Assignment - 1   Numerical methods & Algebra

Q1 $r_1 \rightarrow 12.65cm$ (calculated radius)
$r_1' \rightarrow 12.5cm$ (ideal radius)

i Absolute error $= |12.5 - 12.65|$
$= |0.15cm| = 0.15cm$

ii Relative error $= \frac{|12.5 - 12.66|}{12.5} = \frac{0.15cm}{12.5cm} = 0.012$

iii Percentage error $= \frac{0.15}{12.5} \times 100 = 0.012 \times 100 = 1.2\%$

Q2

x	$\log x$ (y_1)
4	0.60206
4.5	0.65321
5.5	0.74036
6.0	0.77815

a) $x_1 = 4$ $y_1 = 0.60206$
$x = 5$ $y =$
$x_2 = 6$ $y_2 = 0.77815$

$\Rightarrow \frac{x_1 - x}{x - x_2} = \frac{y_1 - y}{y - y_2}$

$\Rightarrow \frac{4-5}{5-6} = \frac{0.60206 - y}{y - 0.77815}$

$\Rightarrow \frac{-1}{-1} = \frac{0.60206 - y}{y - 0.77815} \Rightarrow 0.60206 + 0.77815 = 2y$

$\Rightarrow y = 0.690105$

b) $x_1 = 4.5$ $\quad y_1 = 0.65321$

$x = 5$ $\quad y =$

$x_2 = 5.5$ $\quad y_2 = 0.74036$

$$\Rightarrow \frac{x_1 - x}{x - x_2} = \frac{y_1 - y}{y - y_2}$$

$$\Rightarrow \frac{1 \times \cancel{0.5}}{1 \times \cancel{0.5}} = \frac{0.65321 - y}{y - 0.74036}$$

$\Rightarrow 0.65321 + 0.74036 = 2y$

$\Rightarrow y = 0.6968$

Q3 $x^4 - x - 10 = 0$, $\quad a = 1.5$

$b = 2$

$\Rightarrow f(a) = (1.5)^4 - (1.5) - 10$

$= -6.437 = y_0$

$f(b) = (2)^4 - 2 - 10$

$= 4 = y_1$

NOW, $x_0 = 1.5$ $\quad x_1 = 2$

$y_0 = -6.437$ $\quad y_1 = 4$

$$x_2 = \frac{x_0 + x_1}{2} = \frac{1.5 + 2}{2} = 1.75$$

$y_2 = (1.75)^4 - (1.75) - 10$

$= -2.37109$ (-ve)

$$x_3 = \frac{x_2 + x_1}{2} = \frac{1.75 + 2}{2} = 1.875$$

$$y_3 = (1.875)^4 - (1.875) - 10$$
$$= 0.48461 \ (+ve)$$

$$x_4 = \frac{x_3 + x_2}{2} = \frac{1.875 + 1.75}{2} = 1.8125$$

$$y_4 = (1.8125)^4 - (1.8125) - 10$$
$$= -1.0203$$

$$x_5 = \frac{x_4 + x_3}{2} = 1.8437$$

$$y_5 = (1.84375)^4 - (1.8437) - 10$$
$$= -0.28773$$

$$x_6 = \frac{x_5 + x_4}{2} = \frac{1.8437 + 1.8125}{2} = \cancel{0.0933} \ 1.85937$$

$$y_6 = (1.85937)^4 - (1.85937) - 10$$
$$= 0.09338$$

Root of $x^4 - x - 10$ approx is equal to 1.85937

Q4 $f(x) = x^3 - x - 1 \Rightarrow f'(x) = 3x^2 - 1$

x	$f(x)$
0	-1
1	-1
2	5

$x_0 = 1 \Rightarrow f(x_0) = -1$

$f'(x_0) = 2$

$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{(-1)}{2} = 1.5$

$f(x_1) = (1.5)^3 - (1.5) - 1$
$= 0.875$

$f'(x_1) = 3 \times (1.5)^2 - 1$
$= 5.75$

$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5 - \frac{0.875}{5.75} = 1.3478$

$f(x_2) = 0.10068$
$f'(x_2) = 4.44990$

$\Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3478 - \frac{0.10068}{4.44990} = 1.3252$

