1. $\lim_{h \to 0} \dfrac{2(-3+h)^2 - 18}{h}$

$\Rightarrow = \lim_{h \to 0} \dfrac{2(9-6h+h^2)-18}{h}$

$= \lim_{h \to 0} \dfrac{18 - 12h + 2h^2 - 18}{h}$

$= \lim_{h \to 0} \dfrac{-12h + 2h^2}{h}$

$= \lim_{h \to 0} \dfrac{h(-12+2h)}{h}$

$= \lim_{h \to 0} 2h - 12$

Applying limits.

$= 2(0) - 12$

$= -12 //$

$$\boxed{\therefore \lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h} = -12}$$

2. Determine if the following is continous or discontinuous at (a) $x=4$, (b) $x=6$?

$$g(x)=\begin{cases} 2x & x<6 \\ x-1 & x\geq 6 \end{cases}$$

⇒ a] $x=4$

$\therefore g(4)=8$

$\therefore \lim\limits_{x\to 4^-} g(x)=\lim\limits_{x\to 4^-} 2a$

$= 2(4)$

$\lim\limits_{x\to 4^-} \quad = 8$ //

$\lim\limits_{x\to 4^+} g(x)=\lim\limits_{x\to 4^+} 2x$

$= 2(4)$

$= 8$

$\therefore$ L.H.S = R.H.S.

Hence $\lim\limits_{x\to 4} g(x)$ exisit.

$\boxed{\therefore \text{ The given function is Continuous at } x=4.}$

b] $x=6$

$\therefore g(6)=6-1$

$=5$

$\therefore \lim\limits_{x\to 6^-} g(x)=\lim\limits_{x\to 6^-} 2x$

$=2(6)$

$=12$

$\therefore \lim\limits_{x\to 6^+} g(x)=\lim\limits_{x\to 6^+} x-1$

$=\cancel{x}6-1$

$=5$

$\therefore \lim\limits_{x\to 6^-} g(x)\neq \lim\limits_{x\to 6^+} g(x)$

$\therefore \lim\limits_{x\to 6} g(x)$ does not exist.

$\boxed{\therefore \text{ The given function } g(x) \text{ is discontinous at } x=6.}$

3. $$\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$$

$$\Rightarrow = \lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$$

$$= \lim_{x\to 9} \frac{(x-9)(3+\sqrt{x})}{(9-x)} \qquad [\because (a-b)(a+b) = a^2-b^2]$$

$$= \lim_{x\to 9} \frac{-[\cancel{(9-x)}(3+\sqrt{x})]}{\cancel{(9-x)}}$$

$$= \lim_{x\to 9} -(3+\sqrt{x})$$

\# Applying limits.

$$= -3-\sqrt{9}$$

$$= -3-3$$

$$= -6 \;//$$

$$\boxed{\therefore \lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} = -6}$$

Page No.

Date __/__/__

4. Determine if the following function is continous or discontinuous at (a) $x=-1$, (b) $x=0$, (c) $x=3$?

$$f(x) = \frac{4x+5}{9-3x}$$

$\Rightarrow$ At $x=-1$, $f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{1}{12}$ //

$$\lim_{x\to -1} f(x) = \lim_{x\to -1}\left(\frac{4x+5}{9-3x}\right) = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12} \text{ //}$$

$\boxed{\therefore \text{ The function is continous at } x=1}$ //

At $x=0$, $f(0) = \left[\frac{4(0)+5}{9-3(0)}\right] = \frac{5}{9}$ //

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{4x+5}{9-3x}\right) = \left[\frac{4(0)+5}{9-3(0)}\right] = \frac{5}{9} \text{ //}$$

$\boxed{\therefore \text{ The function is continous at } x=0}$ //

At $x=3$, $f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0} = \infty$

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{4(3)+5}{9-3(3)} = \infty$$

$$\therefore \lim_{x\to 3} = f(3)$$

$\boxed{\therefore \text{ The function is continous at } x=3}$ //

Date __/__/____

5. $\lim_{x\to\infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$\Rightarrow = \lim_{x\to\infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$ $\left[\because \dfrac{e^a}{e^b} = e^{a-b}\right]$

$= \lim_{x\to\infty} \dfrac{\cancel{e^{4x}}[6 - e^{-6x}]}{\cancel{e^{4x}}[8 - e^{2x} + 3e^{-5x}]}$

$= \lim_{x\to\infty} \left[\dfrac{6 - \frac{1}{e^{6x}}}{8 - \frac{1}{e^{2x}} + \frac{3}{e^{5x}}}\right]$

Applying limits.

