PRACTICE QUESTIONS

1) $\int_{1/50}^{2} \frac{e^{2/\omega}}{\omega^2} \, d\omega$

$2/\omega = \cancel{d}\, t.$

$dt = -2/\omega^2 \, d\omega$

$d\omega = -\omega^2/2 \, dt.$

$\int_{1/50}^{2} \frac{-1}{2} e^{t} \, dt \Rightarrow \int_{1/50}^{2} \left[e^{t} \right].$

$\Rightarrow \frac{-1}{2} \left(e^{2/\omega} \right)_{1/50}^{2}$

$\Rightarrow \frac{-1}{2} \left(e^{1} - e^{1/25} \right) \Rightarrow -\frac{1}{2} (2.718 - 1.041)$

$\Rightarrow -0.8345.$

2) $\int \frac{2t^3 + 1}{(t^4 + 2t)^3} \, dt.$

$\Rightarrow$ let $t^4 + 2t = x.$

$dx = 4t^3 + 2 \Rightarrow 2(2t^3 + 1) \, dt.$

$\Rightarrow \int \frac{1}{\cancel{t+4}\, x^3} \frac{dx}{2} \Rightarrow \int \frac{x^{-3}}{2} dx$

$\Rightarrow \frac{x^{-3+1}}{-2 \times 2} \Rightarrow \frac{x^{-2}}{-4} + C$

$\Rightarrow -\frac{1}{4x^2} + C$

3) $f'(x) = x^4 + 3x - 9.$

$\int f'(x)\,dx = f(x)$

$f(x) = \int x^4 + 3x - 9\,dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

4) $f(x) = \begin{cases} 6 & \text{if } x > 1. \\ 3x^2 & \text{if } x \le 1 \end{cases}$

$\int_{-2}^{3} f(x)\,dx \Rightarrow \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$

$\Rightarrow \left(\frac{3x^3}{3}\right)_{-2}^{1} + \left(6x\right)_{1}^{3}.$

$\Rightarrow (1 - (-8) + (18 - 6))$

$\Rightarrow 11$

5) $\int 3(8y-1)e^{4y^2-y} \cdot dy.$

let $4y^2 - y = t.$

$dt = (8y-1)\,dy.$

$\Rightarrow \int 3e^t\,dt$

$\Rightarrow 3e^t$

$\Rightarrow 3(e^{4y^2-y})$

6) $f(x)$ when,

$$f''(x) = 15\sqrt{x} + 5x^3 + 6 \quad , f(1) = \frac{-5}{4} \quad , f(4) = 404$$

$$\Rightarrow \int f''(x)\,dx = f'(x)$$

$$\Rightarrow \int f''(x)\,dx = \frac{2\times 15}{3} x^{3/2} + \frac{5x^4}{4} + 6x + c_1$$

$$\int f''(x)\,dx = 10x^{3/2} + \frac{5x^4}{4} + 6x + c_1$$

$$\Rightarrow \int f'(x)\,dx \Rightarrow f(x)$$

$$f(x) = \frac{2\times 10}{5} x^{\frac{5}{2}} + \frac{5x^5}{20} + \frac{6x^2}{2} + c_1 x + c_2$$

$$= 4x^{5/2} + \frac{1}{4}x^5 + 3x^2 + c_1 x + c_2$$

$$f(1) = 4 + \frac{1}{4} + 3 + c_1 + c_2 = \frac{-5}{4}$$

$$c_1 + c_2 = \frac{-5}{4} - \frac{1}{4} - \frac{28}{4} \Rightarrow \frac{-17}{2}.$$

$$f(4) = 4(4)^{5/2} + (4)^4 + 3(4)^2 + 4c_1 + c_2 = 404$$

$$= 4(32) + 256 + 48 + 4c_1 + c_2$$

$$4c_1 + c_2 = -28$$

$$-3c_1 \Rightarrow 19.5$$

$$c_1 \Rightarrow -6.5$$

$$c_2 \Rightarrow -11$$

7) $f(x+iy) + g(x-iy) = z$

using Langrgian Multiplier :

$$\nabla f(x,y) = \lambda \nabla g(x,y)$$

$$p = \frac{\partial z}{\partial x} = f'(x+iy) + g(x-iy)$$

$$q = \frac{\partial z}{\partial y} = if'(x+iy) - ig(x-iy)$$

$$r = \frac{\partial^2 z}{\partial x^2} = f''(x+iy) + g''(x-iy)$$

$$t = \frac{\partial^2 z}{\partial y^2} = i^2 f''(x+iy) + i^2 g''(x-iy)$$

$$\Rightarrow -f''(x+iy) - g''(x-iy)$$

$r+t=0$ is the required P.d.e.

