Page No.
Date / /

Financial Maths

Assignment - 2

Q1) i) $APR = \dfrac{\text{average annual interest}}{\text{average loan Amount}}$

$= \dfrac{2 \times \text{avg annual interest}}{\text{original loan}}$

$= 2 \times \text{flat rate}$

$\Rightarrow$ Original loan Amount = Rs 20,000

$\Rightarrow 20000 = 12 \times 427.9\, a_{\overline{5}|}^{(12)}$

$a_{\overline{5}|}^{(12)} = 3.8949$

Flat rate = $\dfrac{\text{total interest}}{\text{loan} \times \text{total no of years}}$ =

$= \left(\dfrac{5 \times 12 \times 427.9 - 20000}{20000 \times 5}\right)$

$= 5.67\%$

$\Rightarrow APR \simeq 2 \times 5.67 \simeq 11\%$

$i = 11\%\quad a_{\overline{5}|}^{(12)} = 3.87872$

$i = 10\quad i,\ a_{\overline{5}|}^{(12)} = 3.96154$

Applying interpolation on APR,

$$I = \frac{P - 10\%}{3.8499 - 3.96154} = \frac{11\% - 10}{3.8787 - 3.9651}$$

$i = 10.8\%$

At 10.8%, $12 \times 427.9\, a_{\overline{5}|}^{(12)} = 20000.2$

At 10.9%, $12 \times 427.9\, a_{\overline{5}|}^{12} = 19958.2$

$i = 10.89$ is more close, Hence

$APR = 10.8\%$

(iii) $APR = 10.8\%$

After 1 year, capital left to pay =

$= 12 \times a_{\overline{4}|}^{(12)}$ @ $10.8\% \times 427.9$

$= 16775.98$

= let t denote the term for new loan,

$= 16775.98 = 12(274.49)\, a_{\overline{t}|}^{(12)}$

$$= 16775.98 = \frac{3293.88(1 - 1.108^{-t})}{12(1.108^{1/12} - 1)}$$

Page No.

Date / /

= $1.108^{-t} = 0.475$

= taking log,

= $-t \log(1.108) = \log(0.475)$

= $-t = \frac{-0.744440474}{0.10255658325}$

$t = 7.2588264796$ years

(iii) The original loan → 4 years of payment is remaining,

∴ Net interest $= 427.9 \times 12 \times 4 - 16775.68$
$= 3763.22$

New loan has a term of 7.2588 years

$= 7.2588 \times 12 \times 274.49 - 16775.68$

$= \cancel{7105.33}\ 7104.66832$

∴ They will have to pay ~~(7104.65 - 3763.22)~~

(7104.668 - 3763.22) 3341.23 more

Page No.______
Date __/__/__

(3) Value of 1st NOV 1975 =

$\text{Annuity } 1 = 200 \times v^{(1/4)} \times \ddot{a}_{\overline{22}|}$ at 8%

$= 200 \times 0.9809443 \times 10.2007 \times \frac{0.08}{0.074074}$

$= \underline{2161.366301}$

$\text{Annuity } 2 = 320\,(1+i)^{1/12}\, a^{(4)}_{\overline{16.25}|}$ @ 8%

$= 320 \times 1.006434 \times 9.1842$

$= \underline{2957.87}$

$\text{Annuity } 3 = 180 \times a^{(12)}_{\overline{18.75}|}$

$= 180 \times \frac{1 - (0.92592)^{18.75}}{0.077208}$

$= \underline{1780.66416}$

Now,
let Revised Annuity → A

its value = A = $\frac{6889.87}{1.0194263 \times 10.30889}$

$= \underline{656.5587}$

Shrikrupa

Page No.____
Date __/__/__

(4) $(I\ddot{a})_n \longrightarrow i > 0$

$(I\ddot{a})_{\overline{n}|} = 1 + 2v + 3v^2 + 4v^3 + 5v^4 + \dots + nv^{n-1}$

$I\ddot{a}_{\overline{n}|} = (1+i)\, Ia_{\overline{n}|}$

$= \dfrac{\dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}}{1+i}$

$$\left[I\ddot{a}_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d} \right] \left(\because \frac{i}{1+i} = d \right)$$

(5) For the yr 2014, Time-weighted Rate of Return for each fund =

property fund: $\dfrac{16.4}{12.4} - 1 = 0.3226$ or $32.26\ \%$

equity fund: $\dfrac{15.5}{12.1} - 1 = 0.2810$ or $28.1\ \%$

d) Assuming, investor bought 100 units per quarter in property fund,

E.O.V =

$1240(1+i) + 1310(1+i)^{3/4} + 1480(1+i)^{2/4} + 1580(1+i)^{1/4} = 4 \times 1640$

= on solving we get

= $i = 29.32\%$

b) Assuming that the investor purchases INR 100 worth of units quarter $400 \, x \, s^{(4)} = \frac{\cancel{100} \times 4.720517843}{\cancel{400}\ 4}$

$s^{(4)} = 1.180123$

(5) i) let $x \rightarrow$ initial Annual Repayment

$\rightarrow$ $160000 = x a_{\overline{10}|}$ @ 8%

= $\left[\frac{160000}{a_{\overline{10}|}} = 23,844.65 \right]$

(ii) Out. loan immediately after 4th payment =

$23844.65 \, a_{\overline{6}|}$ @ 8% $= 110231.44187$

$y \rightarrow$ revised Annual Investment.

