Probability & Statistics Assignment 2

Q1)

~~Claim sizes (HI) ⇒ HI ⇒ N(800, 100)~~

~~Claim sizes (CI) ⇒ CI ⇒ N(1200, 300)~~

$X \rightarrow N(800, 100^2)$

$Y \rightarrow N(1200, 300^2)$

$P(Y_1 + Y_2 + Y_3 > x_1 + x_2 + x_3 + x_4 + 800)$

$P(Y_1 + Y_2 + Y_3 - (x_1 + x_2 + x_3 + x_4) > 800)$

$Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4) \sim N(3 \times 1200 - 4 \times 800, (300^2(3) + 4(100^2))$

$\sim N(400, 310000)$

$P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right)$

$P(Z > 0.71842) \rightarrow 1 - P(Z < 0.71842)$

$= 1 - 0.76424$

$= 0.23576$

Q2) $N \sim \text{Neg. Bino.}(K, p)$

$P(N = n) \rightarrow {}^{n+2}C_n (0.9)^3 (0.1)^n$

$= {}^{(n+3)-1}C_{n-1} (0.9)^3 (0.1)^n$

So, $k = n$, $p = 0.1$

$K = 3$, $p = 0.9$

$M_y(t) = M_n(\log M_x t)$

$X \sim Gamma(6,2)$

$y \rightarrow$ Total claim amount

$E(e^{yt}/N=n) = E(e^{x_1t + x_2t \cdots x_nt})$

$= E(e^{tx_1})E(e^{tx_2}) \cdots E(e^{tx_n})$

$= \left(1-\frac{t}{2}\right)^{-6n}$

So,

$E(E(e^{yt}/N=n)) = E\left[\left(1-\frac{t}{2}\right)^{-6n}\right]$

$= E(e^{N\log(1-t/2)^{-6}})$

$= M_n\left[\log\left(1-\frac{t}{2}\right)^{-6}\right]$

$M_n(t) = \frac{pe^t}{1-qe^t} = \left[\frac{pe^{\log(1-t/2)^{-6}}}{1-qe^{\log(1-t/2)^{-6}}}\right]^k$

$\therefore k=3, p=0.9$

$= \left[\frac{0.9(1-t/2)^{-6}}{1-0.(1-t/2)^{-6}}\right]^3$

$$V(Y) = E(N)\cdot V(x) + \left[(E(x))^2 \cdot V(N)\right]$$

$$= \left[\frac{3}{0.9} \times \frac{6}{4}\right] + \left[\left(\frac{8}{2}\right)^2 \times \frac{0.3\times 0.1}{(0.9)^2}\right]$$

$$= \left(1/0.2\right) + \left(6 \times 0.3/0.81\right)$$

$$= 7.22$$

$$SD = \sqrt{Var(Y)} = \sqrt{7.22} = 2.687419$$

Q3) $y(x) = \lambda e^{-\lambda(x-\alpha)}$

$$M_xF = E(e^{tx}) = \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)}\, dx$$

$$M_x(t) = \lambda \int_{\alpha}^{\infty} e^{tx} e^{-\lambda x} \cdot e^{\lambda\alpha}\, dx$$

$$= e^{\lambda\alpha} \lambda \int_{\alpha}^{\infty} e^{-(\lambda-t)x} \cdot dx$$

$$= \frac{e^{\lambda\alpha}\cdot\lambda}{-\lambda \cdot x}\left[e^{-(\lambda-t)x}\right]_{\alpha}^{\infty}$$

$$= e^{t\alpha}\cdot\left(\frac{\lambda}{\lambda-t}\right)$$

$$M_x(t) = \frac{\lambda e^{tx}}{\lambda - t} = \lambda e^{t\alpha}(\lambda - t)^{-1}$$

$$M'_x(t) = \lambda\left[e^{t\alpha}(\lambda - t)^{-2}(1) + (\lambda - t)^{-1} e^{t\alpha}(\alpha)\right]$$

$$= \lambda e^{t\alpha}(\lambda - t)^{-2}(\alpha(\lambda - t) + 1)$$

$$= \lambda(\lambda)^{-2}(\alpha(\lambda + 1))$$

$$= \frac{\lambda\alpha + 1}{\lambda}$$

$$M''_x(t) = \lambda\left[\alpha e^{t\alpha}(\lambda - t)^{-2} + \alpha^2(\lambda - t)^{-1} e^{t\alpha} + e^{t\alpha}(2)(\lambda - t)^{-3} + (\lambda - t)^{-2} e^{t\alpha}\right]$$

$$= \lambda\left[\alpha(\lambda)^{-2} + \alpha^2(\lambda)^{-1} + 2(\lambda)^{-3} + \alpha(\lambda)^{-2}\right]$$

$$= \lambda\left[\frac{2\alpha}{\lambda^2} + \frac{\alpha^2}{\lambda} + \frac{2}{\lambda^3}\right]$$

$$E(x^2) = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\alpha + \frac{1}{\lambda}\right]^2$$

$$= \frac{1}{\lambda^2}$$

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Q4) $G_v(t) = \left(\frac{p}{(1-qt)}\right)^k$