8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$, $r(1)=2$.

$$\frac{dr}{r^2} = \frac{d\theta}{\theta}$$ (using variable seperable)

$$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$$

$\Rightarrow$ $-1r^{-1} = \log\theta + c.$

$\Rightarrow$ $\frac{-1}{r} = \log\theta + c$

$\Rightarrow$ $r = \frac{-1}{\log(1)+c} = 2 \Rightarrow c = \frac{-1}{2}$

$\Rightarrow$ $r = \frac{-2}{+2\log\theta + 1}.$

9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\frac{dv}{dt} + 0.196v = 9.8.$

$P = 0.196$ & $Q = 9.8$

$IF = e^{\int 0.196\,dt} \Rightarrow e^{0.196t}$

$v\,IF = \int Q \cdot IF\,dt.$

$e^{0.196} v = \int e^{0.196t} \cdot 9.8 \cdot dt$

$v.(e^{0.196t}) = \frac{9.8 \cdot e^{0.196t}}{0.196} + c$

$v.\, e^{0.196t} = 50\, e^{0.196t} + c.$

$v = 50 + ce^{-0.196t}$

10) ~~ty' + 2~~ $ty' + 2y = t^2 + t + 1.$, $y(1) = 1/2.$

$y' + \frac{2y}{t} = t + 1 + \frac{1}{t}.$

$u(t) = \int 2/t\,dt = 2\ln t.$

$y(t) = e^{2\ln t} \int e^{-2\ln t}(t - 1 + 1/t)$

$= t^2 \int \left(\frac{1}{t^2}\right)\left(t - 1 + \frac{1}{t}\right)$

$y(t) = t^2\left(\ln t + \frac{1}{t} - \frac{1}{2t^2} + c\right)$

$y(1) = 2$

$1 - \frac{1}{2} + c = 2 \Rightarrow c = \frac{3}{2}.$

$y(t) = t^2\left(\ln t + \frac{1}{t} - \frac{1}{2t^2} + \frac{3}{2}\right)$

11)

$$y' = \frac{xy^3}{\sqrt{1+x^2}} \qquad y(0) = -1.$$

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}.$$

$$\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}}\,dx.$$

let $1+x^2 = t$.

$dt = 2x\,dx$.

$$\int y^{-2}\,dy = \int t^{-\frac{1}{2}} \frac{dt}{2}.$$

$$-y^{-1} = \frac{+2}{2} t^{1/2} + c$$

$$y(t) = \sqrt{t} + c$$

$$y(x) = \sqrt{1+x^2} + c.$$

$$y(0) = \sqrt{1+0} + c = -1$$

$$c = -1.$$

$$\frac{1}{y} = 2.\sqrt{1+x^2} + 1.$$

$$y = \frac{1}{\sqrt{1+x^2} - 1}.$$

12)

12) $f(x) = x^3 - 10x^2 + 6$, $x = 3$.

let $n = 0$, $f(x) = x^3 - 10x + 6$, $f(3) = 3$

$n = 1$, $f'(x) = 3x^2 - 10$, $f'(3) = 17$.

$n = 2$, $f''(x) = 6x$, $f''(3) = 18$

$n = 3$, $f'''(x) = 6$, $f'''(3) = 6$

$n = 4$, $f''''(x) = 0$. $f''''(3) = 0$.

we dont consider n ahead 4 cause all values are 0.

Taylor = $\displaystyle f \sum_{x=0}^{\infty} \frac{f^n(a)(x+a)^n}{n!}$

$= f(a) + f'(a)(x+a) + \frac{f''(a)(x+a)^2}{2!} + \frac{f'''(a)(x+a)^3}{3!}$.

$= 3 + 17(x+3) + \frac{18(x+3)^2}{2} + \frac{6(x+3)^3}{6}$.

$= 3 + 17(x+3) + 9(x+3)^2 + (x+3)^3$

13) $(1+x)^{\mu} = f(x)$

$n=0$, $f(x) = (1+x)^{\mu}$
$n=1$, $f'(x) = +\mu(1+x)^{\mu-1}$.
$n=2$, $f''(x) = +\mu(\mu-1)(1+x)^{\mu-2}$
$n=3$, $f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$
$n=4$, $f''''(x) = \mu(\mu-1)(\mu-2)(\mu-3)(1+x)^{\mu-4}$

Relation for general term $f^n(x)$:

$$\Rightarrow \frac{\mu!\,(1+x)^{\mu-n}}{(\mu-n)!}.$$

$f(0) = 1.$
$f''(0) = \mu$
$f'''(0) = \mu(\mu-1)$
$f''''(0) = \mu(\mu-1)(\mu-2)$
$f''''(0) = \mu(\mu-1)(\mu-2)(\mu-3)$

Mclauren $\rightarrow \sum_{x=0}^{\infty} \frac{f^n(0)\, x^n}{n!}$

$$\Rightarrow \frac{\mu!}{(\mu-n)!} \times \frac{x^n}{n!}$$

$$f(x) = 1 + \mu x + \frac{\mu(\mu-1)x^2}{2!} + \frac{\mu(\mu-1)(\mu-2)x^3}{3!} \ldots$$

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14) $f(x) = \sqrt{1+x}$.

