Assignment

1. $X = 200$

$X \sim \text{Poisson}(\theta)$, $\frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$

$P[x_1 < \theta < x_2] = 0.95$

$P[-1.96 < \theta < 1.96] = 0.96$ [Tables]

95% CI $= \theta \pm 1.96\sqrt{\theta}$

$= 200 \pm 1.96\sqrt{200}$

$= (172.28141, 227.71858)$

2) $X = 1, 2, \ldots n$

i.i.d

Mean $= \mu$

Variance $= \sigma^2$

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$$\bar{X} = \frac{\Sigma x_i}{n}$$

$$S^2 = \frac{1}{n-1}\left(\Sigma x_i^2 - n\bar{x}^2\right)$$

i) $E(\bar{X}) = E\left(\frac{\Sigma x_i}{n}\right) = \frac{\Sigma E(x_i)}{n} = \frac{n\mu}{n} = \underline{\underline{\mu}}$

$$V(\bar{X}) = V\left(\frac{\Sigma x_i}{n}\right) = \frac{1}{n^2} V(\Sigma x_i)$$

$$= \frac{1}{n^2} \Sigma V(x_i) = \frac{\cancel{n}\sigma^2}{\cancel{n^2}n} = \frac{\sigma^2}{n}$$

ii) $E(S^2) = \sigma^2$

$$E(S^2) = E\left[\frac{1}{n-1}\left(\Sigma x_i^2 - n\bar{x}^2\right)\right]$$

$$= \frac{1}{n-1}\left[E\left(\Sigma x_i^2\right) - nE(\bar{x}^2)\right]$$

$$= \frac{1}{n-1}\left[\Sigma E(x_i^2) - nE(\bar{x}^2)\right]$$

$$V(x) = E(x_i^2) - [E(x)]^2$$

$$E(x_i^2) = \mu^2 + \sigma^2$$

$$V(\bar{x}) = E(\bar{x}^2) - [E(\bar{x})]^2$$

$$E(\bar{x}^2) = \mu^2 + \sigma^2/n$$

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$$E(S^2) = \frac{1}{n-1}\left[\Sigma(\mu^2+\sigma^2) - n\left(\mu^2+\frac{\sigma^2}{n}\right)\right]$$

$$= \frac{1}{n-1}(n\mu^2 + n\sigma^2 - n\mu^2 - \sigma^2)$$

$$= \frac{1}{n-1}(n\sigma^2 - \sigma^2)$$

$$= \frac{1}{n-1}\sigma^2(n-1)$$

$$= \underline{\underline{\sigma^2}}$$

iii) $X \sim N(\mu, \sigma^2)$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$V\left(\frac{(n-1)S^2}{\sigma^2}\right) = \frac{(n-1)^2}{\sigma^4} V(S)^2$$

$$\frac{(n-1)^2 V(S)^2}{\sigma^4} = 2(n-1)$$

$$V(S)^2 = \frac{2\sigma^2}{n-1}$$

iv) $n = 10$
$\sigma^2 = 100$

$$P[50 < S^2 < 150] = P[S^2 < 150] - P[S^2 < 50]$$

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$$= P\left[\frac{(n-1)s^2}{\sigma^2} < \frac{9 \times 150}{100}\right] - P\left[\frac{(n-1)s^2}{\sigma^2} < \frac{9 \times 50}{100}\right]$$

$$= P\left[\chi^2_9 < 13.5\right] - P\left[\chi^2_9 < 4.5\right]$$

$= 0.8587 - 0.1244$

$= 0.7343$

3) i) $\prod_{i=1}^{180} P(X = x)$

$$= [P(X=0)]^{86} \cdot [P(X=1)]^{75} \cdot [P(X=2)]^{16} \cdot$$

$$[P(X=3)]^{2} \cdot [P(X=4)]$$

$$= \left[{}^4C_0\, p^0 (1-p)^4\right]^{86} \times \left[{}^4C_1\, p(1-p)^3\right]^{75} \times \left[{}^4C_2\, p^2(1-p)^2\right]^{16}$$

$$\times \left[{}^4C_3\, p^3 (1-p)\right]^2 \times \left[{}^4C_4\, p^4\right]$$

$$L = \text{constant} \times (1-p)^{603} \times p^{117}$$

$$\log L = \log(\text{constant}) + 117 \log(p) + 603 \log(1-p)$$

$$\frac{dl}{dp} = 0 + \frac{117}{p} - \frac{603}{(1-p)}$$

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$$\frac{dl}{dp} = \frac{117}{p} - \frac{603}{(1-p)}$$

