Calculus Assignment - 2

Roshan Mehta (46)

$\cancel{\frac{2}{w} = u}$

Q1) $$\int_{1/50}^{2} \frac{e^{\frac{2}{w}}}{w^2} \cdot dw$$

$\cancel{u = \frac{2}{w^2}}$

$$= \int_{1/50}^{2} e^{2/w} \times \frac{1}{w^2} \cdot dw$$

$$u = \frac{1}{w} = w^{-1}$$

$$du = -w^{-2} \cdot dw$$
$$= -\frac{1}{w^2} \cdot dw$$

$$= \int_{1/50}^{2} e^{2u} \times \cancel{\frac{1}{w^2}} \times -\cancel{w^2} \cdot du$$

$$= -\int_{1/50}^{2} e^{2u} \cdot du$$

$$- \left[\frac{e^{2u}}{2} \right]_{1/50}^{2}$$

$$= - \left[\frac{e^{2 \times \frac{1}{w}}}{2} \right]_{1/50}^{2} = - \left(\left[\frac{e^{2 \times \frac{1}{2}}}{2} \right] - \left[\frac{e^{2 \times \frac{1}{50}}}{2} \right] \right)$$

$$= - \left[\frac{e}{2} - \frac{e^{\frac{1}{25}}}{2} \right]$$

$$= - \frac{(e - e^{\frac{1}{25}})}{2}$$

$$= \frac{e^{1/25} - e}{2}$$

PAGE No
DATE / 202

Q2) $\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$

Q3) $f'(x) = x^4 + 3x - 9$

$f(x) = \int f'(x)$

$\int (x^4 + 3x - 9)\,dx$

$f(x) = x^5 + 3x^2 - 9x$

Q4) $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

$\int_{-2}^{3} f(x)\,dx$

for $x > 1$

$\int_{-2}^{3} 6 \cdot dx = [6x]_{-2}^{3} = [(6\times3) - (6\times-2)]$
$= (18 + 12]$
$= 30$

for $x \leq 1$

$\int_{-2}^{3} 3x^2 \cdot dx = [x^3]_{-2}^{3} = [(3)^3 - (2)^3]$
$= [27 - (-8)]$
$= 35$

Q5) $\int 3(8y-1)e^{4y^2-y} \cdot dy$

$= 3\int(8y-1)e^{4y^2-y} \cdot dy$

$u = 4y^2 - y$

$\frac{du}{dy} = 8y-1$

$dy = \frac{du}{8y-1}$

$= 3\int e^{u} \cdot du$

$= 3e^{u}$

$= 3e^{4y^2-y} + C$

Q6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$, $f(1) = \frac{-5}{4}$, $f(4) = 404$

$f'(x) = \int 15\sqrt{x} + 5x^3 + 6$

$= 3\times 15x^{1/3} + \frac{5x^4}{4} + 6x$

$= 45x^{1/3} + \frac{5x^4}{4} + 6x$

$f(x) = \int 45x^{1/3} + \frac{5x^4}{4} + 6x$

$= 4\times 45\,x^{1/4} + \frac{x^5}{4} + 3x^2 + c$

$$f(x) = 180x^{1/4} + \frac{x^5}{4} + 3x^2$$

$\cancel{f(1) = 180}$

Q7) $$dF(u,v) = \frac{\partial F(u,v)}{\partial u} du + \frac{\partial F(u,v)}{\partial v} dv$$

Q8) $$\frac{dr}{d\theta} = \frac{r^2}{\theta} \qquad r(1) = 2$$

$\cancel{r(\theta) = -\frac{1}{\ln(\theta) + C}}$

$$\int_2^r u^{-2} \cdot du = \int_1^\theta v^{-1} dv$$

$$\frac{1}{4}\Big[\ \Big]_2^r = \Big[\ln u\Big]_1^\theta$$

$$\frac{1}{4}\Big[\ \Big]_2^r = -\Big[\ln u\Big]_1^\theta$$

$$\frac{1}{r} - \frac{1}{2} = -\left[\ln\theta - \ln 1\right] = \ln\theta$$

$$\frac{1}{r} = -\ln\theta + \frac{1}{2}$$

PAGE No
DATE : / 202

$$\frac{1}{\gamma} = -\ln(\theta) + \frac{1}{2}$$

$$\frac{1}{\gamma} = \frac{1 - 2\ln(\theta)}{2}$$

$$\gamma = \frac{2}{1 - 2\ln(\sigma)}$$

$$0 < \theta$$

$$1 - 2\ln\theta \neq 0$$

$$1 \neq 2\ln(\theta)$$

Q9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\int \frac{dv}{dt} =$ $IF = e^{\int 0.196 dt}$

