PROBABILITY & STATS ASSIGNMENT

Q. 1

(i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$$\bar{X} = \frac{x_1 + x_2 + x_3 \dots + x_n}{n}$$

$= 4+7+9+9+11+15+18+18+18+25 / 10$

$= 13.4$

(ii) Median: $\left(\frac{n+1}{2}\right)^{th}$ term + $\left(\frac{n}{2}\right)^{th}$ term

$\Rightarrow$ (5)th term & (6)th term

$\Rightarrow$ 11 + 15 / 2 = 26/2 $\Rightarrow$ 13

(iii) Mode = 18 (highest frequency)

(iv) standard deviation:

~~x~~	X	f	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2 f$
~~14~~	4	1	13.4	-9.4	88.36
~~36~~	7	1		-6.4	40.96
~~4~~	9	2		-4.4	38.72
	~~9~~ 11	1		-2.4	5.76
	15	1		1.6	2.56
	18	3		4.6	63.48
	25	1		11.6	134.56
					324.4

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N-1}} = \sqrt{\frac{324.4}{10}} = 5.69$$

(v) $Q_1 = \left(\frac{N+1}{4}\right)^{th}$ term $\Rightarrow 2.75 \Rightarrow 7 + \frac{3 \times 9}{4} = 13.75$

$Q_3 = \frac{3}{4}\left(\frac{N+1}{4}\right)^{th}$ term $\Rightarrow 8.25 = ~~18~~.\ 22.5$

$IQR = Q_3 - Q_1 = 8.75$

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(ii)

number of hours (x)	number of people (f)	(cf)	$x_i f_i$	$(x-\bar{x})^2 f$
0	5	5	0	20
1	11	16	11	11
2	26	42	52	0
3	15	57	45	15
4	3	60	12	12
			120	58.

1) Mean $\frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{120}{60} = 20$

2) Median : $\left(\frac{60+1}{2}\right) = (30.5)^{th}$ item.

$= \text{value of } \frac{(30^{th} + 31^{th} \text{ item})}{2}$.

$= (2+2)/2 = 2$.

3) Mode = 2 (26 is highest frequency.

4) $\sigma = \sqrt{\frac{\sum_{i=1}^{n} f_i(x_i - \bar{x})^2}{N \text{ or } 1}}$

$= \sqrt{\frac{58}{60}} = 0.9831$.

5) $Q_3 = \frac{3}{4}(60+1) = (45.75)^{th}$ term $= 3$.

$Q_1 = \frac{61}{4} = 1$

$IQR = Q_3 - Q_1 = 3 - 1 = 2$.

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Q.2 $X \sim Poi(\lambda)$

$\lambda = 0.5$

(i) No. claims $\Rightarrow (X = 0)$

$P(X=0) = \frac{0.5^0 e^{-0.5}}{0!}$ $\quad \left(P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}\right)$

$= 0.60653$

(ii)(a) Probability of one or more claims in one year.

$\cancel{P(X \geq 1) = 1 - P(X < 1)}$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - 0.60653.$

$= 0.39347.$

~~(iii)~~ (b) This will be a binomial distribution :

$X \sim Bin(3, 0.39347)$

$P(X=x) = \binom{n}{x} p^x q^{1-x}$

$= {}^3C_1 (0.39347)(0.60653)^2$

$= 0.43425.$

(iii) For calculating the waiting period T (in terms of years)