$G_v(t) = \sum t^v \cdot P(v=v)$

$P(v=4) = \frac{\sqrt{k+4}}{\sqrt{5}\cdot\sqrt{k}} \cdot p^k \cdot q^k$ $\because G_v(t) = p^k(1-qt)^2$

$= \frac{\sqrt{k+2}}{4!\sqrt{k}} p^k q^k$

$= \frac{(k+3)!}{4!(k-1)!} p^k (1-p)^k$

Q5) $a = 2$

$a \int_{10}^{20}\int_{0}^{5} (x^2 + xy)\,dx\,dy = 1$

$\therefore \int_{10}^{20} \left(\frac{x^3}{3} + \frac{x^2 y}{2}\right)_0^5 dy = \frac{1}{a}$

$\therefore \left(\frac{125y}{3} + \frac{25y^2}{4}\right)_{10}^{20} = \frac{1}{a}$

$\therefore a = \left[\frac{125\times 20}{3} + \frac{25\times 400}{4} - \frac{125\times 10}{3} - \frac{25\times 100}{4}\right]$

$\therefore a = \left[\frac{125\times 10}{3} + 2500 - 625\right]$ $\cancel{= 2290}.666666$

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$E(Y/x)$ $\quad a = \frac{3}{6875}$

$$f_x(y) = \frac{f(x,y)}{f(x)}$$

$$f(x) = \frac{3}{6875}\int_{10}^{20} x^2 + xy \cdot dy$$

$$= \frac{3}{6875}(10x^2 + 150x)$$

$$= \frac{30x}{6875}(x+15)$$

$$f_{y/x} = \frac{3}{6875} \times \frac{x(x+y)}{30(x)} \times \frac{\cancel{x} \cdot 6875}{x+15} = \frac{x+y}{10(x+15)}$$

$$E(Y/x) = \int_{10}^{20} \frac{(x+y)}{10(x+15)} \, dy$$

$$= \frac{1}{10(x+15)}\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{10}^{20}$$

$$= \frac{1}{x+15}\left(15x + \frac{700}{3}\right)$$

$P(Y > \frac{1}{3}X + 10)$.

$$f(y) = \frac{3}{6875}\int_0^5 x^2 + xy \cdot dx$$

$$= \frac{3}{6875}\left(\frac{x^3}{3} + \frac{x^2 y}{2}\right)_0^5$$

$$= \frac{3}{6875}\left(\frac{125}{3} + \frac{25y}{2}\right)$$

So,

$$\frac{3}{6875}\left(\int_{\frac{1}{3}x+10}^{20} \frac{125}{3} + \frac{25}{2}y \cdot dy\right)$$

$$= \frac{3}{6875}\left(\frac{125y}{3} + \frac{25y^2}{4}\right)_{\frac{1}{3}x+10}^{20}$$

$$= \frac{3}{6875}\left(\frac{1250}{3} + 1875 - \frac{25x^2}{36} - \frac{125x}{7} - \frac{25x}{6}\right)$$

Q6)

$X \sim Poi(\lambda)$ $\quad$ $Y \sim Poi(\mu)$

$M_x(t) = e^{\lambda}(e^{t-1})$ $\quad$ $M_y(t) = e^{\mu}(e^{t-1})$

So, $Z = X - Y$

$$M_Z(t) = E(e^{tz})$$

$$= \frac{E(e^{tx})}{E(e^{ty})}$$

$$= \frac{e^{\lambda}(e^{t-1})}{e^{\mu}(e^{t-1})} = e^{(\lambda-\mu)(e^{t-1})} \sim Poi(\lambda - \mu)$$

Now, Let $Z = X + Y$

$$E(e^{tZ}) = e^{\lambda(e^t-1)} \cdot e^{\mu(e^t-1)}$$
$$= e^{(\lambda+\mu)(e^t-1)}$$

So, by uniquenes $Z \sim Poi(\lambda+\mu)$

Q7)

