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# Probability & Statistics - I

## Assignment - 1

(i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25.

$$\text{Mean} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$\Rightarrow \text{Mean} = \frac{134}{10}$$

$$\Rightarrow \text{Mean} = 13.4$$

$$\text{Median} = \frac{11+15}{2} = \frac{26}{2}$$

$$\Rightarrow \text{Median} = 13$$

$$\text{Mode} = 18$$

$(13.4 - 4)^2 = 88.36$
$(13.4 - 7)^2 = 40.96$
$(13.4 - 18)^2 = 21.16$
$(13.4 - 25)^2 = 134.56$
$(13.4 - 9)^2 = 19.36$
$(13.4 - 11)^2 = 5.76$
$(13.4 - 15)^2 = 2.56$

$$\text{S.D.} = \sqrt{\frac{88.36 + (19.36 \times 2) + 40.96 + 5.76 + (21.16 \times 3) + 2.56 + 134.56}{9}}$$

$$\text{Standard deviation} = 6.4498$$

$25^{th}$ percentile $= \frac{4+9}{2} = 9$

$75^{th}$ percentile $= \frac{18+18}{2} = 18$

$$\text{Interquartile range} = 18 - 9 = 9$$

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| | | | | | | Total |
|---|---|---|---|---|---|---|
| No. of people ($x_i$) | 5 | 11 | 26 | 15 | 3 | |
| No. of households ($f_i$) | 0 | 1 | 2 | 3 | 4 | 40 |
| $x_i f_i$ | 0 | 11 | 52 | 75 | 12 | 120 |
| $(x_i - \bar{x})^2$ or $x^2$ | 49 | 1 | 196 | 9 | 81 | |

ii)

$$\text{Mean}(\bar{x}) = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{120}{10}$$

$$\text{Mean} = 12$$

$$\text{Median} = \frac{10}{2} = 5^{th} \text{ observation}$$

$$\text{Mode} = 3$$

$$3\,\text{Median} = \text{Mode} + 2\,\text{Mean}$$

$$\Rightarrow 3\,\text{Median} = 3 + 2(12)$$

$$\Rightarrow \text{Median} = \frac{27}{3}$$

$$\Rightarrow \text{Median} = 9$$

$$\text{Std. dev.} = \sqrt{\frac{\Sigma x_i^2 f_i}{\Sigma f_i}}$$

$$= \sqrt{\frac{1 + (196 \times 2) + (9 \times 3) + (81 \times 4)}{10}}$$

$$= 8.625$$

$25^{th}$ percentile = 3 . $75^{th}$ percentile = 15

Interquartile range = 15 - 3 = 12

A3) i. $P(\text{no claim}) = P(X=0)$, $X \sim \text{Poisson}(0.5)$

$= 0.60653$ [From table]

ii. Let $Y$ = No. of years with a claim.

$P(Y=1) = 3(0.3935)(0.6065)^2$

$= 0.434$

iii. $P(\text{no claim})^2 = (0.60653)^2$

$= 0.368$

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A3) i) $\alpha = 2$ & $\lambda = 1/2$

Since $\alpha = k$ & $\lambda = 1/\theta$

$k = 2$ & $\theta = 2$.

Mean $= k\theta = 2 \times 2$

Mean $= 4$

Variance $= k\theta^2 = 2 \times (2)^2$

Variance $= 8$

S.d. dev. $= \sqrt{8} = 2\sqrt{2}$ △

ii) The above gamma distribution has a skewed shape

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Q) No. of Males = 10 , No. of females = 8.
No. of qualified males = 6. , No. of qualified females = 7.
Total = 18 workers.

Probability for 2 people selected ~~at random~~ $= {}^{18}C_2$
$= 306/2$
$= 153.$

Probability of atleast one female to be selected (from 8 females) $= {}^{8}C_1 + {}^{8}C_2$
$= 36.$

Probability of atleast one female to be selected from total
$= \frac{36}{153}$ ~~or $\frac{36 \times 2}{306}$~~

$= \frac{4}{17}.$

Probability of selecting qualified females from all females
$= {}^{8}C_7 = {}^{8}C_1$
$= 8.$

Required probability $= \frac{8}{4/17}$

$= \frac{8 \times 17}{4} = \frac{136}{4}$

$= 34$

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A5) % of packages delivered via level 1 = 40%.
% of ..... via level 2 = 50%.
∴ % of ..... via level 3 = 100 − (40+50)
= 10%.

Probability that a package was delivered via level 2 = 50%.
= 0.5 = $\frac{1}{2}$.

Probability of a package arriving late using level 2 = 40%.

Required probability = 40% × $\frac{1}{2}$

= $\frac{40^{2}}{100} \times \frac{1}{2}$

= 0.2 ⇒

or

20% ⇒

A7). i). $F(3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.65 + 0.15 + 0.35$

$= 0.55$ Ans

ii). $E(X) = (1 \times 0.05) + (2 \times 0.15) + (3 \times 0.35) + (4 \times 0.4) + (5 \times 0.05)$

$= 3.25$ Ans

iii). $V(X) = (1-3.25)^2 \times 0.05 + (2-3.25)^2 \cdot 0.15 + (3-3.25)^2 \cdot 0.35 + (4-3.25)^2 \cdot 0.4 + (5-3.25)^2 \cdot 0.05$

$= 0.25 + 0.23 + 0.02 + 0.22 + 0.15$

$= 0.87$ Ans.

iv).

Coefficient of skewness $= \left(\frac{1-3.25}{0.87}\right)^3 + \left(\frac{2-3.25}{0.87}\right)^3 + \left(\frac{3-3.25}{0.87}\right)^3 + \left(\frac{4-3.25}{0.87}\right)^3 + \left(\frac{5-3.25}{0.87}\right)^3$

$= (-17.29) + (-2.96) + (-0.02) + 0.64 + 8.13$

$= -11.5$ Ans

The given distribution is negatively skewed.

A13) $a = 75$

$b = 120$.

$$P(80 < x < 100) = (100 - 80)\frac{1}{(120 - 75)}$$

$$= \frac{20}{45}$$

$$= \frac{4}{9} \text{ or } 0.44$$

A14) $X \sim \text{Gamma}(5, 0.4)$.

$\alpha = 5$ $\quad \beta = 0.4$ $\quad x = 22.89$

$$f(x) = \frac{1}{0.4^5 \Gamma(5)} (22.89)^{5-1} e^{-\frac{22.89}{0.4}} \qquad x > 0, \ \alpha \& \beta > 0.$$

$$= \frac{1}{(0.4)^5 (24)} (22.89)^4 e^{-\frac{22.89}{0.4}}$$

$$= 0.$$

$$= 1.5688e^{-19}$$

$$P(X < 22.89) = \int_0^{22.89} 1.5688e^{-19}$$

$$P(X < 22.89) = 1$$