$f(x_3) = 0.00205$
$f'(x_3) = 4.2684$

$\Rightarrow$ $x^4 = 1.3250 - \frac{0.00205}{4.2684} = 1.324519726$

$\Rightarrow$ $f(x_4) = -0.00008452$

Root of eqn $x^3 - x - 1$ is 1.324519726

Q5 $A^2 = A$

$$\Rightarrow 7A - (I+A)^3 = 7A - [I^3 + A^3 + 3I^2A + 3IA^2]$$
$$= 7A - [I + A(A^2) + 3IA + 3IA]$$
$$= \cancel{7}\cancel{8}\ \cancel{7A} - I - \cancel{A} + \cancel{3A} - \cancel{3A}$$
$$= -I$$

Q6 $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$\Rightarrow$ $|A| = 1[0-6] + 1[-4-0] + 2[4-0]$
$= -6 - 4 + 8 = -2$

$$\text{Adj } A = \begin{bmatrix} -6 & +4 & 4 \\ +1 & -1 & -1 \\ -6 & +2 & 4 \end{bmatrix}^T$$

$$= \begin{bmatrix} -6 & 1 & -6 \\ -4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$\Rightarrow$ $A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

Q7 $\vec{a} = 8\hat{i} - 9\hat{j}$ $\qquad$ $\vec{b} = 7\hat{i} - 9\hat{j}$

$\Rightarrow \quad \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$|\vec{a}| = \sqrt{64+81} = \sqrt{145}$

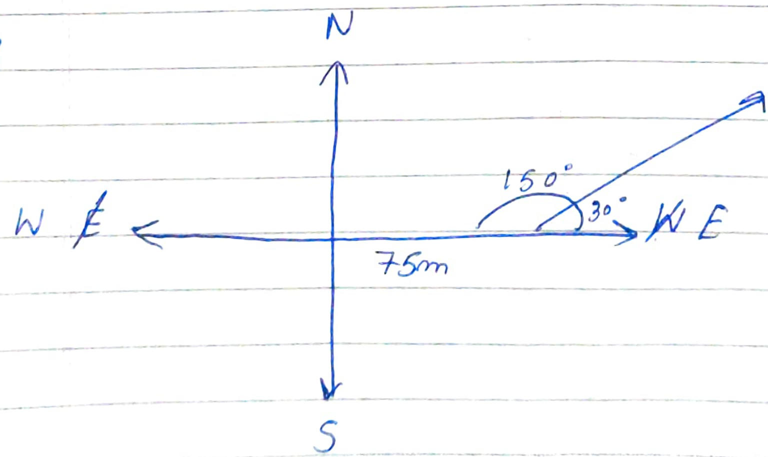
$|\vec{b}| = \sqrt{49+81} = \sqrt{130}$

$\Rightarrow \quad \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{8\times 7 + (-9\times -9)}{\sqrt{145}\times\sqrt{130}}$

$= \dfrac{\overset{56}{\cancel{63}}+81}{\sqrt{145\times 130}} = \dfrac{137}{\sqrt{8850}}$

$\Rightarrow \quad \theta = \cos^{-1}\left[\dfrac{137}{\sqrt{8850}}\right] = 3.7585^\circ$

Q8



$\Rightarrow \quad R^2 = A^2 + B^2 + 2AB\cos 30^\circ$

$\Rightarrow \quad R = \sqrt{75^2 + 25^2 + 2[75][25]\times \frac{\sqrt{3}}{2}}$

$\Rightarrow \quad R = 97.46m$

Q9 ~~104~~, 101, 104, ..., 998

$\Rightarrow a_n = 998$

$\Rightarrow a + [n-1]d = 998$

$\Rightarrow n-1 = \dfrac{998-a}{d} \Rightarrow n = \left[\dfrac{998-a}{d}\right] + 1 = 300$

So, $S_{300} = \dfrac{300}{2}[101+998] = 164850$

Q10 $ar^5 = 32$
$ar^7 = 128$

So $ar^5 = 32 \Rightarrow a = \dfrac{32}{r^5}$

$\Rightarrow \dfrac{32}{\cancel{r^5}} \times r^{\cancel{7}\,2} = 128 \Rightarrow r^2 = \dfrac{\cancel{128}\,4}{\cancel{32}\,1} \Rightarrow r = 2$