$= \dfrac{6 - \frac{1}{\infty}}{8 - \frac{1}{\infty} + \frac{3}{\infty}}$

$= \dfrac{6}{8}$

$= \dfrac{3}{4}$ //

$\boxed{\therefore \lim_{x\to\infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}} = \dfrac{3}{4}}$

Date __/__/__

6. Given function

$$g(y) = \begin{cases} y^2+5 & \text{if } y<-2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$$

Compute the following limits.

a) $\lim_{y\to 6} g(y)$

b) $\lim_{y\to 2} g(y)$

$\Rightarrow$ Given function

$$g(y) = \begin{cases} y^2+5 & \text{if } y<-2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$$

a) $\lim_{y\to 6} g(y) = \lim_{y\to 6} 1-3y$

$= \cancel{\lim_{y\to}} \ 1-3(6)$

$= 1-18$

$= -17$ //

$$\boxed{\therefore \lim_{y\to 6} g(y) = -17}$$

Date ___/___/___

b) $\lim_{y \to -2} g(y) = \lim_{x \to -2} 1-3y$

$= 1 - 3(-2)$

$= 1+6$

$= 7$ //

$$\boxed{\therefore \lim_{y \to -2} g(y) = 7}$$

7. Use the product rule to derivative of;

$f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

Differentiating w.r.t. t

$\therefore f'(t) = (4t^2-t)\frac{d}{dt}(t^3-8t^2+12) + (t^3-8t^2-12)\frac{d}{dt}(4t^2-t)$

$\therefore f'(t) = (4t^2-t)(3t^2-16t) + (t^3-8t^2+12)(8t-1)$

$\therefore f'(t) = 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12$

$$\boxed{\therefore f'(t) = 20t^4 - 132t^3 + 24t^2 + 96t - 12}$$ //

Date __/__/__

8. Use the Quotient Rule to find the derivative of;

$$R(\omega) = \frac{3\omega + \omega^4}{2\omega^2+1}$$

⇒ Differentiating w.r.t.x.

$$R'(\omega) = \frac{(2\omega^2+1)\frac{d}{d\omega}(3\omega+\omega^4) - (3\omega+\omega^4)\frac{d}{d\omega}(2\omega^2+1)}{(2\omega^2+1)^2}$$

$$\therefore R'(\omega) = \frac{2\omega^2+1(3+4\omega^3) - (3\omega+\omega^4)(4\omega)}{(2\omega^2+1)^2}$$

$$\therefore R'(\omega) = \frac{6\omega^2 + 8\omega^5 + 3 + 4\omega^3 - [12\omega^2 + 4\omega^5]}{(2\omega^2+1)^2}$$

$$\therefore R'(\omega) = \frac{6\omega^2 + 8\omega^5 + 3 + 4\omega^3 - 12\omega^2 - 4\omega^5}{(2\omega^2+1)^2}$$

$$\therefore R'(\omega) = \frac{4\omega^5 - 6\omega^2 + 3 + 4\omega^3}{(2\omega^2+1)^2}$$

$$\boxed{\therefore R'(\omega) = \frac{4\omega^5 + 4\omega^3 - 6\omega^2 + 3}{(2\omega^2+1)^2}}$$

Date __/__/__

Saathi

9. Differentiate;

$$f(x) = 2e^x - 8^x$$

$\Rightarrow$ Differentiating w.r.t. x.

