = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$\because \lim_{x\to 0} f(x) = f(0)$ $\therefore f(x)$ is continuous at $x=0$

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(c) $x = 3$

$$f(3) = \frac{4(3)+5}{9-3(3)} = \frac{12+5}{9-9} = \frac{17}{0}$$

$\therefore f(x)$ doesn't exist

$\therefore f(x)$ is discontinuous at $x = 3$

(6) $$g(y) = \begin{cases} y^2+5 & ; y < -2 \\ 1-3y & ; y \geq -2 \end{cases}$$

(a) $$\lim_{y \to 6} g(y) = \lim_{y \to 6} 1-3y = 1-3(6) = 1-18 = \boxed{-17}$$

(b) $$\lim_{y \to -2^-} g(y) = \lim_{y \to -2^-} y^2+5$$

$$= 4+5 = \boxed{9}$$

$$\lim_{y \to -2^+} g(y) = \lim_{y \to -2^+} 1-3y$$

$$= 1-3(-2) = 1+6 = \boxed{7}$$

(7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

Diff w.r.t. t

$$f'(t) = (4t^2-t)(3t^2-16t) + (t^3-8t^2+12)(8t-1)$$
$$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12$$
$$f'(t) = 20t^4 - 132t^3 + 24t^2 + 96t - 12$$

(8) $$R(w) = \frac{3w + w^4}{2w^2+1}$$

Diff w.r.t. w

$$R'(w) = \frac{(2w^2+1)(3+4w^3) - (3w+w^4)(4w)}{(2w^2+1)^2}$$

$$= \frac{6w^2 + 8w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{4w^4 + 4w^2 + 1}$$

$$= \frac{4w^5 + 4w^3 - 6w^2 + 3}{4w^4 + 4w^2 + 1}$$

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⑨ $f(x) = 2e^x - 8^x$

Diff w.r.t x

$f'(x) = 2e^x - 8^x \log 8$

⑩ $y = z^3 - e^z \ln(z)$

Diff wrt z

$$\frac{dy}{dz} = 5z^4 - \left[e^z \frac{1}{z} + \ln(z)\, e^z \right]$$

$$= 5z^4 - e^z \left(\frac{1}{z} + \ln(z) \right)$$

⑪

(a) $f(x) = (6x^2+7x)^4$

Diff wrt x

$f'(x) = 4(6x^2+7x)^3 \cdot (12x+7)$

(b) $f(t) = 5 + e^{4t+t^7}$

Diff w.r.t t

$f'(t) = e^{4t+t^7} \cdot (4+7t^6)$

(12)

(a) $z = \ln(7-x^3)$

Diff w.r.t x

$$\frac{dz}{dx} = \frac{-3x^2}{7-x^3}$$

Further diff w.r.t x

$$\frac{d^2z}{dx^2} = \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{49 - 14x^3 + x^6} = \frac{-42x - 3x^4}{x^6 - 14x^3 + 49} = \frac{-42x - 3x^4}{(7-x^3)^2}$$

(b) $$g(v) = \frac{2}{(6+2v-v^2)^4} = 2(6+2v-v^2)^{-4}$$

Diff w.r.t v

$$g'(v) = 2\left[-4(6+2v-v^2)^{-5} \cdot (2-2v)\right]$$

$$= -16(1-v)(6+2v-v^2)^{-5}$$

Further diff w.r.t v

$$g''(v) = -16\left[(1-v).-5(6+2v-v^2)^{-6} \cdot (2-2v) + (6+2v-v^2)^{-5}.(-1)\right]$$

$$= -16\left[-10(1-v)(1-v)(6+2v-v^2)^{-6} - (6+2v-v^2)^{-5}\right]$$

$$= -16\left[-10(1-v)^2(6+2v-v^2)^{-6} - (6+2v-v^2)^{-5}\right]$$

$$= -16\left[\frac{-10(1-v)^2}{(6+2v-v^2)^6} - \frac{1}{(6+2v-v^2)^5}\right]$$

$$= \frac{-16}{(6+2v-v^2)^5}\left[\frac{-10(1-v)^2}{(6+2v-v^2)} - 1\right]$$

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(c) $f(t) = \ln(1+t^2)$

Diff w.r.t t

$$f'(t) = \frac{1}{(1+t^2)}(2t) = \frac{2t}{1+t^2}$$

Further diff w.r.t t

$$f''(t) = \frac{(1+t^2)(2) - 2t(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$= \frac{2-2t^2}{(1+t^2)^2}$$

$$= \frac{2(1-t^2)}{(1+t^2)^2}$$

(13) $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

For the extreme values $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$x^2 + x - 3x - 3 = 0$

