Numerical Methods of Algebra

Assignment - 1

1. Actual area $\Rightarrow \pi r^2$

$\Rightarrow \pi (12.5)^2$

$\Rightarrow$ **490.8736 cm²**

In 2nd case : approx area $\Rightarrow \pi r^2$ (where $r = 12.65$)

$\Rightarrow \pi (12.65)^2$

$\Rightarrow$ **502.7255 cm²**

* Absolute error $\Rightarrow$ Approx. area − actual area $\Rightarrow$ **11.8489**
* Relative error $\Rightarrow$ $\frac{\text{Absolute area}}{\text{Actual area}}$ $\Rightarrow$ **0.02413**
* Percentage error $\Rightarrow$ Relative error $\times 100$ $\Rightarrow$ **2.413%**

2.

(i) $x_1 = 4$ $\quad y_1 = 0.60206$

$x_2 = 6$ $\quad y_2 = 0.7781513$

Using linear interpolation $\longrightarrow x_2 - x_1 = y_2 - y_1$

$2 = 0.7781513 - 0.60206$

$= \frac{0.1760913}{2} \Rightarrow$ **0.8804565**

(ii) $x_1 = 4.5$ $\quad y_1 = 0.6532125$

$x_2 = 5.5$ $\quad y_2 = 0.7403627$

$1 \Rightarrow 0.7403627 - 0.6532125$

$\Rightarrow$ **0.871502**

$y \Rightarrow 0.6532125 + 0.0871502 \Rightarrow$ **0.6967876**

3. $x^4 - x - 10 = 0$

$a = 1.5$; $b = 2$

x	y
$a = x_0 = 1.5$	-6.4375
$b = x_1 = 2$	4

(i) $x_2 \Rightarrow \frac{x_0 + x_1}{2} \Rightarrow$ **1.75** ; $y \Rightarrow$ **−2.37109**

(ii) $x_3 \Rightarrow \frac{x_1 + x_2}{2} \Rightarrow$ **1.875** ; $y \Rightarrow$ **0.48462**

(iii) $x_4 \Rightarrow \frac{x_2 + x_3}{2} \Rightarrow$ **1.8125** ; $y_4 \Rightarrow$ **−1.0202**

$x_5 = \frac{x_4 + x_3}{2} \Rightarrow \boxed{1.84375}$; $y_5 = \boxed{-0.28773}$

$x_6 \Rightarrow \frac{x_5 + x_3}{2} \Rightarrow \boxed{-0.28773}$; $y_6 \Rightarrow \boxed{0.0933}$

4. $x^3 - x - 1 = 0$

$f'(x) = 3x^2 - 1$

x	0	1	2
y	-1	-1	5

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2$

$\rightarrow x_1 = x_0 - f(x_0)$

$\Rightarrow 1 - (-1/2) \Rightarrow \boxed{1.5}$; $y_1 \Rightarrow \boxed{0.875}$

$\rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$\Rightarrow 1.5 - \left(\frac{0.875}{0.75}\right) \Rightarrow \boxed{1.34783}$

$f(x_2) = \boxed{0.100699}$; $f'(x_2) = \boxed{4.44998}$

$\rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$\Rightarrow \boxed{1.3252}$; $y_3 \Rightarrow \boxed{0.00205}$

$f'(x_3) = \boxed{4.26846}$

$\rightarrow x_4 = x_3 - \left(\frac{f(x_3)}{f'(x_3)}\right) \Rightarrow \boxed{1.32472}$; $y_4 = \boxed{0.0000087}$

5. $A^2 = A$

$7A - (I + A)^3 = 7A - \left[I^3 + A^3 + 3A^2I + 3I^2A\right]$

$\Rightarrow 7A - \left[I^3 + A^2(A) + 3A + 3A\right]$

$\Rightarrow 7A - (I + A^2 + 6A)$

$\Rightarrow 7A - I + 7A$ $\quad [\because A^2 = A]$

$\Rightarrow \boxed{-I}$

6. $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & 1 \end{bmatrix}$; $|A| \Rightarrow 1(0-6)+1(4-0)+2(4-0)$

$\Rightarrow -6+1(4)+2(4) \Rightarrow \boxed{6}$

$A_{11} \Rightarrow (-1)^2(0-6) \Rightarrow -6$

$A_{12} \Rightarrow (-1)^3(4-0) \Rightarrow -4$

$A_{13} \Rightarrow (-1)^4(4-0) \Rightarrow 4$

$A_{21} \Rightarrow (-1)^3(-1-2) \Rightarrow 3$

$A_{22} \Rightarrow (-1)^4(1-0) \Rightarrow \boxed{1}$

$A_{23} \Rightarrow (-1)^5(1-0) \Rightarrow \boxed{-1}$

$A_{31} \Rightarrow (-1)^4(-6-0) \Rightarrow \boxed{-6}$

$A_{32} \Rightarrow (-1)^5(6-8) \Rightarrow -(-2) \Rightarrow \boxed{2}$

$A_{33} \Rightarrow (-1)^6(-0+4) \Rightarrow \boxed{4}$

$Adj[A] \Rightarrow \begin{bmatrix} -6 & -4 & 4 \\ 3 & 1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$

