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Calculus: Assignment-1

1. $$\lim_{h\to 0} \frac{2(-3+h)^2-18}{h}$$

$$= \lim_{h\to 0} \frac{2(h-3)^2-18}{h}$$

$$= \lim_{h\to 0} \frac{2(h^2+9-6h)-18}{h}$$

$$= \lim_{h\to 0} \frac{2h^2+\cancel{18}-12h\cancel{-18}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2h-12)}{\cancel{h}}$$

$$= 2(0)-12 = \underline{-12}.$$

2. $$g(x) = \begin{cases} 2x & x<6 \\ x-1 & x\geq 6 \end{cases}$$

(a) $x = 4$

$g(4) = 2(4) = 8$

$$\lim_{x\to 4^-} g(x) = \lim_{x\to 4^-} 2x = 8$$

$$\lim_{x\to 4^+} g(x) = \lim_{x\to 4^+} (x-1) = 3$$

$\therefore$ ~~lim~~ $\underset{\text{at } x=4}{\text{LHL}} \neq \underset{\text{at } x=4}{\text{RHL}}$

fn is discontinuous at $x=4$.

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(b) $x = 4$

$g(6) = 6 - 1 = 5$

$$\lim_{x \to 6^-} g(x) = \lim_{x \to 6^-} 2x = 2(6) = 12$$

$$\lim_{x \to 6^+} g(x) = \lim_{x \to 6^+} x - 1 = 5$$

$\therefore$ $\underset{\text{at } x=6}{LHL} \neq \underset{x=6}{RHL}$ function is discontinuous at $x = 6$.

3. $$\lim_{x \to 9} \frac{x - 9}{3 - \sqrt{x}}$$

$$= \lim_{x \to 9} \frac{(\sqrt{x})^2 - (3)^2}{3 - \sqrt{x}}$$

$$= \lim_{x \to 9} \frac{\cancel{(\sqrt{x} - 3)}(\sqrt{x} + 3)}{-\cancel{(\sqrt{x} - 3)}}$$

$$= -(\sqrt{x} + 3) = -(3 + 3) = \underline{\underline{-6}}$$

4. $$f(x) = \frac{4x + 5}{9 - 3x}$$

(a) $x = -1$

$$f(-1) = \frac{4(-1) + 5}{9 - 3(-1)} = \frac{-1}{12}$$

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~~f(x)~~ $\lim_{x \to -1} \frac{4x+5}{9-3x} = \frac{1}{12}$

~~limit~~

$\therefore \lim_{x \to -1} f(x) = f(-1)$

$\therefore$ function is continuous at $x = -1$.

(b) $x = 0$

$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$\lim_{x \to 0} \frac{4x+5}{9-3x} = \frac{5}{9}$

fn. is continuous at $x = 0$

(c) $x = 3$

$f(3) = \frac{4(3)+5}{9-3(3)} =$ not defined

$\lim_{x \to 3} \frac{4x+5}{9-3(x)} =$ ~~not def~~ N.D.

$\Rightarrow$ fn. is discontinuous at $x = 3$.

5. $$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$\cancel{\lim_{x \to \infty} \frac{24e^{4x} + 2e^{-2x}}{32e^{4x} - 2e^{2x} - 3e^{-x}}}$$

$$\lim_{x \to \infty} \frac{\cancel{e^{-2x}}(6e^{6x} - 1)}{\cancel{e^{-2x}}(8e^{6x} - e^{4x} + 3e^{x})}$$

$$= \lim_{x \to \infty} \frac{\cancel{e^{6x}}\left(6 - \frac{1}{e^{6x}}\right)}{\cancel{e^{6x}}\left(8 - \frac{1}{e^{2x}} + \frac{3}{e^{5x}}\right)}$$

$$= \lim_{x \to \infty} \frac{6}{8}$$

$$= \boxed{\frac{3}{4}}$$

[$\frac{1}{e^{6x}}$, $\frac{1}{e^{2x}}$ & $\frac{1}{e^{5x}}$ will be 0 as $x = \infty$]

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6. $g(y) = \begin{cases} y^2+5 ; & y < -2 \\ 1-3y ; & y \geq -2 \end{cases}$

(a) $\lim_{y \to 6} g(y) = \lim_{y \to 6} 1-3y$

$= 1 - 3(6) = 1 - 18 = -17$

(b) $\lim_{y \to -2} g(y) = \lim_{y \to -2} 1-3y$

$= 1 - 3(-2) = 1 + 6 = 7$

7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$= (8t-1)(t^3 - 8t^2 + 12) + (3t^2 - 16t)(8t-1)$