~~has a~~ we will calculate exponential distribution

$\Rightarrow X \sim Exp(0.5)$

$P(T>2) = 1 - P(T \leq 2)$

$= 1 - F(2)$

$= 1 - (1 - e^{-\lambda x})$

$= 1 - 1 + e^{-0.5 \times 2}$

$= e^{-1}$

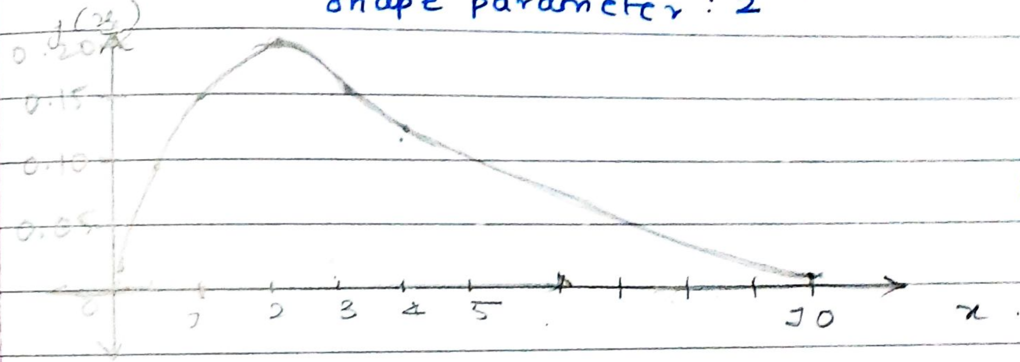
$= 0.36788.$

Q.3 Gamma $(a, \lambda) \Rightarrow$ Gamma $\left(2, \frac{1}{2}\right)$

(i) $E(x) = \dfrac{a}{\lambda} = \dfrac{2}{1/2} = 4$

$\sigma = \sqrt{V(x)} = \sqrt{\dfrac{a}{\lambda^2}} = \sqrt{\dfrac{2}{1/4}} = 2\sqrt{2} = 2.828$

(ii) Graph (scale parameter : 1/2
shape parameter : 2)



Its positively skewed.
(x can't take -ve values
& σ is large relative to mean.

$f_x(x) = \dfrac{(0.5)^2 x^1 e^{-0.5x}}{\Gamma(a) = 1!}$ $= \dfrac{1}{(1/2)}$

$f_x(1) = 0.25 e^{-0.5} = 0.15$, $f_x(0.5) = 0.092$, $f_x(0.1) = 0.024$

$f_x(2) = 0.25 \times 2 \times e^{-1} = 0.18$

$f_x(3) = 0.25 \times 3 \times e^{-0.5 \times 3} = 0.16$, $f_x(4) = 0.13$, $f_x(5) = 0.102$

$f_x(10) = 0.016$

iii $F_x(x) = \displaystyle\int_0^x f_x(x)\,dx$

$= \displaystyle\int_0^x \frac{\lambda^a x^{a-1} e^{-\lambda x}}{\Gamma(a)}\,dx \Rightarrow \int_0^x \frac{(0.5)^2 x e^{-0.5x}}{1!}$

$= (0.5)^2 \displaystyle\int_0^x x e^{-0.5x}\,dx$

$= 0.25 \left[\dfrac{x \cdot e^{-0.5x}}{-0.5} - \displaystyle\int \frac{e^{-0.5x}\,dx}{-0.5}\right] \Rightarrow 0.25 \left[\dfrac{x \cdot e^{-0.5x}}{-0.5} - \dfrac{e^{-0.5x}}{0.25}\right]_0^x$

$F_x(x) = +0.25\left(-\dfrac{x \cdot e^{-0.5x}}{+0.5} - \dfrac{e^{-0.5x}}{0.25} - 0 + \dfrac{1}{0.25}\right)$

$= 1 - e^{-1/2 x}\left(1 + \frac{1}{2}x\right)$

	Total	Fully Qualified
Males	10	6
Females	8	7

P(choosing 2 random workers) = $^{18}C_2$

$= 153$

P(2 Qualified females) $= \frac{^{7}C_2}{^{18}C_2} = \frac{21}{153} = \frac{7}{51}$

$\underbrace{\hphantom{P(2 \text{ Qualified females})}}_{E \cap F)}$

$P(\underbrace{X \geq 1}_{F}) = 1 - P(X=0)$

$= 1 - \left(\frac{8}{18}\right)^0 \left(\frac{10}{18}\right)^{18}$

$= 1 - (10/18)^{18}$

$\frac{P(E \cap F)}{P(F)} = \frac{7/51}{1-(5/9)^{18}} = 0.1372$

Q.5

$\Rightarrow$ $P(L_1) = 40\%$.

