1) Let x_i be claim size on home insurance
y_i " " " " car insurance

So, AIQ

$$X \sim N(800, 100^2)$$

$$Y \sim N(1200, 300^2)$$

$$P(y_1 + y_2 + y_3 > x_1 + x_2 + x_3 + x_4 + 800)$$

$$P(y_1 + y_2 + y_3 - (x_1 + x_2 + x_3 + x_4) > 800)$$

$$(y_1 + y_2 + y_3) - (x_1 + x_2 + x_3 + x_4) \sim N(3 \times 1200 - 4 \times 800, 300^2(3) + 100^2(4))$$

$$\sim N(400, 310000)$$

$$P\left(z > \frac{800 - 400}{\sqrt{310000}}\right)$$

$$P(z > 0.71842) \longrightarrow 1 - P(z < 0.71842)$$

$$1 - P(z < 0.72)$$

$$1 - 0.76424$$

$$\longrightarrow \boxed{0.23576}$$

(2) $N \sim$ Neg. Binomial (k, p)

$$P(N=n) = {}^{n+2}C_n \, (0.9)^3 (0.1)^n$$

$$= {}^{(n+3)-1}C_{n-1} \, (0.9)^3 (0.1)^n$$

So,

$K = 3$, $P = 0.9$

$M_Y t = N_n (\log M_X t)$

$X \sim$ Gamma $(6, 2)$

$Y \rightarrow$ Total claim amount

(i) <u>MGF of Y</u>

$$M_Y(t) = E\,(E\,k^{yt} / N = n)$$

$$E\,(e^{yt}/N=n) = E\,(e^{x_1 t + x_2 t + \dots x_n t})$$

$$= E\,(e^{t x_1}) \cdot E\,(e^{tn}) \dots E\,(e^{t x n})$$

$$\Rightarrow \left(1 - \frac{t}{2}\right)^{-6n}$$

So, $E\,(E\,(e^{yt}/N=n)) = E\left(\left(1-\frac{t}{2}\right)^{-6n}\right)$

$$= E\left(e^{N \log\left(1-\frac{t}{2}\right)^{-6}}\right)$$

$$= M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

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$$M_N(t) = \left(\frac{Pe^t}{1-qe^t}\right)^k \Longrightarrow \left(\frac{Pe^{\log(1-\frac{t}{2})^{-6}}}{1-qe^{\log(1-\frac{t}{2})^{-6}}}\right)^3$$

$$V(y) = E(N)V(x) + (E(x))^2 \; V(N)$$

$$\Longrightarrow \left[\frac{3}{0.9} \times \frac{6}{4}\right] + \left(\frac{36}{2}\right)^2 \times \frac{(3)(10.1)}{(0.9)^2}$$

st. deviation $\longrightarrow$ 2.887

(5) $f(x,y) = ax(x+y)$

(i) $P\left(y > \frac{1}{3}x + 10\right)$

$$a\int_{10}^{20}\int_{0}^{5}(x^2+xy)\,dx\,dy = 1$$

$$\int_{10}^{20}\left[\frac{x^3}{3} + \frac{x^2}{2}y\right]_0^5 dy = \frac{1}{a}$$

$$\int_{10}^{20}\left(\frac{125}{3} + \frac{25}{2}y\right)dy = \frac{1}{a} \Rightarrow \left(\frac{125}{3}y + \frac{25y^2}{4}\right)_{10}^{20} = \frac{1}{a}$$

$$a = \left(\frac{2500}{3} + 2500 - \frac{1250}{3} - 625\right) = 1$$

$$a\left(\frac{1250}{3} + 1875\right) = 1$$

$$a\left(\frac{6815}{3}\right) = 1$$

$$\boxed{a = \frac{3}{6875}}$$

$$833.33 + 2500 - (416.667 + 625) = \frac{1}{a}$$

$$\boxed{a = 0.00043636}$$

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(iii) $\cancel{P\left(y > \frac{x}{10} + 10\right)}$ $E(y/x)$

$$f_{y/x} = \frac{f(x,y)}{f(x)}$$

$$f(x) = \frac{3}{6875}\int_{10}^{20}(x^2+xy)\,dy \Rightarrow \frac{3}{6875}\left[x^2y + \frac{xy^2}{2}\right]_{10}^{20}$$

$$\Rightarrow \frac{3}{6875}\left[20x^2 + x(200) - 10x^2 - 50x\right]$$

$$= \frac{3}{6875}\left[10x^2 + 150x\right] \Rightarrow \frac{30x}{6875}\left[x+15\right]$$

$$f_{y/x} = \frac{\cancel{3}}{\cancel{6875}}\frac{x(x+y)}{\cancel{3}0x} \times \frac{6875}{(x+15)} \Big/ \frac{\cancel{3}}{\cancel{6875}}\frac{(\cancel{x}(x+y))}{10\cancel{x}}\frac{\cancel{(6875)}}{(x+15)}$$