$x(x+1) - 3(x+1) = 0$

$(x+1)(x-3) = 0$

$\therefore x+1 = 0$ or $x-3 = 0$

$x = -1$ $x = 3$

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For $x=-1$;

$f''(-1) = 6(-1)-6$

$= -6-6$

$= -12 < 0$

$\therefore x=-1$ is the maxima

$\therefore f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$

$\therefore$ Maximum value $= 17$

For $x=3$;

$f''(x) = 6(3)-6$

$= 18-6$

$= 12 > 0$

$\therefore x=3$ is the minima

$\therefore f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= -15$

$\therefore$ Minimum value $= -15$

(14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$f''(x) = 24x - 36$

For the extreme values, $f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$x^2 - 3x + 2 = 0$

$x^2 - x - 2x + 2 = 0$

$x(x-1) - 2(x-1) = 0$

$(x-1)(x-2) = 0$

$x-1=0$ or $x-2=0$

$x=1$ $\quad$ $x=2$

For $x=1$;

$f''(1) = 24 - 36$

$= -12 < 0$

$\therefore x=1$ is the maxima

$\therefore f(1) = 4 - 18 + 24 - 7$

$= 3$

$\therefore$ Maximum value $= 3$

For $x=2$;

$f''(2) = 24(2) - 36$

$= 48 - 36 = 12 > 0$

$\therefore x=2$ is the minima

$\therefore f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 32 - 72 + 48 - 7 = 1$

$\therefore$ Minimum value $= 1$

⑤ $$\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$= \lim_{x\to\infty} \frac{6e^{4x} - \frac{1}{e^{2x}}}{8e^{4x} - e^{2x} + \frac{3}{e^{x}}}$$

$$= \lim_{x\to\infty} \frac{6 - \frac{1}{e^{6x}}}{8 - \frac{1}{e^{2x}} + \frac{3}{e^{5x}}} \qquad [\div \text{ by } e^{4x}]$$

$$= \frac{\lim_{x\to\infty} 6 - \frac{1}{e^{6x}}}{\lim_{x\to\infty} 8 - \frac{1}{e^{2x}} + \frac{3}{e^{5x}}}$$

$$= \frac{\lim_{x\to\infty} 6 - e^{-6x}}{\lim_{x\to\infty} 8 - e^{-2x} + 3e^{-5x}}$$

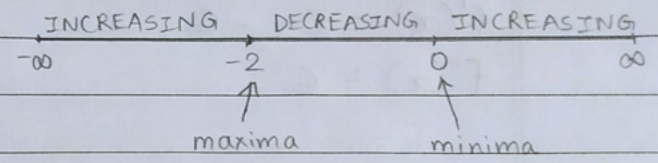
$$= \frac{6 - e^{-\infty}}{8 - e^{-\infty} + 3e^{-\infty}}$$

$$= \frac{6 - 0}{8 - 0 + 0} = \frac{6}{8} = \boxed{\frac{3}{4}}$$

(15) $f(x) = x^2(x+3) = x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$

$f''(x) = 6x + 6$



For the extreme values, $f'(x) = 0$

$\therefore 3x^2 + 6x = 0$

$x(x+2) = 0$

$\therefore x = 0$ or $x + 2 = 0$

$x = -2$

For $x = 0$;

$f''(0) = 6(0) + 6$

$= 6 > 0$

$\therefore x = 0$ is the minima

$\therefore F(0) = (0)^3 + 3(0)^2$

$= 0$

Minimum value $= 0$

For $x = -2$;