EOV = $y a_{\overline{6}|} = 110231.44187$ @ 10%

= 25309.7156

Page No.______

Date ___/___/___

iii) The loan outstanding immediately after 7th payment

$= 25309.72\, a_{\overline{3}|}$ @10%

$= 25309.72 \times 2.4869$

$= 62942.742668$

$y \rightarrow$ revised Annual installment

$y\, a_{\overline{3}|} = 62942.74$ @9%

$y = \frac{62942.74}{a_{\overline{3}|}} = \frac{62942.74}{2.5313}$

$= 24865.77$

(8) i) PV of out-lays

$2.7 + a_{\overline{3}|} @ i$

Exp. PV of income is:-

$= 1.03v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) @ p$

Equating the PV of out lays is to PV of income =>

$2.7 + 0.2a_{\overline{3}|} = 0.1v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) @ i$

(ii) Based on the info present value of company's income is

$= 0.1v^3 + 0.85\bar{a}_{\overline{10}|}v^3$

$= 0.1 \times 0.7513 + 0.85 \times 0.7513 \times 8.4469 - 2.7 - 0.2 \times 2.04868$

$= 1.0089$

= Since NPV is the 1 so it is good to invest in the project.

(7) (i) TWRR

$(1+i)^3 = \frac{33}{30.5} \times \frac{41.05 - 4.5}{39} \times \frac{45.6}{41.05} \times \frac{47}{45.6 - 2.5}$

= On Evaluating,

$i = 0.07095128$

TWRR = 7.095128 %

MWRR

$= 30.5(1+i)^3 + 6(1+i)^{2.2} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} - 47$

= let $1+i = x$

$= 30.5x^3 + 6x^{2.25} + 4.5x^{1.5} - 2.5x^{0.5} - 47 = 0$

$1+i = 1.07204$

$i = 0.7204 = 7.204\%$

(ii) Linked rate of Return

EOV → Yr 2011

$30.5(1+i) + 6(1+i)^{0.25} = 38.5$

let $1+i = x$

= So on solving we get

$x = 1.062$

$i = 0.062$ 6.2 %

Page No.

Date / /

year 2012 → EOV

$$38.5(1+i) + 4.5(1+i)^{1/2} = 45$$

= again on consider $(1+i) \rightarrow x$

$$i = 0.0492 \text{ or } 4.92\%$$

= EOV → year 2013 →

|||rly,

$$45x - 2.5x^{0.5} - 47 = 0$$

$$x = 1.10279$$

$$i = 0.10279 \text{ or } 10.279\%$$

= Linked Rate of Return

$$= (1+i)^3 = (1.0626) \times (1.0492) \times 1.10279$$

$$= i = 0.0712898$$

$$= i = 7.12898\%$$

(iii) TWRR eliminates the effect of the cashflow amount and timing and i.e. gives a fairer basis for assessing the investment performance

Shrikrupa

for the fund. NHWRR is sensitive to the amount of the net cashflows.

10) i) let loan amount be L

$L = 180000\, a_{\overline{15}|} + 20000 (Ia_{\overline{15}|})$ @ 12%.

$L = 180000 \times 6.8109 + 20000 \times \frac{4.887260487}{0.12}$

$\underline{L = 2040588.7479}$

(Total house cost = $\frac{2040588.7479}{0.8}$ =

= $\underline{2550735.93487}$

ii) The loan outstanding at the begining of 9th year

$L = 180000 v + 180000 v^2$

$L(9) = 340000\, a_{\overline{7}|} + 20000 (Ia)_{\overline{7}|}$ @ 12%

$L(9) = 1551692 + 324158.33$

$= 1875850.33$

Loan Schedule

year	loan o/s	loan Installment
9	1875850.33	360000
10	1740952.37	380000
11	1569866.65	

Interest	Capital Repayment
225102.04	134897.96
208914.28	171085.72

(iii) let the new installment be Rs P

$1569866.65 = Pa_{\overline{5}|}$ @ 10%

$P = 414126.87$

Extra payment by Mr. Sam if the loan continued with the first bank I.