		X			
		1	2	3	4
Y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

$$V(X/Y=2) = E(X^2/Y=2) - [E(X/Y=2)]^2$$

$$E(x^2/Y=2) = \frac{\sum x^2 \cdot P(X=x, y=2)}{P(Y=2)}$$

When, $x=1$

$$\frac{1^2 P(x=1, y=2)}{P(y=2)} \text{ i.e } \frac{1(0)}{0.6} + \frac{4(0.3)}{0.6} + \frac{9(0.1)}{0.6} + \frac{16(0.2)}{0.6}$$

$$= 8.83333$$

$$E(X/Y=2) = \frac{\sum_x x P(x=x, y=2)}{P(y=2)}$$

$$= \frac{1(0)}{0.6} + \frac{2(0.3)}{0.6} + \frac{3(0.1)}{0.6} + \frac{4(0.2)}{0.6}$$

$$= 2.83333$$

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$$V(X/Y=2) = 8.8333 - (2.8333)^2$$
$$= 0.80757$$

$$f(u,v) = \frac{48}{67}(2uv - u^2) \qquad 0<u<1,\ \frac{u}{2}<v<2$$

$$E(U/V=v) = ?$$

$$\frac{f(u,v)}{f(v)} \rightarrow f\ U/V=v$$

$$\therefore f(v) = \frac{48}{67}\int_0^1 (2uv - u^2)\,du$$
$$= \frac{48}{67}\left(u^2v - \frac{u^3}{3}\right)_0^1$$
$$= \frac{48}{67}(v - 1/3)$$
$$= \frac{16}{67}(3v-1)$$

$$\frac{f(u,v)}{f(v)} = \frac{\cancel{48}^{3}\,2uv^2 - u^2}{\cancel{67}\ 3v-1} \times \frac{\cancel{67}}{\cancel{16}_1}$$
$$= \frac{3}{3v-1}(2uv^2 - u^2)$$

$$\therefore E(U/V=v) = \frac{3}{3v-1}\int_0^1 u(2uv^2 - u^2)\,du$$
$$= \frac{3}{3v-1}\left[\frac{2u^3v^2}{3} - \frac{u^4}{4}\right]_0^1$$
$$= \frac{8v^2-3}{4(3v-1)}$$

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Q8) $X \sim Gamma(3,2)$ $E(x) = 3/2$, $V(x) = 3/4$

$E(Y/X=x) = 3x+1$ $Var(Y/X=x) = 2x^2+5$

$E(Y) = E(E/Y=x) \Rightarrow E(3x+1)$

$= 3(E(x))+1$

$= 3\times 3/2+1$

$= \frac{11}{2}$

$V(Y) = E(V(X/Y)) + V(E(X/Y))$

$= E(2x^2+5) + V(3x+1)$

$= 2E(x^2)+5+9V(x)$

$V(x) = E(x^2) - [E(x)]^2$

$E(x^2) = 3/4 + 9/4 = 3$

$\therefore V(Y) = 2(3)+5+9(3/4)$

$= \frac{71}{4}$

Q9) $E(x) = 3$ $E(Y) = 4$

$V(x) = 4$ $V(Y) = 1$

Co-relation co-eff $= -0.3\sqrt{4} \rightarrow Cov(x,y)$

$= -0.6$

$Z = x+y$

$Cov(X+Y) = Cov(X,X) + Cov(X,Y)$

$= V(x) + Cov(X,Y)$

$= 4 + (0.6)$ $= 3.4$

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$$Var(Z) = Cov(X+X, X+Y)$$
$$= Cov(X,X) + 2Cov(X,Y) + Cov(Y,Y)$$
$$= V(X) + 2Cov(X,Y) + V(Y)$$
$$= 4 + 2(-0.6) + 1$$
$$= 5 - 1.2$$
$$= \underline{\underline{3.8}}$$

Q10) $f(x,y) = kx^{-\alpha} e^{-y/\beta}$

$$= k\int_1^{\infty}\int_1^{\infty} x^{-\alpha} e^{-y/\beta}\, dx\, dy = 1$$

$$\therefore k\int_1^{\infty} e^{-y/\beta} \left[\frac{x^{-\alpha+1}}{1-\alpha}\right]_1^{\infty} = 1$$

$$\therefore \frac{k}{1-\alpha}\int_1^{\infty} e^{-y/\beta} = 1$$

$$\therefore \frac{k}{\alpha-1}\left(\frac{e^{-y/\beta}}{-1/\beta}\right)_1^{\infty} = 1$$

$$\therefore \frac{-k}{\alpha-1}(\beta)(e^{-1/\beta}) = 1$$

$$\therefore k = \frac{\alpha-1}{\beta(e^{-1/\beta})}$$