$f''(-2) = 6(-2) + 6$

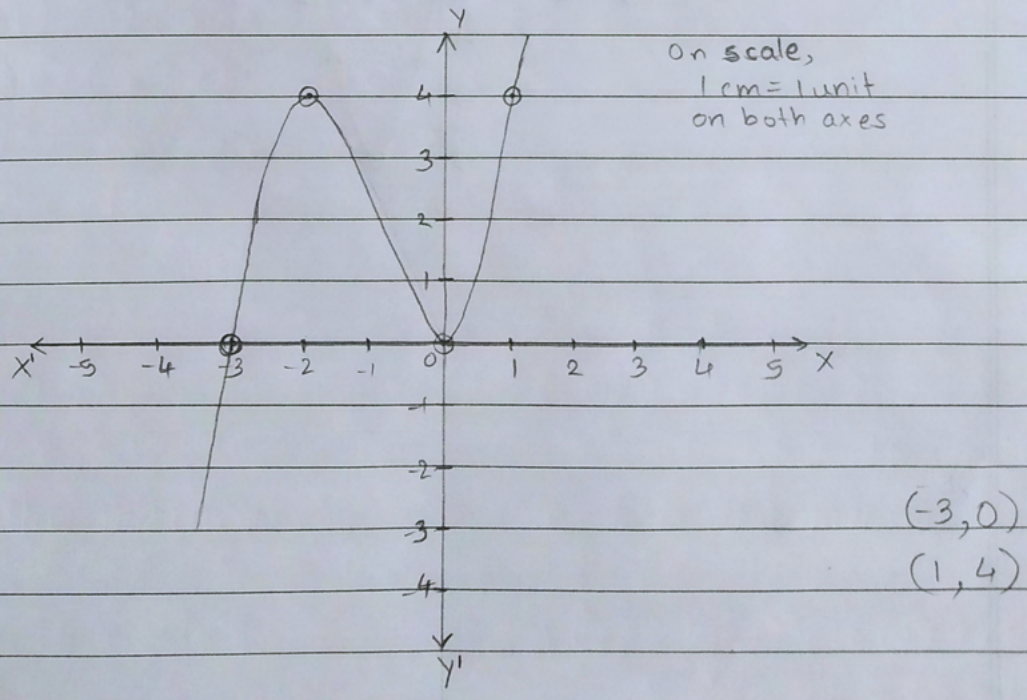
$= -12 + 6 = -6 < 0$

$\therefore x = -2$ is the maxima

$\therefore F(-2) = (-2)^3 + 3(-2)^2$

$= -8 + 12 = 4$

Maximum value $= 4$



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(16) $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

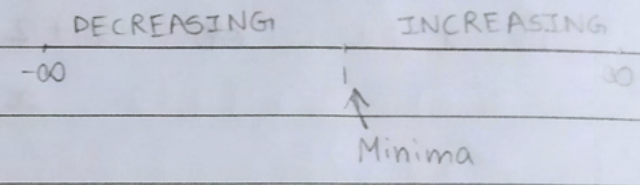
$f''(x) = 6$

For the extreme values, $f'(x) = 0$

$6x - 6 = 0$

$x - 1 = 0$

$x = 1$

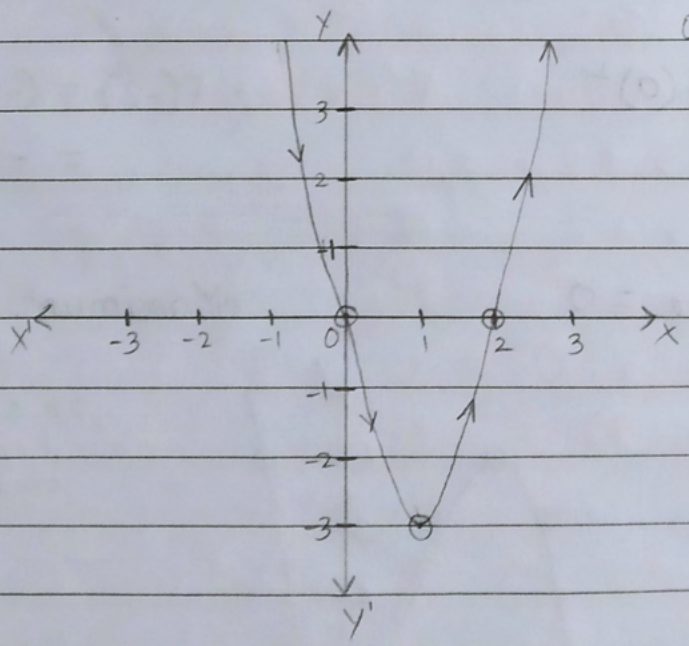


$f''(1) = 6 > 0$

$\therefore x = 1$ is the minima

$f(1) = 3(1)^2 - 6(1) = 3 - 6 = -3$

Minimum value $= -3$



On scale
1 cm = 1 unit
on both axis

(-1, 9)
(0, 0)
(2, 0)
(3, 9)