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PROBABILITY & STATISTICS 1, ASSIGNMENT 1.

Q1. Calculate the mean, median, mode standard deviation and inter quartile range of the following data :

i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$$\text{mean} = \frac{\Sigma x}{n} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$= \frac{134}{10} = 13.4$$

$$\text{median} = \frac{n/2^{th} + (n+2)/2^{th}}{2} \text{ term}$$

$$= \frac{10/2 + 12/2}{2} = \frac{5^{th} + 6^{th}}{2} \text{ term}$$

$$= \frac{11+15}{2} = 13$$

mode = 18 as the frequency of the number is highest which is 3.

$$\text{Variance} = \frac{\Sigma (x-\bar{x})^2}{n-1}$$

x	4	7	9	9	11	15	18	18	18	25
$(x-\bar{x})^2$	88.36	40.96	19.36	19.36	5.76	2.56	21.16	21.16	21.16	134

$$\Sigma (x-\bar{x})^2 = 374.4$$

$$\text{Variance} = \frac{374.4}{9} = 41.6$$

$$\text{S.D.} = \sqrt{\text{Variance}} = \sqrt{41.6} = 6.4498$$

$$IQR = \frac{3(n+1)}{4}^{th} - \frac{(n+1)}{4}^{th} \text{ term}$$

$$= 8.25^{th} - 2.75^{th} \text{ term}$$

$$= 18 + \frac{(18-18)}{100} \times 25 - 7 + \frac{(9-7)}{100} \times 75$$

$$= 18 - 8.5$$

$$= 9.5$$

ii)

x	f	cf	fx	cfx	$x-\bar{x}$	$(x-\bar{x})^2$
0	5	5	0	0	-2	4
1	11	16	11	11	-1	1
2	26	42	52	63	0	0
3	15	57	45	108	1	1
4	3	60	12	120	2	4

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{120}{60} = 2$$

$$\text{Median} = \frac{(n/2)^{th} + (n+2)/2^{th}}{2} \text{ term}$$

$$= \frac{60/2^{th} + 62/2^{th}}{2} \text{ term}$$

$$= \frac{(30 + 31)^{th}}{2} \text{ term} = \frac{2+2}{2} = 2$$

Mode = 2 as the frequency of that number is highest which is 26.

$$\text{Variance} = \frac{\Sigma f(x-\bar{x})^2}{\cancel{n-1}\,\Sigma f} = \frac{58}{60} = 0.9667$$

$$SD = \sqrt{Var} = \sqrt{0.967} = \cancel{1.054}\ 0.9831$$

$$IQR = Q3 - Q1 = \frac{3(n+1)}{4} - \frac{(n+1)}{4} = 45.75^{th} - 15.25^{th}$$

$$= 3 - 1 = 2$$

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Q2 Poison (0.5) = Poisson (μ)

i) Determine the probability that no claims arise in a single year.

~~$\lambda = 0$~~ $x = 0$

$$P(X=0) = \frac{e^{-0.5} \times 0.5^{0}}{0!} = 0.60653$$

ii) Determine the probality that in three consecutive years, there is one or more claims in one of the years and no claims in each of the other two years.

$$P(X=x) = P(X \geq 1) + P(X=0) + P(X=0)$$

iii)

Q3. $X \sim \text{gamma}(\alpha, \lambda)$

$\alpha = 2$, $\lambda = 1/2$; $0 < x < \infty$.

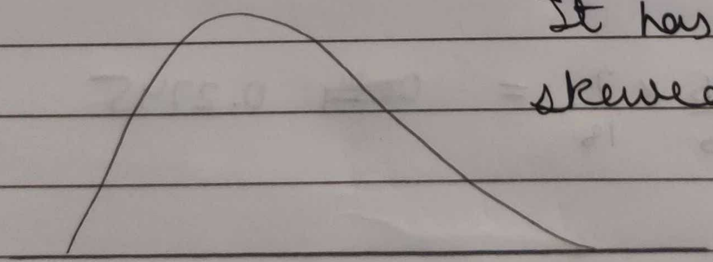
i) State mean and standard deviation of this distribution.

$$E(X) = \frac{\alpha}{\lambda} = \frac{2}{1/2} = 4$$

$$V(X) = \frac{\alpha}{\lambda^2} = \sigma^2$$

$$\sigma = \sqrt{\frac{\alpha}{\lambda^2}} = \sqrt{\frac{2}{0.25}} = \sqrt{8} = 2.828$$

ii) Hence comment briefly on its shape



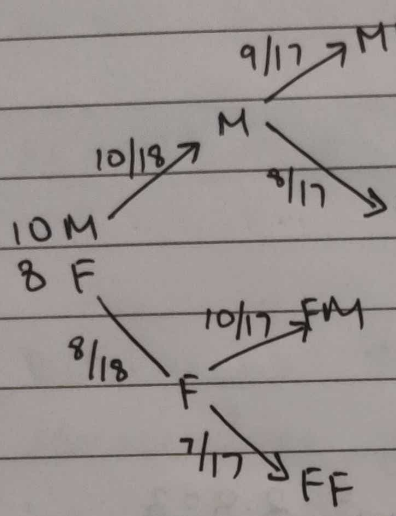
It has a positively skewed curve.