$n=1,\ f'(x) = \frac{1}{2}.(1+x)^{1/2-1}$, $f'(0) = \frac{1}{2}$.

$n=2,\ f''(x) = -\frac{1}{4}.(1+x)^{-3/2}$, $f''(0) = \frac{-1}{4}$

$n=3,\ f'''(x) = +\frac{3}{8}.(1+x)^{-5/2}$, $f'''(0) = \frac{3}{8}$

$n=4,\ f''''(x) = -\frac{15}{16}(1+x)^{-7/2}$, $f''''(0) = \frac{-15}{16}$.

Mclauren $\rightarrow$ $\sum_{x=0}^{\infty} \frac{f^n(0)\,x^n}{n!}$

$\Rightarrow f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f''''(0)x^4}{4!} \ldots\ldots$

$\Rightarrow 1 + \frac{1}{2}x + -\frac{1}{4}.\frac{x^2}{2!} + \left(\frac{3}{8}\right)\frac{x^3}{3!} + \left(\frac{-15}{16}\right)\frac{x^4}{4!} \ldots\ldots$

$\Rightarrow 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}.x^3 - \frac{5}{108}x^4 \ldots$

15) $f(x) = e^{-6x}$, $x = -4$

$n=0$, $f(x) = e^{-6x}$, $f(4) = e^{-24}$
$n=1$, $f'(x) = -6e^{-6x}$, $f(4) : -6e^{-24}$
$n=2$, $f''(x) = +36e^{-6x}$, $f(4) = +36e^{-24}$
$n=3$, $f'''(x) = -216e^{-6x}$, $f(4) = -216e^{-24}$
$n=4$, $f''''(x) = 1296e^{-6x}$, $f(4) = 7296e^{-24}$

General term $\rightarrow f^n(x)$

$$f^n(x) = e^{-6x} (\cancel{x=6} \sum_{x=0}^{n} (-6)^n e^{-6x}.$$

Taylor $\Rightarrow$ $$\sum_{x=0}^{\infty} \frac{f^n(4)(x+4)^n}{n!}$$

$$\Rightarrow \frac{(-6)^n e^{-24} (x+4)^n}{n!}$$

$$\Rightarrow e^{-24} - 6e^{-24}(x+4) + \frac{36e^{-24}(x+4)^2}{2!} + \frac{-216e^{-24}(x+4}{3!}$$

$$\Rightarrow e^{-24} - 6e^{-24}(x+4) + 38e^{-24}(x+4)^2 - 36e^{-24}(x+4)^3 + 54(x+$$

16) $f(x) = \ln(4x+3)$, $x = 0$

$n=0$, $f(x) = \ln(4x+3)$

$n=1$, $f'(x) = (4x+3)^{-1}(4)$

$n=2$, $f''(x) = -(4)^2(4x+3)^{-2}$

$n=3$, $f'''(x) = (4x)^3(-1)(-2)(4x+3)^{-3}$

$n=4$, $f''''(x) = (-1)(-2)(-3)(4x+3)^{-4}(4)^4$

General term $f^n(x)$

$$f^n(x) = \sum_{x=\cancel{0}\,1}^{\infty} -(\cancel{-}4)^n (4x+3)^{-n} \times n!$$

$$f^n(0) = \sum_{x=1}^{\infty} -(-4)^n (3)^{-n} \times \cancel{n!}\,(n-1)!$$

Mclauren : $\displaystyle\sum_{x=0}^{\infty} \frac{f^n(0)\,x^n}{n!}$

$\Rightarrow$ $\ln(\cancel{3}) + \dfrac{4x}{3} + \dfrac{(16)(-1)x^2}{3^2\,2!} + \dfrac{(64)}{3^3} \times \dfrac{x^3}{3!} \times 2! + \dfrac{(-256)}{3^4} \times \dfrac{x^4}{4!} \times 3!$

$\Rightarrow$ $\ln(3) + \dfrac{4x}{3} - \dfrac{\cancel{16}\,8x^2}{3^2} + \dfrac{\cancel{64}x^3}{3^4} - \dfrac{64x^4}{3^4} \ldots\ldots$

$\Rightarrow$ $\cancel{f(0)}$