$= \int -0.196 \cdot dt$
$= -0.196t$

$ve^{(-0.196t)} = -9.8 \times \frac{e^{(-0.196n)}}{-0.196}$

$= \frac{9800}{196} e^{-0.196}$

$= 50 + e^{0.196} + c$

PAGE No
DATE / 202

Q10) $ty' + 2y = t^2 - t + 1$

Q11) $t\frac{dy}{dx} + 2y = t^2 - t + 1$

Q12) $f(x) = x^3 - 10x^2 + 6 \quad x = 3$

$$\sum_{n=0} \frac{f^n(a)}{n!}(x-a)^n, \; 6^n$$

$f(x) = x^3 - 10x^2 + 6$
$f'(x) = 3x^2 - 20x$

$f''(x) = 6x - 20$

$f'''(x) = 6$

$f'''' = 0$

$$f(x) = x^3 - 10x^2 + 6 = \frac{-57}{0!}(x-3)^0$$
$$+ \frac{-33}{1!}(x-3)^1 + \frac{-2}{2!}(x-3)^2 + \frac{6}{3!}(x-3)^3$$

$\therefore f(x) = x^3 - 10x^2 + 6$
Taylor $\rightarrow f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$

Q13) $f(x) = (1+x)^{\alpha}$

$$(1+x)^{\alpha} = 1 + \alpha x + \frac{1}{2}(\alpha-1)\alpha x^2 + \frac{1}{6}(\alpha-2)(\alpha-1)\alpha x^3 + \dots$$

$$(1+x)^{\alpha} = 1 + \sum_{k=1}^{n} \binom{\alpha}{k} x^k + o(x^n)$$

Q14) $\sqrt{1+x}$ $\quad x = 0$

$f(x) = 1$

$f(x) = \cancel{f(0)} \ (1+x)^{1/2}$

$a = \frac{1}{2}$

$f'(x) = a(1+x)^{a-1}$

$f''(x) = a(a-1)(1+x)^{a-2}$

$f^{k}(x) = a(a-1)\dots\dots(a-k+1)(1+x)^{a-k}$

$$\left| \frac{\binom{a}{k+1} x^{k+1}}{\binom{a}{k} x^k} \right| = \left| \frac{a-k}{k+1} x \right|$$

$a = \frac{1}{2}$

PAGE No
DATE : / 202

2) $\frac{S(n-1)}{(n)^2 n^3}$ + ...

$$\binom{1/2}{k} = \frac{1}{k!}\,\frac{1}{2}\left(\frac{1}{2}-1\right)\cdots\left(\frac{1}{2}-k+1\right)$$

$$= \frac{(-1)^{k-1}}{k!} \times \frac{1}{2^k}\,\frac{1.\,2.\,3.\,\ldots\,(2k-2)}{2.\,4.\,6(2k-2)}$$

$$= \frac{(-1)^{k-1}}{k!}\,\frac{1}{2^{k}}\,\frac{(2k)!\;2}{(k-1)!\;(2k-1)(2k)}$$

$$= \frac{(-1)^{k-1}\,2k!}{2^{2k}(k!)^2(2k-1)}$$

Q15) $f(x) = e^{-6x}$ $\quad x = -4$

$$\sum_{n=0}^{N} \frac{f^{(n)}a}{n!}(x-a)^n$$

$$f(x) = e^{-6x}$$

$$f'(x) = -6e^{-6x}$$

$$f''(x) = 36e^{-6x}$$

$$f'''(x) = -216e^{-6x}$$

$$f''''(x) = 1296e^{-6x}$$

$$= \frac{e^{-6x}}{0!} - \frac{6e^{-6a}}{1!}(x-a)^1 + -\frac{36e^{-6a}}{2!}(x-a)^2 +$$

$$-\frac{216e^{-6x}}{3!}(x-a)^3 + \frac{1296e^{-6x}}{4!}(x-a)^4$$