Q11 $(4+3x)^5 = 1 \times 4^5 \times (3x)^0 + 5 \times 4^4 \times (3x)^1 + 10 \times 4^3 \times (3x)^2 + 10 \times 4^2 \times (3x)^3 + 5 \times 4^1 \times (3x)^4 + 1 \times 4^0 \times (3x)^5$

$= 1280 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

Q12 $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow A - \lambda I = \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$\Rightarrow \quad |A - \lambda I| = 0$

$\Rightarrow \quad (2-\lambda)\{(-5-\lambda)(3-\lambda)-0\} + 3\{2\times(3-\lambda)\}$

$\Rightarrow \quad (2-\lambda)\{-15+5\lambda-3\lambda+\lambda^2\} + 3\{6-2\lambda\}$

$\Rightarrow \quad -30+10\lambda-6\lambda+2\lambda^2+15\lambda-5\lambda^2+3\lambda^2-\lambda^3+18-6\lambda$

$\Rightarrow \quad -\lambda^3+0\lambda^2+13\lambda-12=0$

$\Rightarrow \quad -\lambda^3+13\lambda-12=0 \Rightarrow \lambda^3-13\lambda+12=0$

$[\lambda=1] \rightarrow$ Satisfy

So

$$\begin{array}{c|cccc}
1 & 1 & 0 & -13 & 12 \\
 & 1 & 1 & 1 & -12 \\
\hline
 & \cancel{0}1 & 1 & -12 & 0
\end{array}$$

$\Rightarrow \quad \lambda^2+\lambda-12=0$

$\Rightarrow \quad \lambda+4\lambda-3\lambda-12=0$

$\Rightarrow \quad \lambda(\lambda+4)-3(\lambda+4)=0$

$\Rightarrow \quad (\lambda-3)(\lambda+4)$

$\Rightarrow \quad \lambda=1, 3, -4$

Q13 $f(x) = x^3 - 7x^2 + 8x - 3$ $\qquad x_0 = 5$

$f'(x_0) = 3x^2 - 14x + 8$

$f(5) = 5^3 - 7 \times 5^2 + 8 \times 5 - 3$
$= -13$

$f'(5) = 3 \times 5^2 - 14 \times 5 + 8$
$= 13$

$x_1 = x_0 - \frac{(-13)}{13} = 5 + 1 = 6$

~~$y_1 =$~~ $f(x_1) = 6^3 - 7 \times 6^2 + 8 \times 6 - 3$
$= 9$

$f'(x_1) = 3 \times 6^2 - 14 \times 6 + 8$
$= 32$

$x_2 = 6 - \frac{9}{32} = 5.71875$

~~$f(x_2)$~~

Q14 $(x^2 - x + 1)^4$, Let $a = (1 - x)$

$\Rightarrow (a + x^2)^4 = 1 \times a^4 \times (x^2)^0 + 4 \times a^3 \times (x^2)^1 + 6 \times a^2 \times (x^2)^2$
$+ 4 \times a^1 \times (x^2)^3 + 1 \times a^0 \times (x^2)^4$

$= a^4 + 4a^3 + 6a^2 x^4 + 4ax^6 + x^8$
$= (1-x)^4 + 4(1-x)^3 + 6(1-x)^2 x^4 + 4(1-x)x^5 + x^8$

Q15 $e^{-x} = 3\log(x)$

$\Rightarrow$ $f(x) = 3\log(x) - e^{-x}$

$\Rightarrow$

x	$f(x)$
$\cancel{0}$	
1	$-0.367879441 2$
2	1.944106

So, $x_2 = \frac{x_0 + x_1}{2} = \frac{1+2}{2} = 1.5$

$f(x_2) = 0.9932651642$

$x_3 = \frac{x_0 + x_2}{2} = \frac{\cancel{2}\,1 + 1.5}{2} = \cancel{1}\,1.25$

$f(x_3) = 0.382926$

$x_4 = \frac{x_0 + x_3}{2} = \frac{1+1.25}{2} = 1.125$

$f(x_4) = 0.0286966$

$x_5 = \frac{x_0 + x_4}{2} = \frac{1+1.125}{2} = 1.0625$

$f(x^5) = -0.163717$

$x_6 = \dfrac{x_4 + x_5}{2} = \dfrac{1.125 + 1.0625}{2} = 1.09375$

$f(x_6) = -0.066122$

Root of $f(x) = 3\log(x) - e^{-x}$ is 1.0937528