NMA Assignment

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Q1) radius measured = 12.65 cm
actual length = 12.5 cm
calc area of circular plate = ?

area of circular plate $A(r) = \pi r^2$
$A'(r) = 2\pi r$

Change in area $= A'(12.5)(0.15)$
$= 3.75\pi \, cm^2$

$A(12.65) - A(12.5)$
$= 160.0225\pi - 156.25\pi$
$= 3.7725\pi \, cm^2$

i) Absolute error
$= 3.7725\pi - 3.75\pi$
$= 0.0225\pi \, cm^2$

ii) Relative Error
$= \dfrac{\text{Absolute error}}{3.7725\pi}$

$= \dfrac{0.0225\cancel{\pi}}{3.7725\cancel{\pi}} = 0.006 \, cm^2$

iii) Percentage Error
$= 0.006 \times 100$
$= 0.6\%$

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2) logarithmic table

x	4.0	4.5	5.5	6.0
$f(x) = \log(x)$	0.60206	0.6532	0.74036	0.7781513

a. $f(x) =$

2i) $x_1 = 4$ $\quad y = 0.60206$

$x = 5$ $\quad y = ?$

$x_2 = 6$ $\quad y_2 = 0.7781513$

$$y = \frac{y_2(x - x_1) + y_1(x_2 - x_1)}{(x_2 - x_1)}$$

$$= \frac{0.7781513(1) + 0.60206(1)}{2}$$

$$\boxed{y = 0.6901}$$

ii) $x_1 = 4.5$ $\quad y_1 = 0.6532125$

$x = 5$ $\quad y = ?$

$x_2 = 5.5$ $\quad y_2 = 0.7403627$

$$y = \frac{y_2(x - x_1) + y_1(x_2 - x_1)}{x_2 - x_1}$$

$$\underline{y = 0.6901}$$

Q4) root of equation ~~$x^4 - x - 10$~~

$f(x) = x^3 - x - 1$ (Newton Raphson Method)

$f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

$f(1) = -1 < 0$

$f(2) = 5 > 0$

x	0	1	2
$f(x)$	-1	-1	5

∴ The root lines are between 1 and 2.

$x_0 = \frac{1+2}{2} = 1.5$

I^st iteration:

$y(x_0) = f(1.5) = 0.875$

$f'(x_0) = f'(1.5) = 5.75$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 1.34783$

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II^{nd} iteration:

$f(x_1) = f(1.34783) = 0.10068$
$f'(x_1) = f'(1.34783) = 4.44991$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow x_2 = 1.3252$

III^{rd} iteration:

$f(x_2) = f(1.3252) = 0.00206$
$f'(x_2) = y'(1.3252) = 4.26847$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.32472$

IV^{th} iteration:
$f(x_3) = f(1.32472) = 0$
$f'(x_3) = f'(1.32472) = 4.26463$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \Rightarrow x_4 = 1.32472$

Ans Root of given eqn $f(x) = x^3 - x - 1$ using Newton Raphson method is approximately around ~~1.37~~ 1.32472.

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5) A → square matrix

$A^2 = A$

find value of $7A - (I+A)^3$

I is an identity matrix

→ $7A - (I+A)^3$

$= 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$

$= 7A - (I + A \cdot A^2 + 3A^2 + 3A)$ $\boxed{I = \text{identity}}$

$= 7A - (I + A^2 + 3A + 3A)$ $\boxed{\text{As, } A^2 = A}$

$= 7A - (I + A^2 + 6A)$

$= 7A - (I + 7A)$

$= 7A - 7A - I$

$= -I$

Hence the reqd value is $\underline{\underline{-I}}$

6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$, find the inverse

$A^{-1} = \frac{1}{|A|}(\text{Adj } A)$ $|A| = -2$

$C = \begin{bmatrix} -6 & 4 & 4 \\ \cancel{-1} \; 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$

$\text{Adj } A = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

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$$\rightarrow A^{-1} = \frac{-1}{2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$\rightarrow A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix} \leftarrow \text{ANS}$$