So,

$$E(Y/X) = \int_{10}^{20} y\cdot\frac{(x+y)}{10(x+15)}\,dy \Rightarrow \frac{1}{10(x+15)}\int_{10}^{20}(xy+y^2)\,dy$$

$$\Rightarrow \frac{1}{10(x+15)}\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{10}^{20} \Rightarrow \frac{1}{10(x+15)}\left[200x + \frac{800}{3} - 50x - \frac{1000}{3}\right]$$

$$\frac{1}{10(x+15)}\left[150x + \frac{7000}{3}\right]$$

$$\boxed{\frac{1}{x+15}\left[15x + \frac{700}{3}\right]}$$

(ii) $P\left(y > \frac{1}{3}x + 10\right)$

$$f(y) = \frac{3}{6875}\int_0^5 x^2 + xy \, dx \Rightarrow \frac{3}{6875}\left[\frac{x^3}{3} + \frac{x^2 y}{2}\right]_0^5$$

$$f(y) \Rightarrow \frac{3}{6875}\left[\frac{125}{3} + \frac{25}{2}y\right]$$

$$\text{So, } \frac{3}{6875}\int_{\frac{1}{3}x+10}^{20}\left(\frac{125}{3} + \frac{25}{2}y\right) dy \Rightarrow \frac{3}{6875}\left[\frac{125}{3}y + \frac{25}{4}y^2\right]_{\frac{1}{3}x+10}^{20}$$

$$\Rightarrow \frac{3}{6875}\left[\frac{125}{3}(20) + \frac{25}{\cancel{4}}(\overset{100}{\cancel{400}}) - \frac{125}{3}\left(\frac{1}{3}x + 10\right) - \frac{25}{4}\left(\frac{1}{3}x+10\right)^2\right]$$

$$\frac{3}{6875}\left[\frac{2500}{3} + 2500 - \frac{125x}{9} - \frac{1250}{3} - \frac{25}{4}\left(\frac{1}{9}x^2 + 100 + \frac{2x}{3}\right)\right]$$

$$\frac{3}{6875}\left[\frac{2500}{3} + 2500 - \frac{125}{9}x - \frac{1250}{3} - \frac{25x^2}{36} - 625 - \frac{25x}{1}\right]$$

$$= 3\left(1250 + 18\right]$$

(3) $f(x) = \lambda e^{-\lambda(x-\alpha)}$

$$MGF = E[e^{tx}] \Rightarrow \int_\alpha^\infty e^{tx}\lambda e^{-\lambda(x-\alpha)} dx$$

$$M_x t = \lambda\int_\alpha^\infty e^{tx} e^{-\lambda x} e^{\lambda\alpha} dx \Rightarrow e^{\lambda\alpha}\lambda\int_\alpha^\infty e^{-(\lambda-t)x} dx$$

$$\frac{e^{\alpha d}\, d}{t-d}\left[e^{-(d-t)x}\right]_{\alpha}^{\infty} \Rightarrow \frac{e^{d\alpha}\, d}{t-d}\left[-e^{-(d-t)\alpha}\right]$$

$$\frac{e^{d\alpha}}{d-t}\, e^{-d\alpha}\cdot e^{t\alpha} \Rightarrow e^{t\alpha}\,\frac{d}{d-t}$$

(ii) $M_x(t) = \dfrac{d e^{t\alpha}}{d-t} \Rightarrow d e^{t\alpha}(d-t)^{-1}$

$M_x^2(t) = d\left[e^{t\alpha}(d-t)^{-2}(+1) + (d-t)^{-1} e^{t\alpha}(\alpha)\right]$

$= d\left[(d-t)^{-1} e^{t\alpha}(\alpha) + e^{t\alpha}(d-t)^{-2}\right]$

$= d e^{t\alpha}(d-t)^{-2}\left[\alpha(d-t)+1\right]$

$= d(d)^{-2}\left[\alpha(d)+1\right]$

$= \dfrac{1}{d}\left[d\alpha+1\right] \Rightarrow \dfrac{d\alpha+1}{d}$

$E(x^2) = M_\alpha''(t) \Rightarrow d\left[\alpha e^{t\alpha}(d-t)^{-2} + \alpha^2(d-t)^{-1}e^{t\alpha} + e^{t\alpha}(+2)(d-t)^{-3} + \alpha(d-t)^{-2}e^{x}\right]$