$$\frac{dl}{dp} = 0$$

$$\frac{117}{p} = \frac{603}{1-p}$$

$117(1-p) = 603p$

$117 - 117p = 603p$

$p = \frac{117}{720}$

$\hat{p} = 0.1625$

ii)

x_i	O_i	E_i	
0	86	$180 \times P(x=0) = 88.55$	
1	75	68.73	
2	16	20.0	
3	2	2.58	< 5
4	1	0.125	< 5

x_i	O_i	E_i	
0	86	88.55	
1	75	68.73	
2	19	22.705	

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$(O_i - E_i)^2 / E_i$

0.0737
0.5722
0.6078

$\chi^2_{cal} = 1.2537$

$\chi^2_{tab} = \chi^2_1 = 3.841$

$\chi^2_{cal} < \chi^2_{tab}$

$\therefore$ Do not reject H_0 at 5% LOS

4)

i) PDF of random variable :

$$f(x) = \frac{CB^3}{(x+B)^4} \; ; \; x>0, \; B>0 \quad (1)$$

$$f(x) = \frac{\alpha \lambda^{\alpha}}{(\lambda + x)^{\alpha+1}} \quad (2) \quad \text{(using Tables)}$$

Comparing (1) & (2)

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$C = 3$

Mean. $E(x) = \frac{\beta}{C-1}$

$= \beta/2$

Variance. $V(x) = \frac{C\beta^2}{(C-1)^2(C-2)} = \frac{3\beta^2}{4\times-1} = \frac{3\beta^2}{-4}$

ii) $E(x) = \beta/2$
$E(x) = \bar{x}$
$\beta/2 = \bar{x}$

$\hat{\beta} = 2\bar{x}$

Bias $= E(\hat{\beta}) - \beta$
$= E(2\bar{x}) - \beta$
$= 2E(\bar{x}) - \beta$
$= \frac{2}{n} \sum E(x_i) - \beta$

$= \frac{2}{n} \times \frac{n\beta}{2} - \beta = \beta - \beta$

$= 0$, unbiased

$$MSE = V(\hat{B}) + (Bias)^2$$
$$= V(\hat{B})$$
$$= V(2\bar{x})$$
$$= 4(\bar{x}) = 4V\left(\frac{\sum x_i}{n}\right)$$
$$= \frac{4}{n^2} \sum V(x_i) = \frac{4}{n^2} \times \frac{3B^2 n}{4} = \frac{3B}{n}$$

iv) $E(\hat{B}_2) = bB/2$

$$= \frac{2}{(1+3/n)} \times B/2 = \frac{nB}{(n+3)}$$

$$Bias = E(\hat{B}_2) - B = \frac{nB - B(n+3)}{n+3}$$
$$= \frac{nB - nB - 3B}{n+3} = \frac{-3B}{n+3}$$

$$MSE = \frac{B^2}{4n} (3b^2 + nb^2 + 4n - 4nb)$$
$$= \frac{B^2}{4n} (b^2(n+3) + 4n(1-b))$$
$$= \frac{B^2}{4n} \left[\frac{4n^2}{(n+3)^2} \times \frac{(n+3)}{n+3} + 4n\left(\frac{3+n-2n}{n+3}\right)\right]$$
$$= B^2 \left[\frac{4n^2}{n+3} + \frac{4n(3-n)}{n+3}\right]$$
$$= B^2 \left[\frac{4n^2 + 12n - 4n^2}{n+3}\right] = \frac{12nB}{n+3}$$

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As $n \to \infty$, M.SE $\to 0$

5) i) $X \sim U(a,b)$

$E(X) = \frac{a+b}{2}$

$\bar{X} = \frac{a+b}{2}$

$a = 2\bar{X} - b$ — (1)

$V(X) = \frac{(b-a)^2}{12}$

$S^2 = \frac{(b-a)^2}{12}$

$12S^2 = (b-a)^2$

$2\sqrt{3}\, S = b-a$

$2\sqrt{3}\, S = 2\bar{X} - a - a$ $\quad (b = 2\bar{X} - a)$

$2\sqrt{3}\, S = 2\bar{X} - 2a$

$\sqrt{3}\, S = \bar{X} - a$

$\hat{a} = \bar{X} - \sqrt{3}\, S$

ii) $b = 2\bar{X} - a$

$b = 2\bar{X} - \bar{X} + \sqrt{3}\, S$

$\hat{b} = \bar{X} + \sqrt{3}\, S$

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iii) Sample of observation = 1, 2, 8, 50, 150