$P(L_2) = 50\%$.

$P(L_3) = 10\%$.

let x be event package is late.

$P(x|L_1) = 20\%$.

$P(x/L_2) = 40\%$.

$P(x|L_3) = 10\%$.

$$P(L_2|x) = \frac{P(x|L_2)\,P(L_2)}{P(x|L_2)P(L_2) + P(x|L_1)P(L_1) + P(x|L_3)P(L_3)}$$

$$= \frac{0.4 \times 0.5}{0.4\times0.5 + 0.1\times0.1 + 0.2\times0.4}$$

$$= \frac{0.2}{0.2 + 0.01 + 0.08}$$

$$= \frac{0.2}{0.29} \Rightarrow \frac{20}{29}.$$

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Q.~~8~~.

$(y = \omega(x))$

Q.6. $x \sim U(1, 2)$ $\qquad x = \omega^{-1}(y)$

$Y = 3/6 - x$

$$f_Y(y) = f_X(\omega^{-1}(y)) \left| \frac{d(\omega^{-1}(y))}{dy} \right|$$

$$x = 6 - 3/Y \qquad \frac{dx}{dy} = \frac{-3}{y^2} = \frac{d(\omega^{-1}(y))}{dy}$$

$$f_X(x) = 1/B-a = 1.$$

$$f_Y(y) = f_X(6 - 3/y)\left(\frac{3}{y^2}\right)$$

$$= \frac{3}{y^2}.$$

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Q.7

x	1	2	3	4	5
$P(x=x)$	0.05	0.15	0.35	0.4	0.05

(i) $F(3) = P(x \leq 3)$

$= P(x=1) + P(x=2) + P(x=3)$

$= 0.05 + 0.15 + 0.35.$

$= 0.55$

(ii) $E(x) = \sum_{i=1}^{5} x_i \cdot p_i(X=x)$

$= 0.05 + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$

$= 0.05 + 0.3 + 1.05 + 1.6 + 0.25$

$= 3.25$

(iii) $V(x) = E(x^2) - (E(x))^2.$

$= 0.05(1) + (0.15)(2)^2 + (0.35)(3^2) + (0.4)4^2 + 0.05(5$

$= 0.05 + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)$

$= 11.45 - (3.25)^2$

$= 0.8875.$

iv coe

(iv) coefficient of skewness $= \dfrac{\bar{X} - \text{Mode}}{\sigma}.$

$= \dfrac{3.25 - 8.0,05}{3.383},$

$= 0.945906$

Q. 8)

$\Rightarrow$ Let X be claims which exceed 1000.

$X \sim Bin(15, 0.2)$

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(X=0) = \binom{15}{0} (0.2)^0 (0.8)^{15}$$

$$P(X=1) = \binom{15}{1} (0.2)(0.8)^{14}$$

$$P(X=2) = \binom{15}{2} (0.2)^2 (0.8)^{13}$$

Probability that fewer than 3 claims exceed 1000

$\Rightarrow P(X=0) + P(X=1) + P(X=2)$

$\Rightarrow$ 0.3980

Q.9 $p = 0.01$.

$x = 7$

$k = 3$.

$$P(X=x) = \binom{x-1}{k-1} (p)^k (1-p)^{x-k}.$$

$$= \binom{6}{2} (0.01)^3 (0.99)^4$$

$$= 0.0000144089$$

.10 $X \sim Poi(\lambda) \Rightarrow X \sim Poi(2)$

$$P(X > 2) = 1 - P(X \leq 2)$$

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}.$$

$$P(X = 0) = \frac{(2)^0 e^{-2}}{0!}. = e^{-2}.$$

$$P(X = 1) = \frac{(2)^1 e^{-2}}{1!} = 2e^{-2}.$$

$$P(X = 2) = \frac{(2)^2 e^{-2}}{2!} = 2e^{-2}.$$

$$P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - (P(X=0) + P(X=1) + P(X=2))$$

$$= 1 - (5e^{-2})$$

$$= 0.3233.$$

Q.11.

