

NUMERICAL METHODS

AND ALGEBRA

ASSIGNMENT 1

DRASHTI SHAH 30

Q1. $\sqrt{7+\sqrt{7+\sqrt{7+\dots}}}$

Let $\sqrt{7+\sqrt{7}} = n$

Squaring both sides

$$7 + \sqrt{7+n} = n^2$$

$$\sqrt{7+n} = n^2 - 7$$

$$7+n = (n^2 - 7)^2$$

$$n^4 - 7n^2 - n + 42 = 0$$

Q2 $7x^2 - 6y^2 = -11x^2 - 4y^2$

$$7x^2 + 11x^2 = -4y^2 + 6y^2$$

$$\frac{x^2}{y^2} = \frac{2}{18}$$

$$\left(\frac{x}{y}\right)^2 = \frac{1}{9}$$

$$\frac{x}{y} = \pm \frac{1}{3}$$

Q3

$$x + 3y = 4$$

$$x^2 + y^2 - 4y = 12 \quad \text{--- (i)}$$

$$x = 4 - 3y \quad \text{--- (ii)}$$

Substituting (ii) in (i)

$$(4 - 3y)^2 + y^2 - 4y = 12$$

$$16 + 9y^2 - 24y + y^2 - 4y = 12$$

$$5y^2 - 14y + 2 = 0$$

$$y = \frac{14 \pm \sqrt{196 - 40}}{10}$$

$$y = 2.6489$$

or

$$y = 0.1510$$

Q4

$$(a) \quad d = 2(3.5)^2 - 16(3.5) + 34$$

$$d = 2.5$$

(millions of particles per cubic foot)

$$(b) \quad \frac{d}{dx}(d) = 4x - 16$$

$$x = 4 \text{ when } \frac{d}{dx} = 0$$

$$\frac{d^2}{dx^2}(d) = 4$$

 $\therefore$ Minimum $x = 4$ per min. (in hundred)

$$(c) \quad 100 = 2x^2 - 16x + 34$$

$$0 = 2x^2 - 16x - 66$$

$$u = \frac{16 \pm \sqrt{256 + 528}}{4}$$

$u = 11$ or $u \neq -3$

$\therefore$ The speed cannot be negative

$\therefore$ The u is 11.

Q5

(a) Company 1 : $1500 + 25n > 0$

(b) Company 2 : $2100 + 22n > 0$

(c) (d) $1500 + 25n < 2100 + 22n$

$3n < 600$

$n < 200$

Q6

$x^3 - 7x^2 + 14x - 6 = 0$

(a) $[0, 1]$

a	b	t	f(t)
0	1	1.5	2.625
0	1.5	0.75	0.984375
0	0.75	0.375	-1.68164
0.375	0.75	0.5625	0.16186
0.5625	0.75	0.65625	0.45547

(b) $[1, 3.2]$

a	b	t	f(t)
3.2	1	2.1	1.791
3.2	2.1	2.65	0.55215
3.2	2.65	2.925	0.8582
3.2	2.925	<u>3.0625</u>	-0.0544

(c) $[3.2, 4]$

a	b	t	f(t)
3.2	4	3.6	0.3336
3.2	3.6	3.4	-0.016
3.4	3.6	3.5	0.125
3.4	3.5	3.45	10.046125
3.4	3.45	3.425	0.01302
3.4	3.425	3.4125	-1.99×10^{-3}
3.4125	3.4125	<u>3.4125</u>	5.35×10^{-3}
3.4125	3.41875		

Q7 $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$
 $f'(x) = 920x^3 + 54x^2 + 18x - 221$

(i) $[-1, 0]$

$x_1 = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = 0 - \frac{(-9)}{(-221)} = \frac{9}{221} \approx 0.040723$$

$$x_2 = -0.040723$$

$$x_3 = -0.0409288$$

$$x_4 = -0.04065988$$

Q8

$$(a) \quad \frac{70+86+81+83+x}{5}$$

$$= \frac{320+x}{5}$$

$$= 4 \quad (\text{mean})$$

$$(b) \quad 90 > \frac{320+x}{5} \geq 80$$

$$(c) \quad 450 > 320+x \geq 400$$

$$130 > x \geq 80$$

Q9

$$x_1 = 4\%$$

$$x_2 = 20\%$$

$$@ \text{IRR}, y=0$$

$$y_1 = 8.54$$

$$y_2 = -11.81$$

$$\frac{x-4}{20-4} = \frac{0-8.54}{-11.81-8.54}$$

$$x = 10.71\%$$

Q10 $f(x) = 32x - 0.21x^2$
 $c(x) = 195 + 12x$

As,

$$R(x) - c(x) = \text{Profit ie. } f(x)$$

$$f(x) = 32x - 0.21x^2 - 195 - 12x$$

$$-0.21x^2 + 20x - 195 = 0$$

$$x = \frac{20 \pm \sqrt{(-20)^2 - 4 \times (0.21) \times 195}}{2 \times 0.21}$$

$$x = 84.2114$$

or

$$x = 11.0266$$

$$f'(x) = 0.42x - 20 > 0$$

$$0.42x > 20$$

$$x > 47.61$$

as the order should be more than 47.61 then the value is 84.2114.