7) def. scalar product, find ∠'s between following vectors. (smallest non-negative ∠)

$$\widehat{AB} = \frac{\overline{AB}}{AB}.$$

$A = 8i - 9j$
$B = 7i - 9j$

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Scalar product :

$$\bar{a}.\bar{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\therefore \bar{A}.\bar{B} = (8)(7) + (-9)(-9)$$

$$= 56 + 81 \quad = \underline{\underline{137}}$$

A

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Q) Sum of all 3 digit nos, that leave remainder 2 when divided by 3

$101, 104, 107 \ldots 998$

$a = 101$ $l = 998$ $d = 104 - 101 = 3$

$l = a + (n-1)d$

$998 = 101 + (n-1)3$

$998 - 101 = 3n - 3$

$897 + 3 = 3n$

$\therefore 3n = 900$

$\boxed{n = 300}$

Sum of all 300 digits are

$S_n = \frac{n}{2}(a + l)$

$S_n = \frac{300}{2}(101 + 998)$

$S_n = 150 \cdot (1099)$

$S_n = 164{,}850$

Therefore the sum is 164,850.

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GP.

10) $6^{th} = 32$ common ratio = ?
$8^{th} = 128$

$GP \rightarrow ar^{n-1}$

$6^{th} \rightarrow ar^5 = 32$
$8^{th} \rightarrow ar^7 = 128 \rightarrow ii)$

$\therefore 6^{th} \rightarrow ar^5 = 32$
$a = \frac{32}{r^5} \rightarrow (i)$

substituting value of (i) in eq(ii)

$ar^7 = 128$
$\frac{32}{r^5} \cdot r^7 = 128$
$32r^2 = 128$
$r^2 = \frac{128}{32} = 4$
$r = \pm 2$

over here, common ratio is 2.

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11) Binomial Th. to expand $(4+3x)^5$

Pascal Δ .

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & 2 & 1 & & & & \\ & & & 1 & 3 & & 3 & 1 & & & \\ & & 1 & 4 & & 6 & & 4 & 1 & & \\ & 1 & 5 & 10 & & & & 10 & 5 & 1 & \end{array}$$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$\therefore$ we get,

$(4+3x)^5 = (4)^5 + (5)(4)^4(3)x + 10(4)^3(3x)^2 + 10(4)^2(3x)^3 + 5(4)(3x)^4 + (3x)^5$

$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$.

12) Find Eigen values & corresponding eigen vectors for matrix A

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Shortcut method,

$$\begin{bmatrix} a & b & c \\ p & q & r \\ u & v & w \end{bmatrix}_{3\times3}$$

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minors of main diagonal

$\lambda^3 + (a + q + w)\lambda^2 + (m_{11} + m_{22} + m_{33})\lambda$

main diagonal

$- |A| = 0$

$m_{11} = \begin{vmatrix} q & r \\ v & w \end{vmatrix} = qw - vr = -15$

$m_{22} = \begin{vmatrix} a & c \\ u & w \end{vmatrix} = aw - cu = 6$

$m_{33} = \begin{vmatrix} a & b \\ p & q \end{vmatrix} = aq - bp = -4$

$= \lambda^3 + (2 + (-5) + 3)\lambda^2 + (-15 + 6 - 4)\lambda$

$- |A|$

$|A| = 2\begin{vmatrix} -5 & 0 \\ 0 & 3 \end{vmatrix} - (-3)\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix}$

$- 0\begin{vmatrix} 2 & -5 \\ 0 & 0 \end{vmatrix}$

$= 2(-15 - 0) + 3(6 - 0) - 0$

$= 2(-15) + 3(6) = -30 + 18$

$= -12$

$\therefore \lambda^3 + 0\lambda^2 + (-13\lambda) + 12 = 0$

$\lambda^3 + 12 - 13\lambda = 0$

one root is 1 as $(1 + 12) - 13 = 0$

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$\lambda^3 - 13\lambda + 12 = 0$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -13 & 12 \\ & \downarrow & +1 & +1 & -12 \\ \hline & 1 & +1 & -12 & 0 \end{array}$$

$\lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$\lambda(\lambda+4) - 3(\lambda+4) = 0$

$(\lambda-3)(\lambda+4) = 0$

$\lambda = -4, \lambda = 3$

roots are $-4, 3, 1$

Eigen vectors

when $\lambda = 3$, we get

$$\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 3. \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

we get,

$$\begin{bmatrix} 2x_1 - 3x_2 + 0x^3 \\ 2x_1 - 5x_2 + 0x^3 \\ 0x_1 + 0x_2 - 3x^3 \end{bmatrix} = \begin{bmatrix} 3x_1 \\ 3x_2 \\ 3x_3 \end{bmatrix}$$