Put $t=0$

$d\left[\alpha(d)^{-2} + \alpha^2(d)^{-1} + (2)(d)^{-3} + \alpha(d)^{-2}\right]$

$d\left[\dfrac{2\alpha}{d^2} + \dfrac{\alpha^2}{d} + \dfrac{2}{d^3}\right]$

$E(x^2) = \dfrac{2\alpha}{d} + \alpha^2 + \dfrac{2}{d^2}$

$$V(X) = E(x^2) - [E(x)^2]^2 \Rightarrow \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\alpha + \frac{1}{\lambda}\right]^2$$

$$\frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\alpha^2 + \frac{1}{\lambda^2} + \frac{2\alpha}{\lambda}\right]$$

$$\cancel{\frac{2\alpha}{\lambda}} + \cancel{\alpha^2} + \frac{2}{\lambda^2} - \cancel{\alpha^2} - \frac{1}{\lambda^2} - \cancel{\frac{2\alpha}{\lambda}} \Rightarrow \boxed{\frac{1}{\lambda^2}}$$

(8) $X \sim \text{Gamma}(3, 2)$ $\quad E(x) = \frac{3}{2} \quad V(x) = \frac{3}{4}$

$E(y/X=x) = 3x+1 \qquad Var(y/X=x) = 2x^2+5$

$$E(Y) = E(E(Y/X=x)) \Rightarrow E(3x+1)$$

$$= 3E(X) + 1$$

$$\Rightarrow 3\left(\frac{3}{2}\right) + 1 \Rightarrow \frac{9}{2} + \frac{1}{1} = \frac{9+2}{2}$$

$$V(Y) = E(V(x/y)) + V(E(x/y))$$

$$= E(2x^2+5) + V(3x+1)$$

$$= 2E(x^2) + 5 + 9V(x)$$

$$V(X) = E(x^2) - (E(x))^2$$

$$\frac{3}{4} + \frac{9}{4} = E(x^2)$$

$$12/4 = E(x^2)$$

$$\Rightarrow \boxed{E(x^2) = 3}$$

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$$v(4) = 2(3) + 5 + 9\left(\frac{3}{4}\right)$$

$$= 11 + \frac{27}{4} \quad = \frac{44 + 27}{4}$$

$$= \boxed{7\,{}^{1}/_{4}}$$

(4) $$G_V(t) = \left(\frac{p}{1-qt}\right)^k$$

$$G_V t = \sum t^v \times P(V = v)$$

$$P(V=4) \Rightarrow \frac{\Gamma k+4}{\Gamma 5\ \Gamma k}\ p^k q^k$$

$$\Rightarrow \frac{\Gamma k+4}{4!\ \Gamma k}\ p^k q^k$$

$$\Rightarrow \frac{(k+3)!}{(k-1)!\ 4!}\ p^k (1-p)^k$$

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(7)

		X			
		1	2	3	4
y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

(i) $V(x/y=2) = E(x^2/y=2) - [E(x/y=2)]^2$

Now: $E(x^2/y=2] \Rightarrow \sum_x x^2 \frac{P(x=x, y=2)}{P(y=2)}$

when $x=1$. when $x=2$ when $x=3$
when $x=4$

$$\frac{(1)^2 P(x=1, y=2)}{P(y=2)} + \frac{(2)^2 P(x=2 y=2)}{P(y=2)} + \frac{(3)^2 \cdot P(x=3, y=2)}{P(y=2)}$$

$$+ \frac{(4)^2 P(x=4, y=2)}{P(y=2)}$$

$$\Rightarrow \frac{(1)(0)}{0.6} + \frac{(4)(0.3)}{0.6} + \frac{(9)(0.1)}{0.6} + \frac{16(0.2)}{0.6}$$

