NUMERICAL METHODS AND ALGEBRA

H.W-1 - LAGRANGE'S INTERPOLATION

1.

x	7	8	9	10	9.5
$f(x)$	3	1	1	9	?

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)}(3) + \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)}(1) + \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)}(1) + \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)}(9)$$

$$= \frac{(1.5)(0.5)(-0.5)}{(-1)(-2)(-3)}(3) + \frac{(2.5)(0.5)(-0.5)}{(1)(-1)(-2)}(1) + \frac{(2.5)(1.5)(-0.5)}{(2)(1)(-1)}(1) + \frac{(2.5)(1.5)(0.5)}{(3)(2)(1)}(9)$$

$$= 3.625$$

x	3	7	9	10	6
f(x)	163	120	72	63	?

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)}(163) + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)}(120) + \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)}(72) + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)}(63)$$

$$= \frac{-1(-3)(-4)}{(-4)(-6)(-7)}(163) + \frac{(3)(-3)(-4)}{(-4)(-2)(-3)}(120) + \frac{(3)(-1)(-4)}{(6)(2)(-1)}(72) + \frac{(3)(-1)(-3)}{(7)(3)(1)}(63)$$

$$= 12 + 130 - 72 + 27$$
$$= 147$$

3.

x	1	2	7	8
$f(x)$	4	5	5	4

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)}(4) + \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)}(5) + \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)}(5) + \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)}(4)$$

$$= \frac{(4)(-1)(-2)}{(-1)(-6)(-7)}(4) + \frac{(5)(-1)(-2)}{(1)(-5)(-6)}(5) + \frac{(5)(4)(-2)}{(6)(5)(-1)}(5) + \frac{(5)(4)(-1)}{(7)(6)(1)}(4)$$

$$= -0.761904476 + 1.6866 + 6.666674 - 1.9047$$

$$= 5.66$$

4.

x	1	2	3	4	7
$f(x)$	2	4	8	16	128

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)}(2) + \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)}(4) + \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)}(8) + \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)}(16) + \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)}(128)$$

$$= \frac{(3)(2)(1)(-2)}{(-1)(-2)(-3)(-6)}(2) + \frac{(4)(2)(1)(-2)}{(1)(-1)(-2)(-5)}(4) + \frac{(4)(3)(1)(-2)}{(2)(1)(-1)(-4)}(8) + \frac{(4)(3)(2)(-2)}{(3)(2)(1)(-3)}(16) + \frac{(4)(3)(2)(1)}{(6)(5)(4)(3)}(128)$$

$$= 32.93333$$

5.

x	10	20	14	16
$f(x)$	0.25	0.2	0.15	0.10

$$f(15) = \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)}(0.25) + \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)}(0.2) + \frac{(15-10)(15-20)(15-16)}{(14-16)(14-20)(14-16)}(0.15) + \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)}(0.10)$$

$$= \frac{(-5)(1)(-1)}{(-10)(-4)(-6)}(0.25) + \frac{(5)(1)(-1)}{(10)(6)(4)}(0.2) + \frac{(5)(-5)(-1)}{(4)(-6)(-2)}(0.15) + \frac{(5)(-5)(1)}{(6)(-4)(2)}(0.10)$$

$$= 0.120333$$

H.W. 2 - REGULAR FALSI METHOD

1. $x^3 + x + 0 = 0$

$f(x) = x^3 + x + 0$

	x	y
	0.5	-0.375
x_0	0.6	-0.1894
x_1	0.7	0.043

$$x_2 = \frac{0.6(0.043) - 0.7(-0.184)}{0.043 - (-0.184)}$$

$$= 0.681057$$

$$y_2 = -0.0030418d$$

$$x_3 = \frac{(0.681057)(0.043) - 0.7(-0.00304180)}{0.043 + 0.00304180}$$

$$= 0.682304$$

$$y_2 = -0.0000 55C36$$

$\therefore$ root is $x = 0.682304$

2. $x^3 - 2x + 0.5 = 0$ $\quad 0 < x < 1$

$f(x) = x^3 - 2x + 0.3$

	x	y	
	0.1	0.801	
$x_0 \leftarrow$	0.2	0.108	$\rightarrow y_0$
$x_1 \leftarrow$	0.3	-0.073	$\rightarrow y_1$

$$x_2 = \frac{0.2(-0.073) - 0.3(0.108)}{(-0.073) - 0.108}$$

$$= 0.259668$$

$$y_2 = -0.00182815$$

$$x_3 = \frac{0.2(-0.00182815) - (0.259668)(0.108)}{-0.00182815 - 0.108}$$

$$= 0.2586748$$

$$y_3 = -0.0000411915$$

root is $x = 0.2586748$

3. $x^3 + 2x - 20 = 0$

$f(x) = x^3 + 2x - 20$

x	y	
2.3	-3.233	
$x_0 \leftarrow$ 2.4	-1.376	$\rightarrow y_0$
$x_1 \leftarrow$ 2.5	0.625	$\rightarrow y_1$