$$f'(x) = 2\frac{d}{dx}e^x - \frac{d}{dx}8^x$$

$$\frac{dy}{dx} = 2e^x - 8^x \cdot \log 8$$

$$\boxed{f'(x) = 2e^x - 8^x \cdot \log 8} \;//$$

10. Differentiate;

$$\cancel{f(x) =}\; y = z^5 - e^z \ln(z)$$

$\Rightarrow$ $y = z^5 - e^z \ln(z)$

Differentiating w.r.t. ~~x~~ z

$$\frac{dy}{dz} = \frac{d}{dz}z^5 - \left[e^z \frac{d}{dz}\ln(z) + \ln(z)\frac{d}{dz}e^z\right]$$

$$\cancel{\frac{dy}{dz} = 5z^4}$$

$$\frac{dy}{dz} = 5z^4 - \left[\frac{e^z}{z} + \ln(z)\cdot e^z\right]$$

Date __/__/__

$$\boxed{\therefore \frac{dy}{dz} = 5z^4 - \frac{e^z}{z} - \ln(z) \cdot e^z}$$ //

11. Using chain rule Differentiate the following;

a. $f(x) = (6x^2 + 7x)^4$

$\Rightarrow f(x) = (6x^2 + 7x)^4$

$\therefore$ Let $y = (6x^2 + 7x)^4$

$\therefore$ Let $u = 6x^2 + 7x$

$y = u^4$

Differentiating w.r.t. ~~x~~ u

$\frac{dy}{du} = 4u^3$

$u = 6x^2 + 7x$

Differentiating w.r.t. x

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = 4u^3 \times (12x + 7)$

$$\boxed{\frac{dy}{dx} = 4(6x^2 + 7x)^3 \cdot (12x + 7)}$$ //

b. $f(t) = 5 + e^{4t+t^7}$

$\Rightarrow \because$ Let $y = 5 + e^{4t+t^7}$

$\because$ Let $u = 4t + t^7$

Differentiating w.r.t. t

$$\frac{du}{dt} = 4 + 7t^6$$

$\therefore y = 5 + e^u$

Differentiating w.r.t. u

$$\frac{dy}{du} = 0 + e^u$$

$$\boxed{\frac{dy}{dt} = \frac{dy}{du} \times \frac{du}{dt} = e^{4t+t^7}(4 + 7t^6)}$$

Page No.

Saathi

Date __/__/__

12. Determine the second derivative of :

a. $z = \ln(7-x^3)$

$\Rightarrow$ Differentiating w.r.t.x

$$\frac{dz}{dx} = \frac{1}{7-x^3}\frac{d}{dx}(7-x^3)$$

$$\frac{dz}{dx} = \frac{-3x^3}{7-x^3}$$

Differenting w.r.t.x

$$\frac{d^2z}{dx^2} = \frac{7-x^3\frac{d}{dx}(-3x^2) - (-3x^2)\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{7x^3(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$\boxed{\frac{d^2z}{dx^2} = \frac{-42x - 3x^4}{(7-x^3)^2}}$$ //

Page No

Saathi

Date __/__/__

b. $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$\Rightarrow$ Differentiating w.r.t. ~~x~~ v.

$$Q'(v) = \frac{(6+2v-v^2)^4 \frac{d}{dv} 2 - 2\frac{d}{dv}(6+2v-v^2)^4}{(6+2v-v^2)^4}$$

$$Q'(v) = \frac{0 - 2.4(6+2v-v^2)^3 \frac{d}{dv}(6+2v-v^2)}{(6+2v-v^2)^4}$$

$$Q'(v) = \frac{-8(6+2v-v^2)^3(2-2v)}{(6+2v-v^2)^4}$$

$$Q'(v) = -8(2-2v)(6+2v-v^2)$$

$$Q'(v) = (-16+16v)(6+2v-v^2)$$

Differentiating w.r.t. ~~x~~ v

$$Q''(v) = (-16+16v)(0+2-2v) + (6+2v-v^2)(16)$$

$$Q''(v) = (2-2v) + 96 + 32v - 16v^2$$

$$\boxed{Q''(v) = -16v^2 + 32v + 96} //.$$

Page No.