$\bar{x} = \cancel{42.2}\ 24.4$

$S = \cancel{63.57}\ 18.147$

$\hat{a} = \bar{x} - 3\sqrt{5}$

$\hat{a} = -7.018$

$\hat{b} = \bar{x} + \sqrt{3}\, S$

$\hat{b} = 55.87$

iv) Sample = $x_1, x_2 \dots x_n$

$a = 0$

$L = \prod_{i=1}^{n} f(x)$

$L = \left(\frac{1}{b-a}\right)^n$

$L = \left(\frac{1}{b}\right)^n \quad [a=0]$

$\ln L = -n \ln b$

$\frac{dl}{db} = \frac{-n}{b}$

$\frac{dl}{db} = 0$, gives $b = \infty$, n.p

$\therefore \frac{d^2 l}{db^2} = \frac{n}{b^2}$

$\frac{n}{b^2} > 0$, Max. = $\hat{b} = \text{Max}(b_i)$

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6) $H_0: p = 0.1$
$H_1: p > 0.1$

i) X: no. of duts in 1^{st} step
Y: no. of duts in 2nd step

$P(\text{type 1 error}) = P(X \geq 3 / p = 0.1) + P(Y \geq 5 / p = 0.1)$
$P(X = 0 / p = 0.1) + P(Y \geq 4 / p = 0.1) \times P(X = 1 / p = 0.1)$

$P(Y \geq 3 / p = 0.1) \times P(X = 2 / p = 0.1)$

$= P(X \geq 2 / p = 0.1) + P(Y > 4 / p = 0.1) \; P(X = 0 / p = 0.1)$

$= \cancel{P(X > 2 / p = 0.1)} + P(Y > 3 / p = 0.1) \; P(X = 1 / p = 0.1)$
$+ P(Y > 2 / p = 0.1) \; P(X = 2 / p = 0.1)$

$= (1 - 0.8891) + (1 - 0.9957) \; (0.2824) + (1 - 0.9744)$
$(0.6590 - 0.2824) + (1 - 0.8891) \; (0.8891 - 0.6590)$

$= 0.1433 = 14.33\%$

ii) P(rejecting H_0) when $p = 0.3$

$= P(X > 2 / p = 0.3) + P(Y > 4 / p = 0.3) \; P(X = 0 / p = 0.3)$

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$+ P(Y>3/p=0.3)\ P(X=1/p=0.3) + P(Y>2/p=0.3)$
$P(X=2/p=0.3)$

$= (1-0.2528) + (1-0.07237)\ (0.0138) + (1-0.4925)$
$(0.0850-0.0138) + (1-0.2528)\ (0.2528-0.0850)$

$= 0.9125 = 91.25\%$

iii) $P(\text{type II error}) = 1 - P(\text{type I error})$
$= 1 - 0.9125$
$= 0.0875$
$= 8.75\%$

iv) $\hat{p} = \frac{40}{120} = \frac{1}{3} = 0.33$

$\Rightarrow \bar{X} \sim N\left(p, \frac{p(1-p)}{n}\right)$

Pivotal quantity $= \frac{X - np}{\sqrt{\frac{p(1-p)}{n}}}$

For 90% one sided CI :

$P\left[\frac{\bar{X}-p}{\sqrt{\frac{p(1-p)}{n}}} < x_1\right] = 0.1$

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$= 90\%$ of CI $= \bar{x} - Z_{0.1}\sqrt{\frac{p(1-p)}{n}}$

$= 0.333 - 1.2816\sqrt{\frac{0.333(0.667)}{120}} = 0.2783$

v) $H_0 = p = 0.278$
$H_1 = p > 0.278$

since $\hat{p} = 0.333 > p$:-

$\frac{x - np_0}{\sqrt{np_0 q_0}} \sim N(0,1)$

$\frac{39.5 - 120(0.278)}{\sqrt{120 \times 0.278 \times 0.722}} = 1.2511$

$Z_{tab} = Z_{10\%} = 1.2816$

$Z_{cal} < Z_{tab} =$ do not reject H_0

vi) (iv) suggests that there is 10% chance, $p < 0.2782$ could be less than 0.278 in 90% of cases

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7)
i) Mean :

Private hospitals $= \frac{\Sigma x_i}{n} = \frac{315.5}{9} = 35.056$

Public hospitals $= \frac{\Sigma x_i}{n} = \frac{270}{9} = 30$

Variance :

Private $= \frac{\Sigma x_i^2 - n\bar{x}}{n-1} = 67.403$

Public $= 64.312$

ii) For using equal mean, we can use 2-sided test.