$2x_1 - 3x_2 = 3x_1$

$-3x^2 = x_1 \rightarrow (1)$

$2x_1 - 5x_2 = 3x_2$

$2x_1 = 8x_2$

$x_1 = 4x_2$

$-3x_3 = 3x_3 \rightarrow$

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Q13) $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$\therefore f(x) = x^3 - 7x^2 + 8x - 3$

$f(5) = -13$

$f'(x) = 3x^2 - 14x + 8$

$f'(5) = 13$

$x_1 = 5 - \left(\frac{-13}{13}\right)$

$= 5 + 1$

$= \underline{\underline{6}}$

$f(6) = 9$ $\qquad x_2 = 6 - \left(\frac{9}{32}\right)$

$f'(6) = 32$

$$\boxed{x_2 = 5.71875}$$

Q14) $(1 - x + x^2)^4$

$\therefore [x^2 + (1-x)]^4$

let $(1-x)$ be y, we get

$= [x^2 + y]^4$

Using Binomial Theorem,

$(a+b)^n = a^n + na^{n-1}b + n(n-1)a^{n-2}b^2/2$

$+ \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} \ldots + b^n$

$\therefore [x^2+y]^4$

$= (x^2)^4 + 4(x^2)^3y + \frac{4\times3}{2}(x^2)^2y^2 + \frac{4\times3\times2}{3\times2}$

$x^2y^3 + y^4$

$= x^8 + 4x^6y + 3x^4 2y^2 + 4x^2y^3 + y^4$

$= x^8 + 4x^6y + 6x^4y^2 + 4x^2y^3 + y^4$

$\therefore [x^2+(1-x)]^4 = x^8 + 4x^6(1-x) + 6x^4(1-x)^2$
$+ 4x^2(1-x)^3 + (1-x)^4$

Q15) $e^{-x} = 3\log x$

$\therefore f(x) = 3\log x - e^{-x}$

$a = 1 \; ; b = 2$

$\therefore f(a) = 0.367879, f(b) = 1.944106$

$\therefore \frac{a+b}{2} = \frac{1+2}{2} = 1.5 = x_1$

$f(x_1) = 0.993265$

$x_2 = \frac{1+1.5}{2} = 1.25$

$f(x_2) = 3\log(1.25) - e^{-1.25}$
$= 0.382925$

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$\therefore x_3 = \frac{1+1.25}{2} = 1.125$

$f(x_3) = 3\log(1.125) - e^{-1.125}$

$= 0.02869664$

$\therefore x_4 = \frac{1+1.125}{2} = 1.0625$

$f(x_4) = 3\log(1.0625) - e^{-1.0625}$

$= -0.16372$

Q3) $x^4 - x - 10 = 0$

$a = 1.5 \; ; b = 2$

$\therefore f(a) = f(1.5) = (1.5)^4 - 1.5 - 10$

$= -6.4375$

$f(b) - f(2) = 2^4 - 2 - 10$

$= 4$

1st iteration:

$x_1 = \frac{2+1.5}{2} = 1.75$

$f(x_1) = f(1.75) = (1.75)^4 - 1.75 - 10$

$= -2.3711$

2nd iteration

$x_2 = \frac{1.75+2}{2} = 1.875$

$f(x_2) = (1.875)^4 - 1.875 - 10$

$= \underline{\underline{0.4846}}$

3rd iteration:

$x_3 = \frac{1.75 + 1.875}{2} = 1.8125$

$f(x_3) = f(1.8125) = (1.8125)^4 - 1.8125 - 10$

$= \underline{\underline{-1.0202}}$

4th iteration:

$x_4 = \frac{1.875 + 1.8125}{2} = 1.84375$

$f(x_4) = 1.84375^2 - 1.84375 - 10$

$= \underline{-0.2877}$

5th iteration:

$x_5 = \frac{1.875 + 1.84375}{2} = 1.859375$

$= 1.859375^4 - 1.859375 - 10$

$= \cancel{0.095}\ \underline{\underline{0.093}}$

root of $x^4 - x - 10 = 0$ is 1.859375
which is approximately $\underline{1.86}$