$$x_2 = \frac{2.4(0.625) - 2.5(-1.376)}{0.625 - (-1.376)}$$

$$= 2.46876$$

$$y_2 = -0.015827$$

$$x_3 = \frac{(2.46876)(0.625) - 2.5(-0.015827)}{0.625 - (-0.015827)}$$

$$= 2.469537$$

$$y_3 = -0.0001747$$

root is $x = 2.469537$

4. $3x - 0.05x - 1 = 0$

$f(x) = 3x - 0.05x - 1$

x	f
0.5	-0.4999619
0.6	-0.1999576
0.7	0.1000746

$$x_2 = \frac{0.6(0.1000746) - 0.7(-0.1999576)}{0.1000746 + 0.1999576}$$

$$= 0.6666439$$

$y_3 = -0.0000003354$

root is $x = 0.6666439$

5. $x^3 - 3x - 5 = 0$

$f(x) = x^3 - 3x - 5$

x	y
2.1	-2.039
2.2	-0.952
2.3	0.267

$$x_2 = \frac{2.2(0.267) - (2.3)(-0.952)}{0.267 - (-0.952)}$$

$$= 2.2780968$$

$y_2 = -0.0115944$

$$x_3 = \frac{(2.278096)(0.267) - (2.3)(-0.011594)}{0.267 - (-0.011594)}$$

$$= 2.2790111$$

$$y_3 = -0.0000966$$

∴ root is $x = 2.2790111$

6. $x \log_{10} x - 1.2 = 0$

$f(x) = x \log_{10} x - 1.2$

	x	y
	2.6	-0.1210692
x_0	2.7	-0.0353178
x_1	2.8	0.0520495

$$x_2 = \frac{2.7(0.0520495) - 2.8(-0.0353178)}{0.0520495 - (-0.0353178)}$$

$$= 2.792159$$

$$y_2 = 0.0457314$$

$$x_3 = \frac{2.7(0.0457314) - 2.792159(-0.0353178)}{0.0457314 - (-0.0353178)}$$

$$= 2.7404563$$

$$y_3 = -0.0006559$$

∴ root is $x = 2.7404563$

7. $x^3 - 4x + 1 = 0$

$f(x) = x^3 - 4x + 1$

	x	y
	0	1
	1	-4
	0.5	-0.875
x_0 –	0.4	-0.536
x_1 –	0.2	0.208

$$x_2 = \frac{0.4(0.208) - (0.2)(-0.536)}{0.208 - (-0.536)}$$

$$= 0.255914$$

$$y_2 = -0.0068950$$

$$x_3 = \frac{(0.255914)(0.208) - (0.2)(-0.006895)}{0.208 - (-0.006895)}$$

$$= 0.254119$$

$$y_3 = -0.000089028$$

$\therefore$ root is $x = 0.254119$

8. $x^3 + 2x^2 - 8 = 0$

$f(x) = x^3 + 2x^2 - 8$

	x	y
	1.4	-1.336
x_0 –	1.5	-0.125
x_1 –	1.6	1.216

$$x_2 = \frac{1.5(1.216) - (1.6)(-0.125)}{1.126 - (-0.125)}$$

$$= 1.5093214$$

$$y_2 = -0.0055865$$

$$x_3 = \frac{(1.5093214)(1.216) - (1.6)(-0.0055865)}{1.216 - (-0.0055865)}$$

$$= 1.5097360\text{8}$$

$$y_3 = -0.0002477$$

$\therefore$ root is $x = 1.5097360$ &

9. $x^3 - 9x + 1 = 0$

$f(x) = x^3 - 9x + 1$

	x	y
	0	1
x_0	0.1	0.101
x_1	0.2	0.792

$$x_2 = \frac{0.1(-0.792) - (0.2)(0.101)}{-0.792 - 0.101}$$

$$= 0.11131019$$

$$y_2 = -0.0004125$$

$\therefore$ root is $x = 0.11131019$

10. $x^3 - x - 4 = 0$

$f(x) = x^3 - x - 4$

	x	y
	1.6	-1.504
$x_0 \rightarrow$	1.7	-0.787
$x_1 \leftarrow$	1.8	0.032

$$x_2 = \frac{1.7(0.032) - (1.8)(-0.787)}{0.032 - (-0.0787)}$$

$$= 1.7960927$$

$$y_2 = -0.001922439$$

$$x_3 = \frac{1.7960927(0.032) - (1.8)(-0.001922439)}{0.032 - (-0.001922439)}$$

$$= 1.7963213$$

$$y_3 = -0.00000453$$

∴ the root is $x = 1.7963213$