Date __/__/__

c. $f(t) = \ln(1+t^2)$

$\Rightarrow$ Differentiating w.r.t. t.

$$f'(t) = \frac{1}{1+t^2} \frac{d}{dt}(1+t^2)$$

$$f'(t) = \frac{2t}{1+t^2}$$

Differentiating w.r.t. ~~x~~ t.

$$f''(t) = \frac{1+t^2 \frac{d}{dt} 2t - 2t \frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$f''(t) = \frac{1+t^2(2) - 2t(2t)}{(1+t^2)^2}$$

$$f''(t) = \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$\boxed{f''(t) = \frac{-2t^2+2}{(1+t^2)^2}}$$ //

Date __/__/__ Saathi

13. Find the maximum and minimum value of the function

$$x^3 - 3x^2 - 9x + 12$$

$\Rightarrow f(x) = x^3 - 3x^2 - 9x + 12$

· Differentiating w.r.t.x.

$f'(x) = 3x^2 - 6x \cancel{x} - 9$

$= 6x - 6$

Consider $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$\cancel{x^2 + x - 3 = 0}$

$x^2 + x - 3x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$

For $x = -1$

$f''(-1) = -6 - 6$

$= -12 < 0$

$f(x)$ attains maximum value at $x = -1$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$

Date ___/___/___

$\boxed{\text{The function } f(x) \text{ has max value } \underline{\underline{17}} \text{ at } x=-1}$

For $x=3$

$f''(3) = 6(3)-6$
$= 12 > 0$

$f(x)$ attains minimum value at $x=3$

$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$
$= 27 - 27 - 27 + 12$
$= 12$

$\boxed{\text{The function } f(x) \text{ has minimum value } \underline{\underline{12}} \text{ at } x=3.}$

14. Find the maximum and minimum value of the function

$$4x^3 - 18x^2 + 24x - 7$$

$\Rightarrow$ $f(x) = 4x^3 - 18x^2 + 24x - 7$

Diff. w.r.t. x.

$f'(x) = 12x^2 - 36x + 24$
$= $ Diff. w.r.t x.

$f''(x) = 24x - 36.$

Date __/__/__

Consider $f'(x) = 0$

$$12x^2 - 36x + 24 = 0$$
$$x^2 - 3x + 2 = 0$$
$$x^2 - x - 2x + 2 = 0$$
$$(x-1)(x-2) = 0$$
$$x = 1 \text{ or } x = 2$$

For $x = 1$

$$f'' \cancel{x}(1) = 24(1) = 36$$
$$= -12 < 0$$

$f(x)$ attains maximum value at $x = 1$

$$f(x) = 4(1)^3 - 18(1)^2 + 24(1) - 7$$
$$= 4 - 18 + 24 - 7$$
$$= 3.$$

The function $f(x)$ has maximum value 3 at $x = 1$

For $x = 2$.

$$f''(2) = 24(2) - 36$$
$$= 12 > 0$$

$f(x)$ attains minmum value at $x = 2$

$$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$$
$$= 36 - 72 + 48 - 7$$
$$= 5.$$

Date __/__/__

The function $f(x)$ has minimum value 5 at $x = \cancel{5}2$.

15. Sketch the curve for the following function;

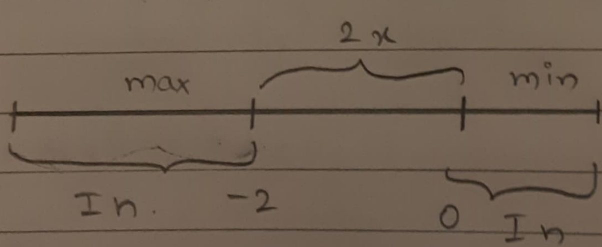
$$f(x) = x^2(x+3)$$

$\Rightarrow$ Diff. w.r.t. x

$$f'(x) = 2x(x+3) + x^2$$
$$= 2x^2 + 6x + x^2$$
$$= 3x^2 + 6x$$

Put $f'(x) = 0$

$$3x^2 + 6x = 0$$
$$\therefore 3(x^2 + 2x) = 0$$
$$x(x+2) = 0$$
$$x = 0 \quad \text{or} \quad \cancel{x+2} \quad x = -2$$