$\Rightarrow 8.8333$

Now: $E(x/y=2) \Rightarrow \sum_x x \frac{P(x=x, y=2)}{P(y=2)}$

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$$\frac{(1)(0)}{0.6} + \frac{(2)(0.3)}{0.6} + \frac{(3)(0.1)}{0.6} + \frac{4(0.2)}{0.6}$$

$$\Rightarrow \frac{0.6+0.3+0.8}{0.6}$$

$$\Rightarrow 2.8333$$

$$\Rightarrow \boxed{0.807}$$

ii) $f(u,v) = \frac{48}{67}(2uv - u^2)$

$0 < u < 1 ; \frac{u}{2} < v < 2$

$E(v \mid V = v) = ?$

$$\frac{f(u,v)}{f(v)} = f\ v / V = v$$

$$f(v) \Rightarrow \frac{48}{67} \int_0^1 (2uv - u^2)\, du \Rightarrow \frac{48}{67}\left[2u^2 v - \frac{u^3}{3}\right]_0^1$$

$$\frac{48}{67}\left[v - \frac{1}{3}\right] \Rightarrow \frac{16}{67}[3v-1]$$

$$f(v) = \frac{16}{67}[3v-1]$$

$$\frac{f(u,v)}{f(v)} = \frac{48}{67} \frac{(2uv^2 - u^2) \times 67}{\times 16(3v-1)}$$

$$\Rightarrow \frac{3}{3v-1}(2uv^2 - u^2) \qquad 2u^2v^2 - u^3$$

Now: $E(U/V=v) = \frac{3}{3v-1}\int_0^1 u(2uv^2-u^2)\,du$

$\frac{3}{3v-1}\left[\frac{2u^3v^2}{3} - \frac{u^4}{4}\right]_0^1 \Rightarrow \frac{3}{3v-1}\left[\frac{2v^2}{3} - \frac{1}{4}\right]$

$\frac{\cancel{3}}{3v-1}\left[\frac{8v^2-3}{\cancel{12}\,4}\right] \Rightarrow \boxed{\frac{8v^2-3}{4(3v-1)}}$

⑥ $X \sim Poi(\lambda)$ $\quad Y \sim Poi(\mu)$

$M_X(t) = e^{\lambda(e^t-1)}$ $\quad M_Y(t) = e^{\mu(e^t-1)}$

So, $Z = X - Y$

$M_Z(t) = E(e^{tZ}) = E(e^{t(X-Y)}) = E(e^{tX}e^{-tY})$

$= E(e^{tX}) * E(e^{-tY}) \Rightarrow \frac{E(e^{tX})}{E(e^{tY})}$

$\Rightarrow e^{(\lambda-\mu)(e^t-1)}$

So, By uniqueness MGF of $Z = e^{(\lambda-\mu)(e^t-1)}$

$\sim$ Pois $(\lambda-\mu)$

Now: let $Z = X + Y$

$E(e^{tZ}) = e^{\lambda(e^t-1)} \times e^{\mu(e^t-1)}$

$\Rightarrow e^{(\lambda+\mu)(e^t-1)}$

So, By uniqueness $Z \sim Poi(\lambda+\mu)$

⑨ $E(X) = 3$, $V(X) = 4$

$E(Y) = 4$, $V(Y) = 1$

correction $= -0.3\sqrt{4}$ $\Rightarrow$ cov (x, y)

$= -0.6$

$Z = X + Y$

(i) $\text{cov}(x, x+y) = \text{cov}(x, x) + \text{cov}(x, y)$

$= V(x) + \text{cov}(x, y)$

$\Rightarrow 4 + (-0.6)$

$= 3.4$

(ii) $\text{Var}(z) = \text{cov}(x+y; x+y)$

$= \text{cov}(x, x) + \text{cov}(x, y) + \text{cov}(y, y)$

$= V(x) + 2\,\text{cov}(x, y) + V(y)$

$= 4 + 2(-0.6) + 1$

$= 5 - 1.2$

$\Rightarrow 3.8$

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(10) $f(x, y) = K x^{-\alpha} e^{-y/B}$

$1 < x < \infty$, $1 < y < \infty$

$\alpha > 1$, $B > 0$

$$K \int_1^\infty \int_1^\infty x^{-\alpha} e^{-y/B} \, dx\, dy = 1$$

$$K \int e^{-y/B} \left[\frac{x^{-(\alpha-1)}}{-\alpha+1} \right] = 1$$

$$\frac{K}{1-\alpha} \int e^{-y/B} [-1] \Rightarrow \frac{K}{\alpha-1} \int e^{-y/B} \, dy = 1$$

$$\frac{K}{\alpha-1} \frac{\left[e^{-y/B} \right]^\infty}{\left(\frac{-1}{B} \right)} \quad -\frac{K}{\alpha-1} (B) (+e^{-1/B}) = 1$$

$$K = \frac{\alpha - 1}{(B)(e^{-1/B})}$$