$\Rightarrow$ Due to memoryless property, the children she has are irrelevant

By using geometric distribution

$P(X=3) = \left(\frac{1}{2}\right)\left(1-\frac{1}{2}\right)^{3-1}$. $\quad (P(X=x) = p(1-p)^{x-1})$

$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Q.12 Pois $(\lambda) \Rightarrow (\lambda = 10)$

$P(x < 0.1) = F(0.1)$

$= 1 - e^{-x \times \lambda}$.

$= 1 - e^{-0.1 \times 10}$.

$= 1 - e^{-1}$

$= 0.63212$.

Q.13. $\alpha = 75$

$\beta = 120.$

$f_x(x) = \frac{1}{\beta - \alpha} = \frac{1}{45}$

$P(80 \le x \le 100) = \int_{80}^{100} \frac{1}{b-a} \, dx.$

$\Rightarrow \frac{1}{45}(100 - 80) = \frac{20}{45} = \frac{4}{9}.$

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Q. 14 $X \sim \text{Gamma}(\underset{\downarrow \atop a}{5}, \underset{\downarrow \atop \lambda}{0.4})$

We know $2\lambda X \sim \chi^2_{10}$

Find : $P(X < 22.89)$

$= P(0.8X < 18.312)$

$= P(\chi^2_{10} < 18.312)$

$18.312 = 0.9988 + \frac{(0.312)}{0.5}(0.9990 - 0.9988)$

$= \cancel{0.9988} + 0.9989248.$

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Q.15 $\frac{k}{(x+5)^4}$, $x>0$.

(i) $k = ?$

$$\int_0^\infty \frac{k}{(x+5)^4}\,dx = 1.$$

$$k\int_0^\infty \frac{(x+5)^{-4+1}}{-3}\,dx = 1.$$

$$\Rightarrow \frac{k}{-3}\left((x+5)^{-3}\right)_0^\infty d = 1.$$

$$\frac{k}{-3}\left(\frac{1}{\infty} - \frac{1}{(0+5)^3}\right) = 1.$$

$$\frac{k}{3}\left(\frac{1}{(5)^3}\right) = 1 \Rightarrow k = 3\times 125 = 375$$

(ii) $F(10) = P(X \le 10) = \int_0^{10} d(t)\,dt$

$$\Rightarrow \int_0^{10} \frac{375}{(x+5)^4}\,dx \Rightarrow \int_0^{10} \frac{375}{-3}\left(\frac{1}{(x+5)^{+3}}\right)$$

$$\Rightarrow \int_0^{10} \frac{375}{-3}\left(\frac{1}{(15)^3} - \frac{1}{(5)^3}\right) \Rightarrow -125\left(\frac{1}{5^3(3)^3}\right) + 125\left(\frac{1}{5^3}\right)$$

$$= 1 - \frac{1}{27} = \frac{26}{27}.$$

(iii) $$E(X) = \int_0^\infty \frac{x.(375)}{(x+5)^4} = 375\int_0^\infty \left(\frac{1}{(x+5)^3} - \frac{5}{(x+5)^4}\,dt\right)$$

$$\Rightarrow 375\left[\frac{(x+5)^{-2}}{-2} - \frac{5}{-3}(x+5)^{-3}\right]_0^\infty$$

$$= 375\left(+\frac{1}{2}\left(\frac{1}{25}\right) + \frac{5}{3}\left(\frac{1}{125}\right)\right] \Rightarrow -5 + \frac{15}{2} = \frac{5}{2}$$