Diff w.r.t. x

$$f''(x) = 6x + 6$$

when $f''(0) = 6(0) + 6$
$$= 6 > 0$$

$$f''(0) > 0$$

Date __/__/__

f(x) attains minimum at $x=0$

$$f(0) = 0^2(0+3)$$
$$= 0$$

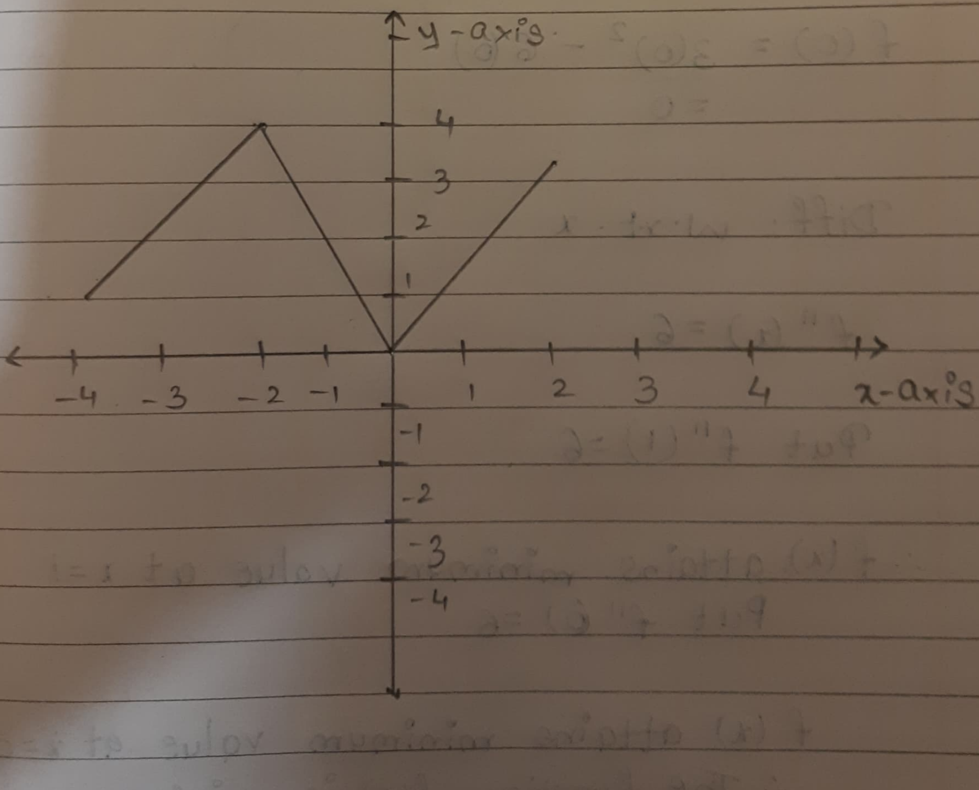
$\therefore$ The function $f'(x)$ has minimum value 0 at $x=0$

$$f''(-2) = 6(-2)+6$$
$$= -6 < 0$$

f(x) attains maximum at $x=-2$

$$f(-2) = (-2)^2(-2+3) = 4$$

$\therefore$ The function $f(x)$ has maximum value 4 at $x=-2$.



Date __/__/__

16. $f(x) = 3x^2 - 6x$

Diff w.r.t. x

$f'(x) = 6x - 6$

Put $f'(x) = 0$

$6x - 6 = 0$

$x - 1 = 0$

$x = 1$

$f(x) = 3x^2 - 6x$

$f(0) = 3(0)^2 - 6(0)$
$= 0$

Diff. w.r.t. x

$f''(x) = 6$

Put $f''(1) = 6$

$\therefore$ f(x) attains minimum value at $x = 1$
Put $f''(0) = 6$

f(x) attains minimum value at $x = 0$
$\therefore$ The function f(x) is minima.

Page No.

Date __/__/__