iii) $F_X(x) = 1 - e^{-\lambda x}$

$F_X(x) = 1 - e^{-0.5x}$

Q4. $P(M) = \frac{10}{18}$, $P(F) = \frac{8}{18}$, $P(MQ) = \frac{6}{18}$, $P(FQ) = \frac{7}{18}$

calculate



$\rightarrow P(MM) = \frac{10}{18} \times \frac{9}{17} = \frac{90}{306}$

$\rightarrow P(MF) = \frac{10}{18} \times \frac{8}{17} = \frac{80}{306}$

$\rightarrow P(FM) = \frac{8}{18} \times \frac{10}{17} = \frac{80}{306}$

$\rightarrow P(FF) = \frac{8}{18} \times \frac{7}{17} = \frac{56}{306}$

$P(x) = P(FF) + P(MF) + P(FM)$

$= \frac{80}{306} + \frac{80}{306} + \frac{56}{306} = \frac{216}{306}$

$P(Q) = \frac{216}{306} \times \frac{7}{18} = 0.2745$

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Q5. $P(L1) = \frac{40}{100}$, $P(L2) = \frac{50}{100}$, $P(L3) = \frac{10}{100}$

$P(L1L) =$ ~~20~~ 0.08, $P(L2L) = 0.2$, $P(L3L) = 0.01$

calculating $P(L2|L)$

$$P(L2|L) = \frac{P(L2L)}{P(L)} = \frac{P(L2L)}{P(L2L) + P(L1L) + P(L3L)}$$

$$= \frac{0.2}{0.2 + 0.08 + 0.01} = 0.6896$$

Q6. $X \sim U(1,2)$, $Y = \frac{3}{6-x}$ determine PDF of Y.

$$X = 6 - \frac{3}{y}$$

Q7.

X	$P(X=x)$	$X \cdot P(X=x)$	$F(X)$	$X^2 \cdot P(X=x)$
1	0.05	0.05	0.05	0.05
2	0.15	0.3	0.35	0.6
3	0.35	1.05	1.4	3.15
4	0.4	1.6	3.0	6.4
5	0.05	0.25	3.25	1.25

i) $F(3) = 1.4$

ii) $E(X) = \sum_{\text{all } x} x \cdot P(X=x)$

$= 0.05 + 0.3 + 1.05 + 1.6 + 0.25$

$= 3.25$

iii) $V(X) = E(X^2) - E(X)^2$

$= 11.45 - (3.25)^2$

$= 11.45 - 10.5625$

$= 0.8875$

iv) Coefficient of skewness

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Q8. $n = 15$, $x = 3$, $P(x) = 0.2$

Calculate $P(X < 3)$

$$P(X<3) = P(0) + P(1) + P(2)$$
$$= \sum_{i=0}^{2} {}^{15}C_{x} \cdot (0.2)^{x} \cdot (0.8)^{15-x}$$

$$= (1 \times 1 \times 0.03518) + (15 \times 0.2 \times 0.04398) + (105 \times 0.04 \times 0.05497)$$
$$= 0.03518 + 0.13194 + 0.230874$$
$$P(X<3) = 0.397994$$

Q9.

Q10) P(success) = P(getting a claim) = $\frac{2}{5}$

$\cancel{x} = \{x > 2\}$

$P(x) = \cancel{\&}\ 1 - P(x<2)$

$= 1 - 0.99207$

$= 0.00793$

Q11) P(success) = P(having a boy) = 0.5

$n = 3$, $x = 3$.

$P(3) = {}^3C_3 \cdot \cancel{B}(0.5)^3 . (0.5)^0$

$P(3) = 1 \times 0.125 \times 1$

$P(3) = 0.125$

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Q12) ~~Ho~~ ~~&~~ $\lambda = 10$, $\alpha = 1$, $t = 0.1$ days.

$$\cancel{P(T < 0.1) = 1 - e^{-\lambda t}}$$
$$\cancel{= 1 - e^{-1}}$$
$$\cancel{= 0.632}12$$

$$F_x(0.1) = \lambda \cdot e^{-\lambda x} = 10 \cdot e^{-1} = 3.67879$$

Q13) $X \sim U(a, b)$; $a = 75$ & $b = 120$; $80 < X < 100$

$$f_x(x) = \frac{1}{b-a} = \frac{1}{120-75} = \frac{1}{45}$$

$$P(80 < x < 100) = \frac{100-80}{45} = 0.444$$

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Q14. If $X \sim$ Gamma $(5, 0.4)$, calculate $P(X < 22.89)$

$$P(x) = \frac{\lambda^{\alpha} \cdot e^{-\lambda x} \cdot x^{\alpha-1}}{\Gamma(\alpha)} \quad \text{where } \Gamma(\alpha) = (\alpha-1)!$$

$$P(x < 22.89) = \cancel{\int_{0}^{22.89}} \int_{22.89}^{0} \frac{0.01024 \times e^{-0.4x} \cdot x^4}{24}$$

$$P(x < 22.89) = \cancel{\int_{22.89}^{0}} \frac{0.01024}{24} \int_{22.89}^{0} e^{-0.4x} \cdot x^4$$

$$\cancel{= 0.00042667 \times \int \left[e^{-0.4x} (4x^3)\right]}$$

Q15) $P(X) = \frac{k}{(x+5)^4}$, $x > 0$

i) k , ii) $F(10)$, iii) $E(X)$