Total installment − loan outstanding at the end of 10th year

= 400000 + 420000 + 440000 + 460000 + 480000 − 1569866.65

= 630133.35

= Extra payment made by Mr Sam if the loan transferred to 2nd loan

= Total installment + 1% processing fees − loan o/s at the end of 10th year

= 2070634.337 + 15698.665 − 1569866.65

= 516466.35

Page No.
Date / /

11) i) Time weighted RR is given by

$$(1+i)^2 = \frac{\cancel{500}^{10}}{460} \times \left(\frac{550+50}{540}\right) \times \frac{x}{\cancel{550}_{11}}$$

$= (1+i)^2 = 1.44$

$= \frac{1.44 \times 46 \times 540 \times 11}{600} = x.$

$= \underline{x = 655.776}$

(ii) Money weighted rate of return is given by

$= 460(1+i)^2 + 40(1+i)^{7/4} - 50(1+i) = 655.776$

$= \cancel{460}$

$\underline{1+i = 1.19854}$

$\underline{i = 19.854\%}$

(iii) effective rate of return or year 2015 is given by solving the eqn of value

$= 460(1+i) + 40(1+i)^{3/4} = 600$

$= \underline{i = 20.2438\%}$

Shrikrupa

= Effective rate of Return for the year 2016.

= $1+i = \frac{655.78}{5TD} = 1.92327$

= $i = 0.92327$

$(1+i)^2 = 1.92327 \times 1.202138$
$= 1.436015121$

$1+i = 1.1983338483$

LIRR = 19.833848%.

iv) When the sub-intervals chosen for calculating LIRR coincide with the interval b/w net cash flow being received then the linked internal rate of Return will be identi~~fied~~ cal to ~~the~~ TWRR.

v) ① TWRR is a better Measure of fund Manage is performance
② MWRR is sensitive to the amount and timing of net cashflows, but fund Manager doesnt have any content on these

Page No.

Date / /

(3) TWRR is calculated using "growth factors" reflecting the change in fund value between the time of consecutive cash flow. Thus TWRR is independent of Amount and timing of cash flows

(4) Hence TWRR is better measure of fund managers performance

(12) i) The discounted payback period of an investment project is the first time when the net PV of cashflow from the project is +ve.

(ii) a) In units of 1000,

$= -800(1+i)^{t} - 50(1+i)^{t-1} + 100\,\bar{s}_{\overline{t-2}|}$ @ 7% $= 0$

$=$ ~~400 (1+~~ $t = 17.767$ years

b) Accumulated Profit after 22 years is found by accumulating the income of after DPP has elapsed at 6% pa

$= 100\,\bar{s}_{\overline{4.33}|}$ @ 6% $= 100 \times \dfrac{(1+0.06)^{4.33} - 1}{\ln(1.06)}$

$=$ Acc Amt = Rs 480074.3066.

iii) Businessman may accumulate his rental income for each year @ 6%.

$$100\bar{s}_{\overline{17}|} = 100 \times \frac{(1.07)^{1} - 1}{\ln 1.07}$$

$$= \underline{102.9708672}$$

The DPP is smallest integer

$$-800(1.07)^{t} - 50(1.07)^{t-1} + 102.9708 \times \left(\frac{1.07^{t-2} - 1}{0.07}\right) > 0$$

$$\{t = 18 \text{ years}\}$$

Balance in his Account after ~~transtartionat~~ 18 years

$= -2703.945824 - 157.9407605 + 2871.665545$

$= \underline{9.77896}$

Amt after 22 yrs =

$= 9.77896(1+i)^{4} + 100\bar{s}_{\overline{47}|}$ @ 6%

$=$ ~~462.8913571~~ $\underline{462801.3571}$

19) i) let $x/12$ be the monthly repayment solving the equation

$$x\, a_{\overline{25}|}^{(12)} = 9880 \ @ \ 7\%$$

$$x\, a_{\overline{25}|} \times \frac{i}{i^{(12)}} = 9880$$

$$\frac{x}{12} = 68.4805758$$

Hence Monthly Repayment Amount =

= $\underline{6848.05758}$

(ii) loan o/s immediately after i.e. payment on 10th march 2029.

$$x\, a_{\overline{34/3}|}^{(12)} = 821.7669096 \left[\frac{1-(1.07)^{-34/3}}{0.067850} \right]$$

$$= \underline{648574.5315}$$

(iii) loan o/s just after the repayment on 10th sept 2026 is made

$$= x\, a_{\overline{83/6}|}^{(12)} \ @ \ 7\%$$

11rly on 10th oct 2026 =

$$x\, a_{\overline{55/4}|}^{(12)} \ @ \ 7\%$$

Shrikrupa

$= x \left[a^{(12)}_{\overline{23/6}|} - a^{(12)}_{\overline{55/4}|} \right]$ @ 7%

$= 8\ 821.766909 (0.0326844761) = 26.85902093$

(v) capital repayment continued in these 12 installments

$= x \left[a^{(12)}_{\overline{22/3}|} - a^{(12)}_{\overline{19/3}|} \right]$ @ 7%

$= 821.766909 \left(\frac{(1/1.07)^{19/3} - (1/1.07)^{22/3}}{0.067850} \right)$

$= 516.1973861$

Total interest in these 12 installment =

$821.766909 - 516.197386 = 305.569523$