$H_0 : \sigma_1^2 = \sigma_2^2$; $H_1 : \sigma_1^2 \neq \sigma_2^2$

$$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1,\, n_2-1}$$

$$= \frac{64.312}{67.403} = 0.9514$$

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$F_{8,8}$ values at 5% levels are 0.2256 and 4.433

$\therefore$ We do not reject

iii) $H_0 : \mu_{PH} = \mu_{PH}$

$H_1 = \mu_{PH} > \mu_{PH}$

$$T_{cal} = \frac{(\bar{X}_2 - \bar{X}_1) - (\mu_2 - \mu_1)}{S_P\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{35.055 - 30}{8.1152\sqrt{\frac{2}{9}}} = 1.32151$$

$T_{tab} = t_{16, 5\%} = 1.746$

$T_{cal} < T_{tab}$ $\therefore$ ~~there~~ insufficient evidence to reject H_0

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Q8) i) An unbiased estimator is an estimator whose mean value is equal to true parameter value.

ii) Bias is difference between the expected value of the estimator & the true parameter value.

iii) $MSE(\hat{\theta}) = Var(\hat{\theta}) + Bias^2$

iv)

v) Both

vi) a) Method of moments
b) MLE

Q9) i) $\dfrac{Z}{\sqrt{C/k}} \sim t_k$;

where $Z \sim N(0,1)$
$C \sim \chi^2$ $k = n-1$

K denotes the degrees of freedom

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ii) Mean of t_k = 0

variance = $\frac{k}{k-2}$

iii) number = 10

$S^2 = 48.667$

$\bar{x} = 50$

a)

For 99% CI = $\bar{x} \pm t_{n-1, 0.5\%} \, S/\sqrt{n}$

$= 50 \pm 3.250 \times \sqrt{\frac{48.667}{10}}$

$= (42.83, 57.1697)$

b) $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim N\left(0, \frac{k}{k-2}\right)$

$t_{n-1} \sim N\left(0, \sqrt{\frac{9}{7}}\right)$

$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim N\left(0, \sqrt{\frac{9}{7}}\right)$

$\quad " \sim N(0,1)$

Critical value level of confidence 2.58

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$$CI = \left(50 - 2.58\sqrt{\frac{48.667}{10} \times \frac{9}{7}}\ ,\ 50 + 2.58\sqrt{\frac{48.667}{10} \times \frac{9}{7}}\right)$$

$CI = (43.556, 56.443)$

Q10) $n = 1000$

$\lambda = 3$

$X \sim Poi(n\lambda)$

$X \sim Poi(3000)$

i) P(Type 1 error) = P(reject H_0 / H_0 is true)

$= P(X > 3100 /\ X \sim Poi(3000))$

Using normal approximation:

$$P(X > 3100) = P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$$

$= 1 - P(Z < 1.825) = 3.39\%$

ii) Type II error is the probability of accepting H_0 when its supposed to be rejected.

iii) Power of test = $P(X > 3100 /\ X \sim N(n\mu, n\mu))$

$= 1 - P(X < 3100 /\ X \sim N(n\mu, n\mu))$

$$= 1 - P\left(Z < \frac{3100 - n\mu}{\sqrt{n\mu}}\right)$$

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iv) $\bar{X} \sim N(\mu, \frac{\mu}{n})$

$\frac{\bar{X} - \mu}{\sqrt{\mu/n}} \sim N(0,1)$

$= P\left[-2.5758 < \frac{2.9-\mu}{\sqrt{2.9/1000}} < 2.578\right) = 0.99$

$= 99\% \; CI = \frac{2900}{1000} \pm 2.5758\sqrt{\frac{2.9}{1000}}$

$= (2.7613, 3.0387)$

ii) $\Sigma x = 213200$
$n = 12$
$\Sigma x^2 = 4919860000$
$\bar{x} = 17767$
$S = 10144$

$= \Sigma x +$ correct value $-$ incorrect value
$= 213200 + (27500 + 31500) - (11000 + 48000)$
$= 213200$

$\Sigma x_i = \frac{\Sigma x}{n} = \frac{213200}{12} = 17767$

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$$\Sigma x_1^2 = \Sigma x_1^2 - (11000^2 + 48000^2) + 27500^2 + 31500^2$$
$$= 4243360000$$

$$S^2 = \frac{1}{n-1}(x_1^2 - n\bar{x}^2)$$
$$= \frac{1}{11}(4243360000 - (12 \times 17767^2))$$

$$S^2 = 41396776$$
$$S = 6434$$