$= 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12 +$

$24t^3 - 3t^2 - 128t^2 + 16t$

$= 8t^4 - 41t^3 - 123t^2 + 112t - 12$

7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f'(t) = (8t-1)(t^3 - 8t^2 + 12) + (4t^2 - t)(3t^2 - 16t)$

$= 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12 +$

$12t^4 - 64t^3 - 3t^3 + 16t^2$

$= \boxed{20t^4 - 132t^3 + 24t^2 + 96t - 12}$

8. $$R(w) = \frac{3w + w^4}{2w^2 + 1}$$

$$R'(w) = \frac{(3 + 4w^3)(2w^2+1) - (3w+w^4)(4w)}{(2w^2+1)^2}$$

$$= \frac{6w^2 + 3 + 8w^5 + 4w^3 - (12w^2 + 4w^5)}{(2w^2+1)^2}$$

$$= \frac{6w^2 + 3 + 8w^5 + 4w^3 - 12w^2 - 4w^5}{(2w^2+1)^2}$$

$$= \boxed{\frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}}$$

9. $$f(x) = 2e^x - 8^x$$

$$f'(x) = 2e^x - 8^x . \ln 8$$

10. $$y = z^5 - e^z \ln(z)$$

$$\frac{dy}{dz} = 5z^4 - \left[e^z \ln(z) + \frac{e^z}{z}\right]$$

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11. (a) $f(x) = (6x^2+7x)^4$

$f'(x) = 4(6x^2+7x)^3(12x+7)$

(b) $f(t) = (5+e^{4t+t^7})$

$f'(t) = e^{4t+t^7}(4+7t^6)$

12. (a) $z = \ln(7-x^3)$

$\frac{dz}{dx} = \frac{1}{7-x^3} \times -3x$

$\frac{d^2z}{dx^2} = \frac{-1(-3)\times(-3x)}{(7-x^3)^2} = \frac{-9x(7-}{(7-x^3)^2}$

(b) $Q(v) = \frac{2}{(6+2v+v^2)^4}$

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12. (a) $z = \ln(7 - x^3)$

$$\frac{dz}{dx} = \frac{1}{7 - x^3}(-3x^2)$$

$$= \frac{3x^2}{x^3 - 7}$$

$$\frac{d^2z}{dx^2} = \frac{6x(x^3 - 7) - (3x^2)(3x^2)}{(x^3 - 7)^2}$$

$$= \frac{6x^4 - 42x - 9x^4}{(x^3 - 7)^2}$$

$$= \frac{-3x^4 - 42x}{(x^3 - 7)^2}$$

$$= -\frac{3x(x^3 + 14)}{(x^3 - 7)^2}$$

(b) $Q(v) = \frac{2}{(6 + 2v + v^2)^4}$

$$Q'(v) = \frac{-8}{(6 + 2v + v^2)^5}(2 + 2v)$$

$$= -\frac{16(v + 1)}{(6 + 2v + v^2)^5}$$

$$\phi''(v) = -16\left[\frac{(x^2+2x+6)^5 - 5(x+1)(2x+2)(x^2+2x+6)^4}{(x^2+2x+6)^{10}}\right]$$

$$= \frac{80(x+1)(2x+2)}{(x^2+2x+6)^6} - \frac{16}{(x^2+2x+6)^5}$$

$$= \frac{80(2x^2+4x+2) - 16(x^2+2x+6)}{(x^2+2x+6)^6}$$

$$= \frac{16(10x^2+20x+10) - 16(x^2+2x+6)}{(x^2+2x+6)^6}$$

$$= \frac{16(9x^2+18x+4)}{(x^2+2x+6)^6}$$

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(c) $f(t) = \ln(1+t^2)$

$f'(t) = \frac{1}{1+t^2}(2t)$

$= \frac{2t}{t^2+1}$

$f''(t) = \frac{(t^2+1)(2) - (2t)(2t)}{(t^2+1)^2}$

$= \frac{2t^2+2-4t^2}{(t^2+1)^2}$

$= \frac{-2(t^2-1)}{(t^2+1)^2}$

13. $f(x) = x^3 - 3x^2 - 9x + 12$

diff. wrt x

$f'(x) = 3x^2 - 6x - 9$

put $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$x^2 - 3x + x - 3 = 0$

$x(x+1) - 3(x+1) = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$

$f''(x) = 6x - 6$

$f''(-1) = 6(-1) - 6 = -12 < 0$

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$\cancel{f''(3)} \Rightarrow f(x)$ is maximum at $x = -1$

Maximum value $= f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12 = -4 + 21$

$= \underline{\underline{17}}$.