$A^{-1} \Rightarrow \frac{1}{|A|} \times adj(A) \Rightarrow \frac{1}{6}\begin{bmatrix} -6 & -4 & 4 \\ 3 & 1 & -1 \\ -6 & 2 & 4 \end{bmatrix} \Rightarrow \boxed{\begin{bmatrix} -1 & -\frac{2}{3} & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{6} & -\frac{1}{6} \\ -1 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}}$

7. $\vec{a} = 8\hat{i} - 9\hat{j}$

$\vec{b} = 7\hat{i} - 9\hat{j}$

$\vec{a}.\vec{b} = ab\cos(\theta) \Rightarrow (8\hat{i}-9\hat{j})(7\hat{i}-9\hat{j})$

$\Rightarrow 56+81 \Rightarrow \boxed{137}$

$a = \sqrt{64+72} = \boxed{\sqrt{145}}$

$b = \sqrt{49+81} = \boxed{\sqrt{130}}$

$\vec{a}\,\vec{b} = ab\cos\theta$

$\cos\theta \Rightarrow \frac{\vec{a}\,\vec{b}}{ab} \Rightarrow \frac{137}{\sqrt{130}\times\sqrt{145}} \Rightarrow \boxed{0.972022420}$

3. $\vec{R} = \vec{a} + \vec{b}$

$R^2 = A^2 + B^2 + 2AB\cos\theta \quad (\text{when } \theta = 30^\circ)$

$R^2 = (75)^2 + (25)^2 + 2(75)(25)\cos 30^\circ$

$R^2 \Rightarrow \boxed{9497.595284}$

9. $d = 3$; $A.P. = 101, 104, 107, \ldots 998$

$a = 101$

$t_n \Rightarrow a+(n-1)d = 101 + (n-1)3 = 998$

$n = \frac{897+3}{3} \Rightarrow \boxed{300}$

$S_n \Rightarrow \frac{n}{2}(a+l)$

$\Rightarrow \frac{300}{2}(1099) \Rightarrow 150(1099) \Rightarrow \boxed{164850}$

10. $t_6 = 32$

$ar^5 = 32$

$ar^7 = 128$

$\Rightarrow \frac{ar^7}{ar^5} = \frac{128}{32} \Rightarrow r^2 = 4$

$r \Rightarrow \boxed{\pm 2}$

11. $(3x+4)^5$

$\Rightarrow (3x)^5 + 5(3x)^4 4 + 10(3x^3) \times 4^2 + 10(3x^2) \times 4^3 + 5(3x)(4)^4 + 4^5$

$\Rightarrow 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$,

12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A - \lambda I \Rightarrow \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$|A - \lambda I| \Rightarrow 2-\lambda((-5-\lambda)(3-\lambda)) + 3[6-2\lambda] + 0$

$\Rightarrow$

13. $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(5) \Rightarrow 125 - 175 + 40 - 3 \Rightarrow \boxed{-13}$

$f'(5) \Rightarrow 75 - 70 + 8 \Rightarrow \boxed{13}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow 5 - \frac{(-13)}{13} \Rightarrow \boxed{6}$

$f(x_1) = 9$

$f'(x_1) = 32$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 6 - \frac{9}{32} \Rightarrow \boxed{5.71875}$

$f(x_2) = \boxed{0.84787}$

17 $(x^2-x+1)^4 = ((x-1)^2+x)^4$

14. $(x^2-x+1)^4 = ((x-1)^2+x)^4$

Using Pascal's triangle,

$[(x-1)^2+x]^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$

$(x^2-x+1)^4 = (x-1)^8 + 4x(x-1)^6 + 6x^2(x-1)^4 + 4x^3(x-1)^2 + x^4$.

15. $e^{-x} = 3\log(x)$

$3\log(x) + e^{-x} = 0$

x	y
0.1	-2.0951625
0.2	-1.278179
0.3	-0.8278180
0.4	-0.5235
0.5	-0.29656
0.6	-0.11673
0.7	-0.03188
~~0.8~~	

Where $x_0 = 0.6$ $y_0 = 0.11673$

$x_1 = 0.7$ $y_1 = 0.03188$

$x_2 \Rightarrow \frac{x_0+x_1}{2} = \boxed{0.65}$

$y_2 \Rightarrow \boxed{-0.03921}$

$x_3 \Rightarrow \frac{x_1+x_2}{2} \Rightarrow 0.675$

$y_3 \Rightarrow \boxed{-0.00293}$

$x_4 \Rightarrow \frac{x_3+x_1}{2} \Rightarrow \boxed{0.6875}$

$y_4 \Rightarrow \boxed{0.01465}$

$x_5 \Rightarrow \frac{x_4+x_3}{2} \Rightarrow \boxed{0.68125}$

$y_5 \Rightarrow \boxed{0.0059.}$