$f''(3) = 6(3) - 6 = 18 - 6 = 12 > 0$

$\Rightarrow f(x)$ is minimum at $x = 3$

Minimum value =

$f(3) = \cancel{(3)^3} - \cancel{3(3)^2} - 9(3) + 12$

$= -27 + 12$

$= \underline{\underline{-15}}$.

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

diff. wrt x

$f'(x) = 12x^2 - 36x + 24$

put $f'(x) = 0$

$\Rightarrow 12x^2 - 36x + 24 = 0$

$12(x^2 - 3x + 2) = 0$

$= x^2 - x - 2x + 2 = 0$

$= x(x-1) - 2(x-1) = 0$

$= (x-2)(x-1) = 0$

$x = 2 \; ; \; x = 1$

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$f''(x) = 24x - 36$

$f''(2) = 24(2) - 36 = 48 - 36 = 12 > 0$

$\Rightarrow f(x)$ is minimum at $x = 2$

minimum value $= f(2) = 4(2)^3 - 18(4) + 48 - 7$

$= 32 - 72 + 48 - 7$

$= \underline{\underline{1}}.$

$f''(1) = 24(1) - 36 = -12 < 0$

$\Rightarrow f(x)$ is maximum at $x = 1$.

maximum value $= f(1) = 4(1)^3 - 18(1) + 48 - 7$

$= \underline{\underline{27}}.$

15. $f(x) = x^2(x+3) = x^3 + 3x^2$

diff wrt x

$f'(x) = 3x^2 + 6x$

$= 3x(x+2)$

put $f'(x) = 0$

$\rightarrow x = 0$ or $x = -2$.

-ve	+ve	+ve
dec.	inc.	inc.

-2 0

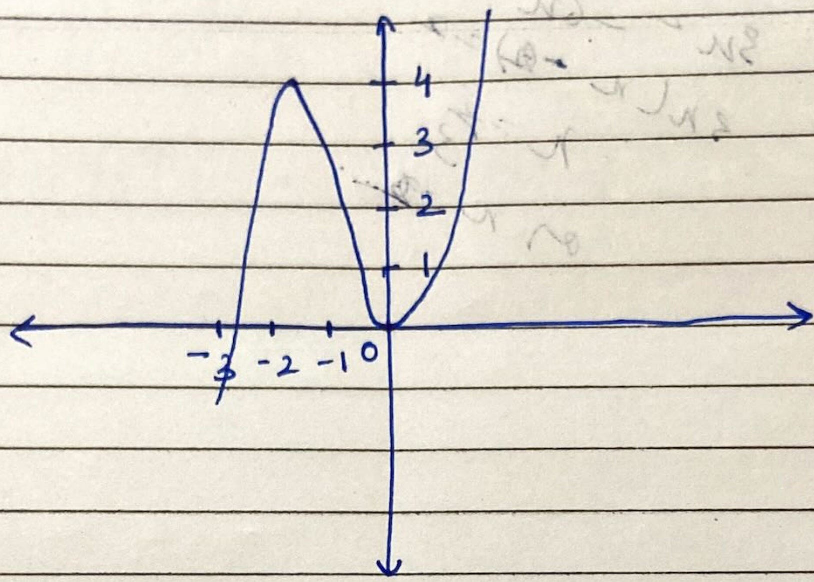
$f(0) = 0 \Rightarrow$ y-intercept is 0

$f''(x) = 6x + 6$; $f''(-2) = -ve < 0$; $f''(0) = 6 \rightarrow +ve > 0$

$x = -2$ is a point of maximum &

$x = 0$ is a point of minimum.

$f(-2) = (-2)^2(-2+3) = 4$; $f(0) = 0$



16. $f(x) = 3x^2 - 6x$

diff. wrt x

$f'(x) = 6x - 6$

put $f'(x) = 0$

$6x - 6 = 0$

$6(x-1) = 0$

$x = 1$

decreasing | increasing

$-\infty$ | $-ve$ | 1 | $+ve$ | ∞

$f''(x) = 6$

$f''(1) = 6 > 0$

$\rightarrow f(x)$ is minimum at $x = 1$.

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$f(1) = 3(1) - 6(1) = -3$ (min. value)

$f(0) = 0$